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Hyperbolic Conservation Laws

October 22th – October 28th, 2000

The conference was organized by C. Dafermos (Providence), D. Kröner (Freiburg) and R. LeVeque (Seattle). It was joined by forty-eight participants from ten countries (Germany:16, USA:13, France:6, Norway:4, Italy:3, Brazil:2, Greece, Spain, Sweden, Israel:1).
The main purpose of the conference was to bring together different research groups working on theoretical and numerical aspects of conservation laws. In eighteen main lectures during the morning sessions important new results have been presented. Progress has been obtained in many fields of research such as uniqueness and stability issues, diffusive, diffusive-dispersive, relaxation, and kinetic approximations, multifluids, problems with (stiff) source terms, discrete shocks, stability of boundary layers, and degenerate problems.
During accompanying discussions, as well as in short contributions in the afternoons, specific topics such as magnetohydrodynamics, elastodynamics, nonclassical shocks and multiscale approximations have been discussed in more detail.
Two spontaneous problem sessions initiated inspiring discussions on open problems for conservation laws and in particular on the difficulties and new developments for problems with source terms.

Abstracts

Semi-discrete shocks

Sylvie Benzoni-Gavage

Abstract: The existence of stable discrete shocks is a basic issue in the numerical analysis of hyperbolic systems of conservation laws. Despite many efforts, no general enough result is available though. The known results assume that the discrete shock speed is either a rational number (Majda & Ralston, 1979) or a Diophantine number (Liu & Yu, 1999). Motivated by a work of Chow, Mallet-Paret and Shen (1998), we have chosen to focus on semi-discrete shocks. The existence of such stable traveling waves would indeed imply the existence of fully discrete shocks, no matter the nature of their speed. The existence of semi-discrete shocks of small strength can be dealt with by a center manifold argument. For the special "upwind" scheme, the center manifold theorem referred to is available in the delay differential equations theory. For more general schemes, we must prove a center manifold theorem for a differential equation with both delay and advance. This has been done for dissipative and "non-resonant" schemes by P. Huot in his thesis. The stability of small strength semi-discrete shocks can be tackled by a standard energy method ("a la Goodman"). The main purpose of the talk is to introduce a tool encoding the linearized stability of possibly large semi-discrete shocks. This tool is an Evans function of mixed type. It is obtained for the "upwind" scheme by reformulating the eigenvalue equations of a retarded differential operator as a(n infinite dimensional) "dynamical system". The mixed type refers to the use of the adjoint dynamical system in the definition of the Evans function, which relies on the fact that the unstable manifold of a delay differential equation is finite dimensional. The (infinite dimensional) adjoint "dynamical system" is highly non-standard. However, it can (easily) be related to the eigenvalue equations of the adjoint operator, which is nothing but an advanced differential operator. Eventually, the stability condition that we obtain for semi-discrete shocks is the same as for viscous shocks associated with the numerical viscosity matrix. Furthermore, the low-frequency behaviour of our Evans function is certainly related to the spectral requirement in Chow, Mallet-Paret and Shen.

Entropy satisfying flux vector splittings and kinetic BGK models

Francois Bouchut

We establish forward and backward relations between entropy satisfying BGK models such as those introduced previously by the author and the first order flux vector splitting numerical methods for systems of conservation laws. Classically, to a kinetic BGK model that is compatible with some family of entropies we can associate an entropy flux vector splitting. We prove that the converse is true: any entropy flux vector splitting can be interpreted by a kinetic model, and we obtain an explicit characterization of entropy satisfying flux vector splitting schemes. We deduce a new proof of discrete entropy inequalities under a sharp CFL condition that generalizes the monotonicity criterion in the scalar case. In particular, this gives a stability condition for numerical kinetic methods with noncompact velocity support. A new interpretation of general kinetic schemes is also provided via approximate Riemann solvers. We deduce the construction of finite velocity relaxation systems for gas dynamics.

Viscous approximations of hyperbolic systems of conservation laws

Alberto Bressan

For a strictly hyperbolic system of conservation laws, recent work has established the uniqueness and stability of entropy weak solutions of the Cauchy problem with small BV data. A natural conjecture is that such solutions are precisely the unique limits of vanishing viscosity approximations. Toward a proof of this result, the key ingredient is an a priori estimate of the total variation of viscous approximations. Our research (together with S. Bianchini) has shown that the new oscillations produced by interactions of viscous waves of different families, as well as those produced by interactions of waves of the same family, can all be estimated in terms of some new Lyapunov functionals. These replace the classical Glimm wave interaction functional in the presence of viscosity. We thus obtain global BV bounds and convergence of vanishing viscosity approximations for various classes of hyperbolic systems.

