MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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# Analytical and Statistical Approaches to Fluid Models

3.–9. September 2000

This conference was organized by Peter Constantin (Chicago), Alexander Mielke (Stuttgart) and Edriss S. Titi (Irvine).

The aim of the meeting was to bring together experts who work in neighboring, but intellectually distinct fields: partial differential equations, dynamical systems and stochastic differential equations. The common interest of the participants centered on applications in fluid mechanics. Classical fluid mechanics is based on deterministic partial differential equations and has connections to dynamical systems theory. But traditional turbulence theory, as well as more recent models are based on stochastic partial differential equations (SPDE), i.e., PDEs with stochastic coefficients.

In recent years there have been important developments in the physical theoretical approaches to turbulence, originating from Kraichnan's passive scalar model and using mostly field–theoretical methods. They renewed the interest in the more mathematical issues of existence and uniqueness of invariant measures. During the meeting there were extensive discussions concerning the more difficult uniqueness problem. In particular, this led to the presentation of four very recent simultaneous proofs of existence and uniqueness for invariant (ergodic) measures for white–in–time stochastically forced Navier–Stokes equations and other SPDEs by W. E, J.–P. Eckmann, A. Kupiainen and S. Kuksin. A discussion of non–trivial limits due to compressible effects was led by E. Vanden Eijnden.

The stochastic models are surrogates for the deterministic case which still remains unsolved. The closest connection to turbulence in the deterministic Navier–Stokes equation was discussed in a series of three lectures by C.I. Foias which were based on his notion of Vishik–Fursikov statistical solutions.

Problems of pattern formation and dynamics on extended domains were covered abstractly (A. Biryuk, P. Collet) as well as in concrete applications to the Navier–Stokes equations (R. Farwig, T. Gallay).

Further fluid models involved non–Newtonian fluids (J. Necas, T. Hagen), thin–film equations (G. Grün), porous media flow (M. Efendiev), water waves (M. Groves), averaged Euler/Camassa–Holm equation (M. Oliver), geostrophic and climate models (J. Duan, P. Kloeden, G. Sell, B. Turkington).

## Collection of Abstracts

### Estimates for derivatives of the Burgers equations in terms of viscosity

**Andrei Biryuk (Heriot–Watt, UK)**

Consider the one dimensional generalized Burgers equation

$$u_t + f(u)_x = \delta u_{xx} \tag{1}$$

with an $l$–periodic and $C^\infty$–smooth initial state. The function $f$ is also $C^\infty$–smooth and strictly convex ($f'' \geq \sigma > 0$). Equation (1) is studied in the small dissipation regime i.e. $\delta$ is a small positive parameter ($0 < \delta \ll 1$).

We say that quantity $g_\delta$ has order $\delta^{-s}$ if there exist two $\delta$ independent constants $c$ and $C$ such that for $\delta \in (0,1)$ we have: $c\delta^{-s} \leq g_\delta \leq C\delta^{-s}$.

Take time $T = 2l^{3/2}/\sigma|u_0 - \langle u_0\rangle|_{L_2}$. Here $\langle u_0\rangle$ is the mean value of the initial state $u_0$.

It turns out that averaging of $C^m$–norm of solution over $[0,T]$ has order $\delta^{-m}$ and, for $m \geq 1$, the corresponding averaging of the Sobolev $H^m$–norm has order $\delta^{-m+1/2}$. Using explicit expression for a Sobolev $H^m$–norm via Fourier coefficients we can get some information for the behavior of the Fourier coefficients as $\delta \to 0$.

### Derivation of the Euler equations from a caricature of Coulomb interaction

**Yann Brenier (Paris/Nice, France)**

We start with the following caricature of electrodynamics: $N$ particles with positive charge are fixed and equally spaced in a bounded domain $D$. $N$ particles with negative charge are moving around $D$. At each time, there are $N$ springs linking, one–to–one, a positive particle and a negative particle. They have the same stiffness. The dynamics $w$ is made non trivial, by updating the pairing of positive and negative particles so that the total potential energy of the system stays minimal. It is shown that, as the stiffness of the springs and the number of particles $N$ go to infinity (at a much faster, algebraic rate), the negative particles will move as a perfect incompressible fluid provided their initial velocity is given by a smooth divergence free vector field parallel to the boundary $\partial D$. A geometrical interpretation of this theorem is provided: the dynamical system turns out to be a natural discretization of the Euler equations viewed as the geodesic flow on the group of volume preserving diffeomorphisms of $D$. In addition it is (formally) shown how, for finite stiffness, the dynamical system minimizes the Coulomb interaction, through a comparison between the Monge–Ampère and the Poisson equations.

