Mathematisches Forschungsinstitut Oberwolfach

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Komplexe Analysis

27.08.–02.09.2000

Die diesjährige Tagung "Komplexe Analysis" fand unter der Leitung von J.P. Demailly (Grenoble), K. Hulek (Hannover) und T. Peternell (Bayreuth) statt. Insgesamt nahmen 44 Mathematiker aus 9 Ländern teil; es wurden 19 Vorträge gehalten. Einige Schwerpunkthemen waren: Calabi-Yau-Mannigfaltigkeiten, Flächen allgemeinen Typs, Deformation reeller Hyperflächen in komplexen Mannigfaltigkeiten, Hyperbolizität, Modulräume. Neben Diskussionen gab es darüber hinaus eine intensive wissenschaftliche Zusammenarbeit vieler Teilnehmer.

Abstracts

Fibrations in curves of low genus, their invariants and applications

Fabrizio Catanese (Universität Göttingen)

A fibration of a smooth surface X onto a smooth curve B of genus g, $f : X \to B$ is the object of our study. We assume moreover f relatively minimal, $g \geq 2$.

The cases with $g \leq 2$ have been amply studied in the literature, but still existence problems for $g = 2$ are wide open.

Motivated by the need to find constructive methods, particularly suited to the classification of minimal surfaces S of general type with low invariants $K_S^2, \chi = \chi(\mathcal{O}_S) = 1 - q + p_g$, I described work in progress with R. Pignatelli, concerning the case $g \leq 4$. I also reported on some applications, concerning Godeaux Surfaces ($K_S^2 = 1, p_g = 0$) and surfaces with $q = P_g = 1$.

The main tool are some invariants of the relative canonical algebra $\Re(f) = \bigoplus_{n=0}^{\infty} V_n$, where $V_n = f_*(\omega_{X/B})^n$ is a semipositive vector bundle by Fujita's theorem ($\omega_{X/B} = \mathcal{O}_X(K_X - f^*K_B)$). There are two main cases:

(I) The general fibre is non hyperelliptic, then the algebraic structure determines a torsion sheaf $\mathcal{T}_n := \mathrm{coker}(S^n(V_1) \to V_n)$, and for instance, as observed by Reid, $0 \le h^0(\mathcal{T}_2)$ for $g = 3$ yields the Horikawa inequality $K_X^2 \ge 3\chi - 10(b-1)$

(II) Every F is hyperelliptic, then there exists an involution $\sigma : X \to X$ ($\sigma^2 = 1$) commuting with f, and we get a splitting $V_n = V_n^+ \oplus V_n^-$, and similarly for $g = 2$, torsion sheaves $\mathcal{T}_n^{\pm} = \mathrm{coker}(S^n(V_1) \to V_n^{\pm})$, keeping track that $V_1 = V_1^-$.

I did not have time to explain in detail how $h^0(\mathcal{T}_2)$ determines $h^0(\mathcal{T}_n)$ for $g \le 4$, and to describe the precise results obtained for $g = 3, 4$ and general F not hyperelliptic. But I could explain the Structure Theorem.

Theorem. *Let $g = 2$ and $\mathcal{A}$ the subalgebra generated by V_1, V_2. Then $\mathcal{A}^+ = \mathcal{A}_{\mathrm{even}}$, $\mathcal{A}^- = \mathcal{A}_{\mathrm{odd}}$, V_3^+ is a line bundle of degree $K_x^2 - \chi - 7(b-1)$, and $\Re(f)$ is a free $\mathcal{A}-$ module $\Re(f) = \mathcal{A} \oplus \mathcal{A}[-3] \otimes V_3^+$. The ring structure is given through a nowhere vanishing section $\delta \in H^0(B, \mathcal{A}_6 \otimes (V_3^+)^{-2})$.*

Moreover, the algebra $\mathcal{A}$ is completely determined by the rank 2 bundle, V_1, and the extension

$$(*) \quad 0 \to S^2(V_1) \to V_2 \to \mathcal{T}_2 \to 0,$$

where $\mathcal{T}_2$ is locally principal. Thus $(V_1, (*), V_3^+, \delta)$ determine completely $\Re(f)$.

Remarks on the canonical map of surfaces of general type

Ciro Ciliberto (joint work with R. Pardini and F. Torena)

Let X be a smooth, projective, complex surface of general type. Let $\phi \dashrightarrow \Sigma \subset \mathbb{P}^{p_g(X)-1}$ be its canonical map, whose image Σ I assume to be a surface. Let $\epsilon : S \to \Sigma$ be a minimal desingularisation of Σ. Suppose ϕ has degree $d > 1$. Then a classical result of Babbage-Beauville says that:

(i) either $p_g(S) = 0$;

(ii) or S is of general type and $\epsilon : S \to \Sigma$ is the canonical map of S.