On the self-similar solutions of 2-D Riemann problems for hyperbolic conservation laws

Suncica Canic

This talk reports on the recent results related to the study of the existence and properties of self-similar solutions (solution spaces, singularities, structure) to multi-dimensional conservation laws, obtained jointly with Barbara Lee Keyfitz. We have focused on the study of two-dimensional problems and closely examined wave interactions arising in weak shock reflection by a wedge.

We found that for a wide class of two-dimensional conservation laws (including many standard equations of compressible flow [7]) the theory for self-similar solutions of Riemann problems divides naturally into two parts: supersonic and subsonic. The supersonic theory can be completed by generalizations of one-dimensional results (see, for example [6]); the subsonic theory involves solving free-boundary problems for degenerate elliptic equations or equations of mixed type. Our preliminary analysis of the subsonic part of the problem, coupled with the numerical simulations obtained in [8], indicates new features in the solution: hyperbolic "bubbles" imbedded in the subsonic region, and the formation of possible singularities not occurring in one-dimensional problems.

We have recently developed techniques for analyzing the subsonic part of the solution in transonic shock interactions modeled by the transonic small disturbance equation, the steady (with Gary Lieberman, [1]) and the unsteady case (with Eun Heui Kim, [2, 3, 4]). The unsteady transonic small disturbance equation has been used as a model for a benchmark problem in two-dimensional shock interactions: the weak shock reflection by a wedge. Our results provide a complete description of the solution in the case of regular reflection. Irregular reflection (Mach reflection and von Neumann reflection) is under consideration. The crucial part in the analysis is the formulation of the problem as a free-boundary problem for the position of a transonic shock: the Rankine-Hugoniot conditions should be written as a shock evolution condition coupled with the condition describing mass flow across the shock (oblique derivative boundary condition). The main ideas in the proof use compactness arguments which are based on the a priori bounds on the solution (provided by the supsersonic part of the solution and the far-field estimates) and on the regularity estimates which use, in a crucial way, the information obtained from the mass flow through the transonic shock. A generalization of these ideas to a class of shock evolution problems, using the results recently obtained in [5], is under way.

PARTIAL LIST OF THE RELATED REFERENCES WHICH CAN BE OBTAINED FROM WWW.MATH.UH.EDU/~CANIC:

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Uniqueness and stability of Riemann solutions in gas dynamics

GUI-QIANG CHEN

We prove the uniqueness of Riemann solutions in the class of entropy solutions in $L^\infty \cap BV_{loc}$ with arbitrarily large oscillation for the $3X3$ system of Euler equations in gas dynamics. Our proof for solutions with *large* oscillation is based on a detailed analysis of the global behavior of shock curves in the phase space and the singularity of centered rarefaction waves near the center in the physical plane. The uniqueness of Riemann solutions yields their inviscid large-time stability under *arbitrarily large* $L^1 \cap L^\infty \cap BV_{loc}$ perturbation of the Riemann initial data, as long as the corresponding solutions are in L^∞ and have local bounded total variation satisfying a natural condition on its growth with time. No specific reference to any particular method for constructing the entropy solutions is made. Our uniqueness result for Riemann solutions can be easily extended to entropy solutions $U(x,t)$, piecewise Lipschitz in x, for any $t > 0$, with arbitrarily large oscillation. (Joint work with H. Frid and Y. Li).

Challenges in (astrophysical) MHD simulations

Andreas Dedner

Most numerical schemes for the equations of magnetohydrodynamics (MHD) have been developed for the case of an ideal gas. In many applications (e.g. solar physics) this simplification is too far from reality. Therefore these numerical schemes have to be extended to cope with a more general equation of state (EOS). For the Euler equations of gas dynamics two general approaches for such an extension have been proposed recently, which we have extended to MHD. We have compared different techniques of building a second order scheme using piecewise linear reconstruction. Some problems arise from the use of a tabularized EOS, which may become necessary to reduce the computational cost if the evaluation of the EOS is expensive. We suggest the use of an adaptive, hierarchical table. This is joint work with M. Wesenberg.

A serious problem in MHD simulations (independent of the EOS) arises due to the fact that the constraint $\nabla \cdot \vec{B} = 0$ cannot be guaranteed by a numerical scheme. Even if the numerical simulation is started with a solenoidal magnetic field, the numerical approximations lead to errors, which can accumulate in time. This may result in totally unphysical solutions or even a breakdown of the simulation. Two different approaches are often used for the stabilization of the numerical schemes. We suggest a different approach which avoids some of the disadvantages of the other methods. Most importantly, this method is conservative and does not introduce an infinite wave speed. This is joint work with F. Kemm, D. Kröner, C.–D. Munz, T. Schnitzer, M. Wesenberg.