### Global properties of extended systems

**Pierre Collet (Palaiseau, France)**

We consider a damped non linear hyperbolic equation in dimension 1. The semi flow is globally defined and there is an attracting ball for bounded initial conditions (Feireisl). In the spirit of Shannon and Kolmogoroff we look at the $\varepsilon$–entropy per unit length. We prove it exists and is bounded above by $c^{te} \log \varepsilon^{-1}$ for small $\varepsilon$ where $c^{te}$ depends only on the data of the equation. We also prove existence and finiteness of the topological entropy per unit length and show that this number can be recovered using information from a discrete mesh.

(joint work with J.–P. Eckmann)

### Stochastic two–layer quasi–geostrophic flows: is the bottom layer important?

**Jinqiao Duan (Chicago, USA)**

3D rotating Navier–Stokes model reduces to 3D quasi–geostrophic (QG) model when the Rossby number is small. With vertical discretization, 3D QG model simplifies to multi–layer QG model with Ekman dissipation at bottom and random wind forcing on top fluid surface. We show that, under suitable conditions on Ekman number, viscosity, $\beta$–parameter and trace of covariance operator of wind forcing, the long–time dynamics is determined by the top layer, i.e., the bottom layer is slaved by the top layer.

(joint work with I. Chueshov and B. Schmalfuß)

### Invariant measures of the stochastic Navier–Stokes equation

**Weinan E (Princeton/Courant Institute, USA)**

We consider Navier–Stokes equations with white–in–time stochastic forcing. We prove two results concerning the uniqueness of invariant measures. The first result is for any finite dimensional Galerkin approximation. We show that when a minimal number of large scale modes are forced, the system has a unique invariant measure. The second is for the full Navier–Stokes. We show that if the determining modes are forced, then the invariant measure is unique. The first result is proved using Mallivain calculus and the Harris condition. The second is proved by reducing the system to a finite dimensional problem with memory.

(joint work with J. Mattingly and Y.G. Sinai)

### Uniqueness of the invariant measure for a stochastic PDE driven by degenerate noise

**Jean–Pierre Eckmann (Geneva, Switzerland)**

We consider the stochastic Ginzburg–Landau equation on a bounded domain. We assume the stochastic force acts *only* on the high–frequency (spatial) modes. The low–lying frequencies are then only convected through the non–linear coupling with these modes. This problem has a unique invariant measure. The techniques of the proof use a controllability argument together with Malliavin calculus. Note that in this problem, the deterministic problem is already unstable.

(joint work with M. Hairer)

### The long–time behavior of the thermoconvective flow in a porous medium

**M.A. Efendiev (FU Berlin, Germany)**

For the Boussinesq approximation of the equation of coupled heat and fluid flow in a porous medium we show that the corresponding system of partial differential equations possesses a global attractor. We give lower and upper bounds of the Hausdorff dimension of the attractor depending on a physical parameter of the system, namely the Rayleigh number of the flow. Numerical experiments confirm the theoretical findings and raise new questions on the structure of the solutions of the system.

(joint work with J. Fuhrmann and S.V. Zelik)

### Navier–Stokes equations and analytic semigroup theory

**Reinhard Farwig (Darmstadt, Germany)**

Besides Galerkin's approximation analytic semigroup theory is the main tool to construct and analyze instationary solutions to the Navier–Stokes equations. A semigroup approach has the advantage to leave the linear part $u_t - \nu\Delta u + \nabla p$ or $u_t + Au$, when using the Stokes operator $A = -\nu P\Delta$ and the Helmholtz projection $P$, unchanged and to allow an $L^p$–theory, $1 < p < \infty$, including weights from the very beginning. We report about three different directions of progress in this field.