There are plenty of examples of case (i), which should be regarded as the standard case in this situation. As for the non standard case (ii), few examples have been known in the literature so far. In particular, only one family of examples with unbounded invariants, due to Beauville. In this talk I present more families of examples with unbounded invariants and, after having analyzed some of their common features, I prove some classification results.

Logarithmic Jet Bundles and Applications

Gerd Dethloff (Université de Brest, France)

Jet bundles have become an important tool in complex geometry. J.P. Demailly presented a construction of projective jets which are closer to the geometry of holomorphic curves than the usual jets. In order to study the geometry of quasiprojective varieties, it is desirable also to have a logarithmic version of these projective jet bundles and of strictly negatively curved pseudometrics on them.

In the present talk we explain how one can establish this logarithmic generalization of Demailly's construction explicitely via logarithmic coordinates. These coordinates are very important for applications, since they admit explicit computations.

The underlying work of this talk, which is joint work with Steven Lu (University of Waterloo, Canada), will appear in Osaka J. of Math.

The Poincaré series of a quasihomogeneous surface singularity

Wolfgang Ebeling (Universität Hannover)

Let (X, x) be a normal surface singularity with good $\mathbb{C}^*-$action and with orbit invariants $\{g; b; (\alpha_1, \beta_1), \ldots, (\alpha_n, \beta_n)\}$. We consider the Poincaré series $p_A(t)$ of the coordinate algebra A of (X, x). Write

$$p_A(t) = \frac{\phi_A(t)}{\psi_A(t)} \text{ where } \psi_A(t) = (1-t)^{2-r} \prod_{i=1}^{r} (1 - t^{a_i}).$$

If (X, x) is a Kleinian singularity not of type A_{2n} then we derive from the McKay correspondence that $\phi_A(t)$ and $\psi_A(t)$ are the characteristic polynomials of the Coxeter element and the affine Coxeter element respectively.

If (X, x) is a hypersurface singularity or a certain ICIS then we can show that the polynomial $\phi_A(t)$ is in a certain sense dual to the characteristic polynomial of the monodromy operator of the singularity.

There are relations to the mirror symmetry of $K3$ surfaces and to automorphisms of the Leech lattice.

Complex vector bundles and automorphic forms

V. Gritsenko (Lille, St. Petersburg)

The elliptic genus of a compact complex manifold was introduced as a holomorphic Euler characteristic of some formal power series with vector bundle coefficients. EG is an automorphic form in two variables only if the manifold has trivial first Chern class. In physics such a function appears as the partition function of $N = 2$ superconformal field theory.

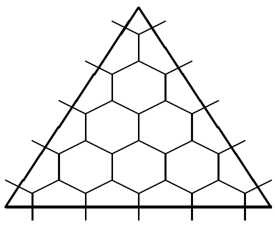
In my talk I define the modified Witten genus of an arbitrary holomorphic vector bundle over a compact complex manifold. It is a Jacobi form of weight zero for arbitrary vector bundles. This construction gives us the Witten genus and the elliptic genus as particular cases. We formulate also some questions about the existence of some vector bundles with prescribed homological invariants.

Toplogical Mirror Symmetry

Mark Gross

We consider the Strominger-Yau-Zaslow conjecture in the topological category, i.e. we study how to dualize proper continuous T^3- fibrations $f : X \to B$ with some regularity assumptions on the singular fibres. After classifying monodromy at semistable singular fibres, we are able to prove that duals to such fibrations exist. We then construct a torus fibration on the quintic threefold in $\mathbb{P}^4$ which has the necessary properties. and whose dual is provably diffeomorphic to the mirror quintic.

This fibration is constructed by brute force; if Ξ is a $4-$simplex, we build a fibration with base $\partial\Xi$ and discriminant locus supported on the $2-$ skeleton of Ξ. On any $2-$ face, the discriminant locus takes the form



One specifies the monodromy of the torus fibration about this graph, and then compactifies. One applies Wall's theorem to prove that we have obtained the quintic and, after dualizing, its mirror.

Moduli spaces for hypersurface singularities via Frobenius manifolds and Gauss Manin connection

Claus Hertling (Bonn)

Theorem. *Fix any isolated hypersurface singularity f. The set of right equivalence classes in the $\mu-$ homotopy class of f is an analytic geometric quotient.*

The first step of the proof consists in writing the set as a quotient of an algebraic variety by an algebraic group, using jets of singularities and jets of coordinate changes. The proof that this quotient is an analytic geometric quotient uses the construction of Frobenius manifolds in singularity theory, which is due to K. Saito and M. Saito (1983), a result of Scherk, and certain period maps from $\mu-$constant strata to a classifying space for polarized mixed Hodge structures.

The base space $\mathcal{M}$ of a semiuniversal unfolding of a singularity f can be equipped with the structure of a Frobenius manifold, that means, with a multiplication on the holomorphic tangent bundle $T\mathcal{M}$, with an Euler field and with a metric on $T\mathcal{M}$, such that they satisfy certain conditions. The multiplication and the Euler field come from Kodaira-Spencer isomorphism and are unique. The metric is not unique. It is induced by the choice of a primitive form of K. Saito, and that comes from the choice of an opposite filtration to a Hodge filtration of a polarized mixed Hodge structure. The construction of the primitive form uses heavily the Gauss-Manin connection.

The definition of a Frobenius manifold was given together with the definition of an $F-$manifold (Manin, Hertling), a manifold with a multiplication on the tangent bundle which satisfies a certain integrability condition.

Deforming manifolds by mean curvature and Ricci flow

Gerhard Huisken (Tübingen)

Mean curvature flow of hypersurfaces and Ricci flow of Riemannian metrics are both parabolic systems of second order with close analogies between them. The Ricci flow asks to extend a given metric $\overline{g}_0$ on a manifold N by solving the equation

$$\frac{d}{dt}\overline{g}_{ij}(p,t) = -2\overline{R}_{ij}(p,t)$$

where $\overline{R}_{ij}$ is the Ricci curvature of the metric $\overline{g}$ at time t, while the mean curvature flow aims to deform an initial hypersurface $F_0 : M^n \to (N^{n+1}, \overline{g})$ in some Riemannian manifold according to the equation

$$\frac{d}{dt}F(p,t) = H(p,t) \cdot \nu(p,t)$$

where H and ν are mean curvature and unit normal. Analogies and recent results for these flows are discussed and it is proposed to study more closely the coupled problem, where a surface moves in a manifold obeying Ricci flow for its metric. It is shown that interesting cancellation occur in this case and some convergence results are given for two dimensional Ricci flow combined with curve shortening flow.

Families of singular rational curves

Stefan Kebekus (Universität Bayreuth)

Let X be a projective variety (not necessarily normal) and $H \subset \mathrm{RatCurves}^n(X)$ a covering family of rational curves. We assume that the following compactness assumption holds which is satisfied e.g. if the curves are of minimal degrees: if $x \in X$ is general point, then the subfamily H_x of curves which contain x is compact. In this setup we show that only finitely many curves associated with H_x are singular at x, and that the singularities are immersed. If a line bundle $L \in \mathrm{Pic}(X)$ exists which intersects the curves with multiplicity 2, then all curves are smooth at x. We believe that this is interesting for the following reasons:

(1) It gives a partial answer to the question to what extent the geometry of minimal rational curves resembles the geometry of lines in $\mathbb{P}_n$.

(2) We can show that the tangent map $H_x \dashrightarrow \mathbb{P}(T_{X,x})$ which associates a curve with its tangent space is finite. This map has been studied extensively by Hwang and Mok.

(3) We show that two sufficiently general points on X define at most one curve.

(4) We give a characterization of $\mathbb{P}_n$ which comprises and extends the known results.

Deforming real hypersurfaces by the trace of the Levi form

W. Klingenberg

Given a closed immersed real hypersurface $F_0 : M^{2n-1} \hookrightarrow \mathbb{C}^n$, we study the following problem for $F : N^{2n-1} \times [0,T) \to \mathbb{C}^n$:

$$\begin{aligned} \tfrac{d}{dt} F(p,t) &= L(p,t)\nu_F(p,t) \\ F(p,0) &= F_0(p). \end{aligned}$$

Here, ν_F denotes the unit normal and L the trace of the Levi form of $N_t = F(N,t)$. We prove the following in collaboration with G. Huisken:

Theorem. *There exists $T > 0$ admitting a solution of the above initial value problem with*

$$\sup_{N_t} |h|^2 + \sup_{N_t} |\bigtriangledown h|^2 \xrightarrow{t \to T} \infty,$$

where h denotes the second fundamental form of N_t.

Theorem. *Let $N_0^3 \hookrightarrow \mathbb{C}^2$ be weakly pseudoconvex. Then the solution F above has strictly pseudoconvex surfaces N_t^3 for all $t \in (0,T)$.*

The first theorem is proved using elliptic regularization. The second theorem uses a Hopf-type maximum principle for the weakly elliptic operator corresponding to L.