- Existence of a bounded analytic semigroup $(e^{-tA})_{t\geq 0}$ in Besov spaces $\overset{\circ}{B}{}^s_{pq}(\mathbb{R}; L^r(\Sigma))$ and in $L^q(\mathbb{R}; L^r(\Sigma))$–spaces in the case of an *infinite cylinder* $\Omega = \mathbb{R} \times \Sigma$ with constant cross section $\Sigma$

- The instationary flow (existence of weak and strong solutions, energy inequalities, decay for $t \to \infty$) in an *aperture domain* prescribing either the flow through the hole or the pressure drop (PhD thesis of M. Franzke)
- A semigroup approach in *weighted Sobolev spaces* using Muckenhoupt weights with singular behavior near the boundary, inside the domain or at infinity (PhD thesis of A. Fröhlich)

**Statistical solutions of the Navier–Stokes equations**

**Ciprian Ilie Foias (Bloomington/College Station, USA)**

A short presentation of the theory of statistical solutions of the Navier–Stokes equations as averages of deterministic solutions, as well as of some simple illustrations of how the theory provides rigorous mathematical proofs of some features of the Kolmogorov and Kraichnan theories for fully developed turbulence.

**Exact controllability and feedback boundary stabilization for Navier–Stokes systems**

**Andrei V. Fursikov (Moscow, Russia)**

Exact controllability problem is as follows: We consider Navier–Stokes equations on a torus $\mathbb{T}$ (i.e. with periodic boundary conditions) with distributed control concentrated on a subdomain $\omega$ of $\mathbb{T}$. Let $(\hat{v}, \nabla\hat{p})$ be a given solution of Navier–Stokes equations without control such that $\hat{v}(0,\cdot) \neq v_0(\cdot)$, where $v_0(\cdot)$ is an initial condition of the Navier–Stokes equations (NSE). The problem is: find a control $u$ ($\text{supp} u \in (0,T) \times \omega$) such that the solution $(v, \nabla p)$ of NSE supplied with $v_0$ satisfies the property: $v(T,x) \equiv \hat{v}(T,x)$. In the talk the solution of exact controllability problem was expounded for 3D and 2D case.

Besides a new formalization of feedback notion was proposed and feedback boundary stabilization for 2D NSE was constructed.

## Steady water waves as a dynamical system

**Mark Groves (Stuttgart, Germany)**

The hydrodynamic problem concerning the irrational flow of a 3D perfect fluid of constant density in a domain bounded above by a free surface and below by a rigid bottom is known as the water–wave problem. Here we examine the *gravity–capillary steady water–wave problem*, in which the fluid is acted upon by the forces of gravity and surface tension and all waves are uniformly translating with constant speed in the horizontal direction $x$. We use the method of *spatial dynamics* and the *Kirchgässner reduction*: we formulate the equations as an infinite–dimensional Hamiltonian system in which an unbounded spatial coordinate is the time–like variable and use a reduction theorem due to A. Mielke to show that it is locally equivalent to a finite–dimensional Hamiltonian system whose solution set can be analyzed.

In the *2D version* of the problem we identify two regions of parameter space in which there are infinitely many *solitary waves*, that is solutions to the equations which decay to zero for large $|x|$. We consider several types of 3D water waves: (a) Waves that are periodic in the direction transverse to $x$. Here we find *generalized solitary waves*, that is 3D waves which decay to a 2D spatially periodic motion at large distances. (b) Waves which are periodic in $x$. Here we find waves which resemble solitary waves in the direction transverse to $x$.

## A waiting time phenomenon for thin film equations

**Günther Grün (Bonn, Germany)**

The lubrication approximation allows us to describe the height of thin films of viscous liquids spreading on plain surfaces by a fourth order degenerate parabolic equation. For the model problem $h_t + \mathrm{div}(h^n \nabla \Delta h) = 0$ we formulate a general criterion on the growth of initial data near the free boundary which guarantees that for sufficiently small times the support locally does not increase. It turns out that this condition only depends on the smoothness of the diffusion coefficient in its point of degeneracy. Our approach combines a new version of Stampacchias's iteration lemma with weighted energy or entropy estimates.