On a non-Kählerian analogue of nonsingular complex projective toric variety

Laurent Meersseman

In the first part of the talk, a construction of compact complex "deeply" not projective manifolds is given; the aim is to describe a large family of examples generalizing those given by Hopf ($S^{2n-1} \times S^1$) and those given by Calabi and Eckmann ($S^{2p-1} \times S^{2q-1}$). "Deeply" not projective means that these manifolds are not projective but moreover that they do not admit any Kählerian modification. They are constructed as quotient space of an open dense subset of the $n-$ projective space by an action of $\mathbb{C}^m$. The crucial fact is that these non projective manifolds are entirely determined when a set of n vectors $\Lambda_1, \dots, \Lambda_n$ of $\mathbb{C}^m$ is given: from this data, the action is uniquely characterized and so are the manifolds. Besides holomorphic properties of the manifolds can be expressed in terms of these Λ_i's; for example, some Hodge numbers, the algebraic dimension, ...

In the second part (which is a joint work with Alberto Verjovsky), the relation to projective toric varieties is explained.

Theorem. *Let N be a manifold as in part I. Then there exists a quasi-regular projective toric variety X and a small pertubation N_0 of N such that $N_0 \to X$ is a principal holomorphic bundle with a compact complex $m-$ torus as fibre and with singular fibres above the singular points of X.*

Theorem. *Conversely, given any quasi-regular projective toric variety X, for any choice of an ample divisor on it, there exists some $m \in \mathbb{N}^*$, and some manifold N as in part I such that N fibres on X with $m-$ torus as fibre as in the previous theorem.*

As examples of these manifolds there are connected sums of products of spheres including $\sharp(8)s^4 \times S^4, \sharp(9)S^3 \times S^5$. As examples of fibrations one can obtain a principal bundle in elliptic curves with $S^3 \times S^3$ (with an adequate complex structure) as total space and any Hirzebruch surface F_a as basis.

Complex symplectic manifolds and projective spaces

Y. Miyaoka (joint work with N. Shepherd-Barron)

A Kählerian manifold M of dimension $2n$ is said to be complex symplectic if M carries a global holomorphic $2-$ form ω which is $d-$ closed and whose highest wedge $\Lambda^n\omega$ is nowhere vanishing.

Our main results are the following

Theorem. *Let M be a compact complex symplectic manifold and $\pi : M \to \hat{M}$ a projective birational morphism onto a normal Kählerian variety. Let $E \subset M$ be an irreducible component of the exceptional locus of π and $Y \subset \hat{M}$ its image. Then Y is a complex symplectic variety and a general fibre of $\pi_{|E}$ is a pseudo-projective space after normalization*

Theorem. *Let M be a projective primitive complex symplectic manifold (i.e. $H^0(M, \Omega^2_M)$ is generated by the symplectic form ω) and $\pi : M \to N$ be a non trivial fibring. (This is a Lagrangian torus fibration by a theorem of Matsushita). Assume that π admits a local section at every $p \in N$. Then N is a pseudo-projective space.*

Here a projective, normal, uniruled variety Z is said to be a pseudo-projective space if every rational curve C can be deformed to a rational curve C' which contains an arbitrarily prescribed pair of points (z_1, z_2) in Z.

Pseudoconvex hypersurfaces in complex manifolds

Takeo Ohsawa (Nagoya University)

Let Ω be a complex manifold, let M be real $\mathcal{C}^\infty$ hypersurface of Ω which is the boundary of a domain D. M is said to be pseudoconvex if D is locally Stein. The Levi form l_M is defined as that of a defining function ρ of D. Take any $T \in \mathcal{C}^\infty(M, TM)$ such that $d^c\rho(T) = 1$ and put $\alpha = -\mathcal{L}_T d^c\rho$, where $\mathcal{L}_T$ denotes the Lie derivative. If a pseudoconvex hypersurface M contains a complex curve C, then $\alpha|C$ is closed, and defines a class in $H^1(C, \mathbb{R})$. In view of Boas-Straube's regularity theorem and Barret's example, a natural question is on the geometry of M along C when the cohomology class $[\alpha|C]$ is zero.

Theorem. $\dim \Omega = 2$ *and* $[\alpha|C] = 0 \Longrightarrow l_M$ *vanishes to* ∞ *order along* C.

As an application we have

Theorem. $\dim \Omega = 2$, *$M \in \mathcal{C}^\infty$, compact, almost strongly pseudoconvex* $\Longrightarrow \exists \phi \in PSH(D)$ *such that $\phi + \log \delta_M$ is bounded and ϕ is strictly plurisubharmonic outside a compact subset of D.*

If $l_M \equiv 0$, M is said to be Levi flat.

Theorem. *Ω compact and of dimension ≥ 2, $M \in \mathcal{C}^\infty$, compact and Levi flat $\Longrightarrow$ the normal bundle of M is not positive.*

Corollary. *There are no $\mathcal{C}^\infty$ compact Levi flat hypersurfaces in $\mathbb{P}^n$ if $n \geq 2$.*

A Numerical Primary Decomposition

Andrew Sommese

Let

$$f = \begin{cases} f_1(x_1, \ldots, x_n) \\ \vdots \\ f_N(x_1, \ldots, x_n) \end{cases}$$

be a system of polynomials on $\mathbb{C}^n$. Let $V(f)$ denote the reduced set of zeroes of f and $f^{-1}(0)$ the possibly non reduced zero scheme. A algorithm (joint work with J. Verschelde and C. Wampler) is given which produces a finite set of solutions $\mathcal{Z}$ of $f = 0$ and decompositions

$$\mathcal{Z} = \bigcup_{i=0}^{\dim V(f)} \mathcal{Z}_i = \bigcup_{i=0}^{\dim V(f)} \bigcup_{j \in \mathcal{Z}_i} \mathcal{Z}_{ij}$$

with unique integers $\nu_{ij} \geq 1$ such that if Z_i is the union of all dimension i irreducible components $\{Z_{ij} | j \in \mathcal{Z}_i\}$ of $V(f)$ and μ_{ij} is the multiplicity of Z_{ij} in $f^{-1}(0)$, then

1) $\mathcal{Z}_{ij}$ consists of degree Z_{ij} "generic points" of Z_{ij}; and

2) $\nu_{ij} \geq \mu_{ij}$ with $\nu_{ij} = 1 \Leftrightarrow \mu_{ij} = 1$.

Moreover equations cutting out any of the Z_{ij} are produced if desired. The algorithm is a "probability one" algorithm based on homotopy continuation, generic slicing, and generic projection. One consequence is the first numeric algorithm to pick out the isolated solutions and only the isolated solutions of a system f. The preprint containing the details for this algorithm is available at `www.nd.edu/~sommese`.

Diffeomorphism type, Braid Monodromy type: Computational Methods

Mina Teicher

In a joint work with Kulikow in 1998 we proved that BMT $\Rightarrow$ Diff (i.e. two surfaces which are of the same Braid Monodromy type are diffeomorphic). In the talk I presented the BMT of a surface and presented an efficient computer algorithm that reduced the complexity of computing the BMT to values that make it computable.

This algorithm can also be applied for the category of 4-manifolds which have a generic projection at a nicely behaved branch curve.

Gauge theoretical Gromov-Witten invariants, Quot spaces and full Seiberg-Witten invariants of ruled surfaces

Ch. Okonek (University of Zürich), Andrei Teleman (CMI, University of Marseille I)

We introduce gauge theoretical invariants for triples (F, α, K) where F is a symplectic almost complex manifold, $\alpha : \hat{K} \times F \to F$ a symplectic almost holomorphic action of a compact Lie group $\hat{K}$ and K is a closed normal subgroup of $\hat{K}$. The invariants are obtained by evaluating canonical cohomology classes on moduli spaces of solutions of certain vortex type equations on Riemannian surfaces. In the case $K = \{1\}$ one obtains spaces of sections on $F-$ bundles over Riemannian surfaces, hence the new invariants can be regarded as generalizations of the classical Gromov-Witten invariants. For the tuple $(\mathrm{Hom}(\mathbb{C}^r, \mathbb{C}^{r_0}),$ natural action of $U(r) \times U(r_0), U(r_0))$ we give explicit descriptions of the moduli spaces in terms of moduli spaces of $\lambda-$ stable holomorphic pairs. In the particular case $r = 1$ we compute the invariants explicitly. Algebraic geometric applications are given:

We show that, via the pushforward construction, the Seiberg-Witten invariants of ruled surfaces can be identified with the newly introduced invariants of the base curve in the studied case $r = 1$.

Finally we discuss the relation of the gauge theoretical invariants associated with a tuple (F, α, K) to the (twisted) Gromov-Witten invariants of the symplectic quotient $\mu_K^{-1}\{0\}/K$.