(joint work with R. Dal Passo and L. Giacomelli)

### On the approximation of the stochastic hyperviscid Burgers equations

**Christoph Gugg (Augsburg, Germany)**

We consider a hyperviscid Burgers equation with additive white–in–time noise, approximate the basic Wiener process by a linear interpolation and prove convergence of the solutions of the equations driven by the noise corresponding to the approximating process towards the solutions of the original equation. The convergence takes place in a $L^p(\Omega)$–space with values in a function space with as much regularity as possible, $\Omega$ is the basic probability space, and is proved under weak conditions on the noise.

### Analytical issues in forced elongation

**Thomas Hagen (TU München, Germany)**

Forced elongation is a quasi–one–dimensional flow in a regime of extension: a thin fluid thread is extruded through a capillary and extended axially by a pulling force at a certain velocity. In the non–isothermal situation, the thin fluid filament solidifies by cooling through the surrounding environment. The mathematical description is based upon the Navier–Stokes equations when "thinness" of the filament is used to reduce the equations to a 1D flow. This ansatz yields equations of the form (involving a moving boundary)

$$\begin{aligned} a_t + (va)_z &= 0 \qquad && a: \text{ cross–sectional area} \\ (av_z)_z &= 0 \qquad && v: \text{ axial velocity} \\ T_t + vT_z + \tfrac{T}{\sqrt{a}} &= 0 \qquad && T: \text{ normalized temperature} \end{aligned}$$

We discuss the following issues for the Matovich–Pearson–filament equations of forced elongation:

- Temperature monotonicity along the thread: It is rigorously proved that (locally in time) the fluid temperature diminishes as the distance to the exit increases.
- The semigroup corresponding to the linearized system is eventually compact, thus allowing stability conclusions through a resolution of the spectrum of its generator.
- The spectrum of the semigroup generator allows an asymptotic description through a logarithmic curve.

(joint work with M. Renardy)

### Stationary solutions of stochastic quasi–geostrophic flows

**Peter Kloeden (Frankfurt, Germany)**

The existence of a stationary solution in the form of a pullback attractor of the random skew product flow generated by the quasi–geostrophic equations with random forcing and a random boundary condition was discussed. The main emphasis for us on different sorts of attractors including pullback attractors for topological skewing product flows generated by ODEs, the generalizations to deterministic PDEs and to random ODEs, to stochastic DEs and stochastic PDE was then indicated.

(joint work with B. Schmalfuß and J. Duan)

### Randomly forced 2D Navier–Stokes equations

**Sergei B. Kuksin (Heriot–Watt, UK)**

We study the 2D Navier–Stokes equation, forced by a random kick force. This equation defines a Markov chain in a function phase space. We prove that this Markov chain has unique invariant measure and is mixing.

### Ergodicity and mixing of 2D stochastic Navier–Stokes equations

**Antti Kupiainen (Helsinki, Finland)**

We consider the Navier Stokes equation on a two–dimensional torus with arbitrary viscosity coefficient $\nu > 0$, and smooth forcing, white noise in time. We require the forcing to be nonzero for a finite number of Fourier components depending on the viscosity. We prove that the stochastic dynamics converges to an invariant measure exponentially fast in time, for arbitrary $L^2$–initial conditions for the vorticity.

### Pattern formation below criticality forced by noise

**Stanislaus Maier–Paape (Augsburg, Germany)**

We address the question of pattern formation of equations of Swift–Hohenberg type perturbed by additive noise. These equations are below criticality, which therefore are stable in the absence of noise. Nevertheless, the stochastic perturbation is strong enough to drive the solution with high probability into regions where patterns are observed, which usually occur in spinodal decomposition.

## Incompressible fluids with viscosity depending on the pressure

**Jindrich Nečas (Prague, Czech Republic)**

We prove the existence of a global solution in time for 3D space–periodic solutions and non–Newtonian, incompressible fluid. The viscosity depends on the pressure and on the second invariant of the velocity gradient giving a shear thinning fluid.

## Symmetries and invariant solutions of turbulent flow

**Martin Oberlack (Darmstadt, Germany)**

The logarithmic law–of–the–wall is still considered as one of the main building blocks in physical and engineering turbulence theory. It has been verified in innumerable experiments and has been implemented in almost all statistical turbulence models. Even data from numerical simulations of the Navier–Stokes equations are usually validated against the law–of–the–wall. The key drawback is that the law–of–the–wall has not been derived from first principles. Using group theoretical methods (Lie–groups) it has been shown that the law–of–the–wall and a very broad variety of new turbulence laws for plane shear flows can be derived from the Navier–Stokes equations alone in the limit of vanishing viscosity. The resulting turbulence laws have clearly been validated against the data from experiments and numerical simulations of Navier–Stokes equations.