Large Complex Structure Limits of K3 Surfaces

P.M.H. Wilson, University of Cambridge

The Strominger-Yau-Zaslow Conjecture leads to the question of the behaviour of the Ricci flat Kähler metric g on a Calabi-Yau $n-$ fold X if one approaches a large complex structure limit point in moduli. In particular, it has been conjectured that under appropriate conditions, the corresponding Riemannian manifolds (X_i, g_i) will converge (in the sense of Gromov-Hausdorff) to a compact metric space (X_∞, d_∞), with X_∞ homeomorphic to S^n and d_∞ induced from a (known) Riemannian metric on $X_\infty \setminus \Delta$ for some codimension 2 set Δ – here X_∞ should be thought of as the base of the special Lagrangian torus fibration predicted by the SYZ conjecture and Δ as the discriminant locus.

In joint work with Mark Gross (DG/0008018), we study this conjecture for $K3$ surfaces. By mirror symmetry for $K3$s and a standard trick, the problem may be reinterpreted in terms of a Kähler degeneration on some fixed elliptic $K3$ surface X, where the Kähler class approaches the wall of the Kähler cone corresponding to the fibration – i.e. holding the volume of X to be one, the area ϵ of the elliptic fibres tends to zero. For small ϵ, we are able to construct a very accurate approximation to the Ricci flat metric (error $= O(e^{-\frac{c}{\epsilon}})$) by glueing a semi-flat metric away from the singular fibres (assumed of type I_1) with Ooguri-Vafa metrics in a neighbourhood of each singular fibre. This glued metric has $\mathrm{Ric} = O(e^{-\frac{c}{\epsilon}})$; by running Yau's argument for the existence of a Ricci flat metric with a given Kähler class, we can show that the glued metric is very close to the Ricci flat one (quantities which get large do so only polynomially in ϵ^{-1}, and these are easily dealt with by the factor $e^{-\frac{c}{\epsilon}}$). The above conjecture for $K3$ surfaces then follows, using this close approximation to the Ricci flat metric.

Holomorphic Curves in Semi-Abelian Varieties

Jörg Winkelmann (Basel, Tokyo)

We (Junjiro Noguchi, Katsutoshi Yamanoi and me) proved a second main theorem for holomorphic curves in semi-abelian varieties.

Theorem. *Let M be a semi-abelian variety with equivariant compactification $M \hookrightarrow \overline{M}$ such that $0 \to (\mathbb{C}^*)^g \to M \to T \to 0$ and $0 \to (\mathbb{P}_1)^g \to \overline{M} \to T \to 0$, $B = \overline{M} \setminus M$. The boundary B is naturally stratified $B = \overset{\circ}{\bigcup} B_i$. Let D be a divisor on $\overline{M}$ such that $B_i \not\subset D_i$ for all i. Let $f : \mathbb{C} \to M$ be holomorphic and non-constant. Then*

$$T_f(r, c_1(D)) = N_{k_0}(r, f^*D) + O(\log r)$$

if f is of finite order and

$$T_f(r, c_1(D)) = N_{k_0}(r, f^*D) + O(\log T_f(r, c_1(D))) + O(\log r)$$

otherwise.

Preprint available as **math.CV/9912086**.

Hyperbolic surfaces in $\mathbb{P}^3$ arising from symmetric squares of curves

M. Zaidenberg (Grenoble)

This is a report on a joint work with B. Shiffman, Intern. J. Math. 11:1, 2000.

After recalling main facts of hyperbolic complex analysis, we give a brief survey on the present day situation in the Kobayashi problem of describing the class of hyperbolic hypersurfaces in $\mathbb{P}^n$. The main progress (Demailly-El Goul, McQuillan) is the theorem which says that a very generic surface of degree $d \geq 21$ in $\mathbb{P}^3$ is Kobayashi hyperbolic. Examples are known starting with degree 11 (Demailly-El Goul, Siu-Yeung). These are based on a careful analysis involving Nevanlinna theory, jet differentials and algebraic foliations.

Here we show how to obtain a simple example of a singular (but smooth after deforming) hyperbolic surface of degree 16 in $\mathbb{P}^3$, taking an explicit smooth plane quartic C and (bi-)canonically embedding the symmetric square C_2 of C into $\mathbb{P}^8$ followed by a generic projection into $\mathbb{P}^3$. Such a surface is hyperbolic if and only if so is its double curve; the latter is indeed hyperbolic bacause it has geometric genus 142. In fact, we show (by computing and comparing the associated numerical invariants) that 16 is the minimal possible degree in this procedure, and that for a genus $g \geq 3$ curve C with general moduli, choosing any very ample divisor on the symmetric square C_2 of C, we always get a hyperbolic surface.