## The averaged Euler equations as a basis for numerical schemes

**Marcel Oliver (Tübingen, Germany)**

We consider the isotropic averaged Euler equations (also called inviscid second grade fluid equations, Camassa–Holm equations, Euler–$\alpha$ equations, or filtered K.–O. equations),

$$\partial_t(1 - \alpha^2(\Delta)u + u\nabla(1 - \alpha^2\Delta)u - \alpha^2(\nabla u)^T\Delta u + \nabla p = 0, \qquad \nabla \cdot u = 0 \qquad (*)$$

These equations regularize the standard Euler equations ($\alpha = 0$) and thus may form a basis for stabilizing or filtering the numerical simulation on the level of the PDE in a structure–preserving way. We discuss the following aspects in two dimensions:

- The equation can be rigorously understood as one particular vortex blob algorithm, which is a true solution to the regularized PDE. Convergence as $\alpha \to 0$ is shown.
- For initial data in the Besov space $B^{2,\infty}_{1/2}$, equation (*) in its vorticity form is an inviscid vorticity limit to the regular Navier–Stokes equations. Due to the matching boundary conditions, the model may provide a way to develop pseudo–viscous point vortex schemes.

### Global climate modeling: A paradigm and an application

### George R. Sell (Minnesota, USA)

We present a new feature of oceanic flows which is based on a gravitational force $\vec{g}(t)$ which is caused by gravitational field of the Sun–Earth–Moon (SEM) acting, in particular, on the ocean. Due to the KAM (Kolmogorov–Arnold–Moser) theory, it is reasonable to assume that $\vec{g}(t)$ is a quasi–periodic function of time $t$, with frequency module $m(\vec{g}) = \{n \cdot \omega : n \in Z^3\}$ where $\omega = (\omega_1, \omega_2, \omega_3)$ is the frequency vector of the three body SEM–problem. This leads to an oceanic model

$$\partial_t w + Aw + L(t)w = F(w) + G(t) \tag{1}$$

where $w = (u, T, S)$, $u$ = velocity, $T$ = temperature and $S$ = salinity. We show that if the averaged equation

$$\partial_t v + Av + \overline{L}v = F(v) + \overline{G} \tag{2}$$

has a hyperbolic stationary solution, and if $L(t) - \overline{L}$ and $G(t) - \overline{G}$ are "small", then the equation (1) has a quasi–periodic solution $w = w(t)$ with $M(w) \subset M(\vec{g})$. We use this result to give an interpretation of the ENSO (El Nino Southern Oscillation) phenomenon in the Pacific Ocean.

### Statistical equilibrium models of coherent structures in two–dimensional and geostrophic turbulence

### Bruce Turkington (Massachusetts, USA)

Gibbsian equilibrium statistical mechanics can be applied to ideal hydrodynamics in 2D and to the asymptotic equations in geophysical fluid dynamics (barotropic, quasi–geostrophic $\beta$–plane equations). The theory predicts the formation of steady flows on the large scales and it links the statistical property of multi–scale fluctuations with the quantitative properties of these flows. But the thermodynamics of these systems is novel. These features, however, are confirmed by comparison with physical observations.

**Generalized flow and turbulent transport**

**Eric Vanden Eijnden (New York, USA)**

The study of passive scalar transport in a turbulent velocity field feeds naturally to the notion of generalized flows. These are families of probability distributions on the space of solutions to the associated ODE's which no longer admit unique solutions. Two most natural regularizations for this problem, namely the regularization via adding a small molecular diffusion and the regularization via smoothing out the velocity field are considered. White–in–time random velocity fields are used as an example to examine the variety of phenomena that take place when the velocity field is not spatially regular. Three regimes characterized by their degree of compressibility are identified in the parameter space. In the regime of intermediate compressibility, different regularizations give rise to different scaling behavior for the structure functions. Physically, this means that the scaling depends on a turbulent Prandtl number measuring the relative strength of molecular diffusion and viscous effects.

(joint work with Weinan E)

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