Edited by Thomas Eckl

Tagungsteilnehmer

Dr. Katja Amerik
Department of Mathematics
Univ. Paris-Sud
Bat. 425
F-91405 Orsay Cedex

Prof. Dr. Marco Andreatta
Dipartimento di Matematica
Universita di Trento
Via Sommarive 14
I-38050 Povo (Trento)

Prof. Dr. Daniel Barlet
Departement de Mathematiques
Universite de Nancy I
Boite Postale 239
F-54506 Vandoeuvre les Nancy Cedex

Dr. Victor V. Batyrev
Mathematisches Institut
Universität Tübingen
72074 Tübingen

Dr. Christina Birkenhake
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2
91054 Erlangen

Prof. Dr. Frederic Campana
Dept. de Mathematiques
Universite de Nancy I
B.P. 239
F-54506 Vandoeuvre-les-Nancy Cedex

Prof. Dr. Fabrizio Catanese
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5
37073 Göttingen

Prof. Ciro Ciliberto
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica
I-00133 Roma

Prof. Dr. Olivier Debarre
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes
F-67084 Strasbourg Cedex

Prof. Dr. Jean-Pierre Demailly
Institut Fourier
UMR 5582 du CNRS
BP 74
100 rue de Maths
F-38402 Saint-Martin d'Heres

Dr. Gerd E. Dethloff
UFR Sciences et Techniques
Departement de Mathematiques
Universite de Bretagne Occidentale
6, avenue Le Gorgeu, BP 452
F-29275 Brest Cedex

Prof. Dr. Klas Diederich
Fachbereich 7: Mathematik
U-GHS Wuppertal
42097 Wuppertal

Prof. Dr. Wolfgang Ebeling
Institut für Mathematik
Universität Hannover
Postfach 6009
30060 Hannover

Thomas Eckl
Mathematisches Institut
Universität Bayreuth
Postfach 101251
95447 Bayreuth

Prof. Dr. A.Valery Gritsenko
U.F.R. de Mathematiques
Universite de Lille I
USTL
F-59655 Villeneuve D'Ascq

Prof. Dr. Mark Gross
Mathematical Institute
University of Warwick
GB-Coventry CV4 7AL

Prof. Dr. Peter Heinzner
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA
44780 Bochum

Dr. Claus Hertling
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn

Prof. Dr.Dr.h.c. Alan T. Huckleberry
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA
44780 Bochum

Prof. Dr. Gerhard Huisken
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
72076 Tübingen

Prof. Dr. Klaus Hulek
Institut für Mathematik (C)
Universität Hannover
Postfach 6009
30060 Hannover

Dr. Stefan Kebekus
Mathematisches Institut
Universität Bayreuth
95440 Bayreuth

Dr. Wilhelm Klingenberg
Inst. für Mathematik
Humboldt-Universität Berlin
Zieglerstr. 13A
10117 Berlin

Dr. Vincent Koziarz
Institut Elie Cartan (Mathematique)
Universite Henri Poincare, Nancy I
Boite Postale 239
F-54506 Vandoeuvre les Nancy Cedex

Dr. Bernd Kreussler
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße
67663 Kaiserslautern

Prof. Dr. Herbert Lange
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2
91054 Erlangen

Dr. Manfred Lehn
Mathematisches Institut
Universität zu Köln
50923 Köln

Prof. Dr. Jürgen Leiterer
Fachbereich Mathematik
Humboldt-Universität Berlin
10099 Berlin

Michael Lönne
Institut für Mathematik (C)
Universität Hannover
Welfengarten 1
30167 Hannover

Dr. Laurent Meersseman
U. E. R. Mathematiques
I. R. M. A. R.
Universite de Rennes I
Campus de Beaulieu
F-35042 Rennes Cedex

Prof. Dr. Yoichi Miyaoka
Dept. of Mathematics
Faculty of Science
Kyoto University
Kitashirakawa, Sakyo-ku
Kyoto 606-8502
JAPAN

Prof. Dr. Takeo Ohsawa
Dept. of Mathematics
Nagoya University
Chikusa-Ku
Nagoya 464-01
JAPAN

Prof. Dr. Christian Okonek
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Prof. Dr. Thomas Peternell
Fakultät für Mathematik und Physik
Universität Bayreuth
95440 Bayreuth

Prof. Dr. Georg Schumacher
Fachbereich Mathematik
Universität Marburg
35032 Marburg

Dr. Bernd Siebert
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA
44780 Bochum

Prof. Dr. Andrew J. Sommese
Department of Mathematics
University of Notre Dame
Notre Dame , IN 46556-5683
USA

Jeroen Spandaw
Institut für Mathematik
Universität Hannover
Welfengarten 1
30167 Hannover

Prof. Dr. Duco van Straten
Fachbereich Mathematik
Universität Mainz
55099 Mainz

Prof. Dr. Mina Teicher
Dept. of Mathematics
Bar-Ilan University
52 900 Ramat-Gan
ISRAEL

Dr. Andrei Teleman
Centre de Mathematiques et d'Informatique
Universite de Provence
39, Rue Joliot Curie
F-13453 Marseille Cedex 13

Prof. Dr. Antonius Van de Ven
Mathematisch Instituut
Rijksuniversiteit Leiden
Postbus 9512
NL-2300 RA Leiden

Prof. Dr. Pelham M.H. Wilson
Center for Mathematical Sciences
University of Cambridge
Wilberforce Road
GB-Cambridge CB3 OWB

Dr. Jörg Winkelmann
Mathematisches Institut
Universität Basel
Rheinsprung 21
CH-4051 Basel

Prof. Dr. Mikhail Zaidenberg
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
B.P. 74
F-38402 Saint-Martin-d'Heres Cedex

E-mail addresses

Katja Amerik	ekaterina.amerik@math.u-psud.fr
Marco Andreatta	andreatt@science.unitn.it
Daniel Barlet	barlet@iecn.u-nancy.fr
	barlet@ciril.fr
Victor V. Batyrev	batyrev@bastau.mathematik.uni-tuebingen.de
Christina Birkenhake	birkenhake@mi.uni-erlangen.de
Frederic Campana	campana@iecn.u-nancy.fr
Fabrizio Catanese	catanese@cfgauss.uni-math.gwdg.de
Ciro Ciliberto	ciliberto@mat.utovrm.it
Olivier Debarre	debarre@math.u-strasbg.fr
Jean-Pierre Demailly	demailly@fourier.ujf-grenoble.fr
Gerd E. Dethloff	Gerd.Detholff@univ-brest.fr
Klas Diederich	klas@math.uni-wuppertal.de

Wolfgang Ebeling	ebeling@math.uni-hannover.de
Thomas Eckl	eckl@btm8x5.mat.uni-bayreuth.de
A.Valery Gritsenko	valery.gritsenko@agat.univ-lille1.fr
Mark Gross	mgross@math.cornell.edu
	mgross@maths.warwick.ac.uk
Peter Heinzner	peter.heinzner@cplx.ruhr-uni-bochum.de
Claus Hertling	hertling@math.uni-bonn.de
Alan T. Huckleberry	ahuck@cplx.ruhr-uni-bochum.de
Gerhard Huisken	gerhard.huisken@uni-tuebingen.de
Klaus Hulek	hulek@math.uni-hannover.de
Stefan Kebekus	stefan.kebekus@uni-bayreuth.de
Wilhelm Klingenberg	klingenb@mathematik.hu-berlin.de
Vincent Koziarz	koziarz@antares.iecn.u-nancy.fr
Bernd Kreussler	kreusler@mathematik.uni-kl.de
Herbert Lange	lange@mi.uni-erlangen.de
Manfred Lehn	manfred.lehn@mi.uni-koeln.de
Jrgen Leiterer	leiterer@mathematik.hu-berlin.de
Michael Lnne	loenne@math.un-hannover.de
Michael McQuillan	mcquilla@maths.ox.ac.uk
	mquillan@ihes.fr
Laurent Meersseman	meersem@maths.univ-rennes1.fr
Yoichi Miyaoka	miyaoka@kurims.kyoto-u.ac.jp
Takeo Ohsawa	ohsawa@math.nagoya-u.ac.jp
Christian Okonek	okonek@math.unizh.ch
Thomas Peternell	thomas.peternell@uni-bayreuth.de
Georg Schumacher	schumac@mathematik.uni-marburg.de
Bernd Siebert	Bernd.Siebert@rz.ruhr-uni.bochum.de
Andrew J. Sommese	sommese.1@nd.edu
Jeroen Spandaw	spandaw@math.uni-hannover.de
Duco van Straten	straten@mathematik.uni-mainz.de
Mina Teicher	teicher@sunglight.cs.biu.ac.il
Andrei Teleman	teleman@cmi.univ-mrs.fr
Antonius Van de Ven	evn@leidenuniv.nl
Pelham M.H. Wilson	pmhw@pmms.cam.ac.uk
Jrg Winkelmann	jw@cplx.ruhr-uni-bochum.de
	winkel@math.unibas.ch
Mikhail Zaidenberg	Mikhail.Zaidenberg@ujf-grenoble.fr