Asymptotic stability of viscous shock waves near states of non-convexity or non-strict hyperbolicity

Heinrich Freistühler, Christian Fries

We consider a hyperbolic-viscous system of conservation laws

$$u_t + f(u)_x = \mu u_{xx},$$

and solutions $u(x,t) = \phi(x - st)$ that represent viscous shock waves connecting two nearby states u_-, u_+. I. e., $\phi : \mathbf{R} \to \mathbf{R}^n$ solves

$$\mu\phi' = f(\phi) - s\phi - q, \quad \phi(\pm\infty) = u_\pm,$$

where u_-, u_+, $q \in \mathbf{R}^n$, $s \in \mathbf{R}$ satisfy $f(u_-) - su_- = f(u_+) - su_+ = q$ and

$$|u_+ - u_-| \ll 1.$$

The shock waves we are interested in are Laxian or overcompressive, i. e., the characteristic speeds $\lambda_1 \leq \ldots \leq \lambda_n$ (= eigenvalues of f') at u_-, u_+ satisfy

$$\begin{aligned} \lambda_i(u_\pm) < s \quad &\text{for} \quad 1 \leq i < n_1, \\ \lambda_i(u_-) > s > \lambda_i(u_+) \quad &\text{for} \quad n_1 \leq i \leq n_2, \\ \lambda_i(u_\pm) > s \quad &\text{for} \quad n_2 < i \leq n, \end{aligned}$$

with some $n_1 \leq n_2$. We prove the stability, for $t \to \infty$, of these traveling wave solutions under further natural assumptions.

The result applies to systems with generic rotational equivariance, such as nonlinear elastodynamics, electromagnetism, or magnetofluiddynamics (with "artificial" viscosity). The "full" versions of these systems (plane waves in several space dimensions) have states of non-strict hyperbolicity, while for the corresponding "coplanar" subsystems, the same states are points of (strict hyperbolicity but) non-convexity. The result means the stability of small "non-classical" shocks near these points.

Decay of solutions of conservation laws, decay of almost periodic solutions of conservation laws

Hermano Frid

We consider the asymptotic behavior of solutions of systems of inviscid or viscous conservation laws in one or several space variables, which are almost periodic in the space variables in a generalized sense introduced by W. Stepanoff and Wiener, which extends the original one of H. Bohr. We prove that if $u(x,t)$ is such a solution whose inclusion intervals at time t, with respect to $\varepsilon > 0$, satisfy $l_\varepsilon(t)/t \to 0$ as $t \to \infty$, and so that the scaling sequence $u^T(x,t) = u(Tx,Tt)$ in pre-compact as $T \to \infty$ in $L^1_{loc}(\mathbf{R}^{d+1}_+)$ then $u(x,t)$ decays to its mean value $\bar{u}$, which is independent of t, as $t \to \infty$. The decay considered here is in L^1_{loc} of the variable $\xi = x/t$, which implies, as we show, that $M_x(|u(x,t) - \bar{u}|) \to 0$, as $t \to \infty$, where M_x denotes taking the mean value with respect to x. In many cases we show

that the solutions are almost periodic in the generalized sense if the initial data are. We also show, in these cases, how to reduce the condition on the growth of the inclusion intervals $l_\varepsilon(t)$ with t, as $t \to \infty$, for fixed $\varepsilon > 0$, to a condition on the growth of $l_\varepsilon(0)$ with ε, as $\varepsilon \to 0$, which amounts to impose restrictions only on the initial data. We show with a simple example the existence of almost periodic (non-periodic) functions whose inclusion intervals satisfy any prescribed growth condition as $\varepsilon \to 0$. The applications given here include inviscid and viscous scalar conservation laws in several space variables, some inviscid systems from gas dynamics and chromatography, and many viscous 2×2 systems such as those of nonlinear elasticity and Eulerian isentropic gas dynamics, with artificial viscosity, among others. In the case of inviscid scalar equations, the class of initial data for which decay results are proved includes, in particular, all the L^∞ generalized limit periodic functions. Our procedures can be easily adapted to provide similar results for semilinear and kinetic relaxations of systems of conservation laws.

Shallow water with topography

Thierry Gallouet

In my talk, I present a way to take into account some source terms in hyperbolic systems of conservation laws. The main objective is to obtain a convergent scheme (as space and time steps go to 0) which gives also "satisfying approximate solutions" for large space and time steps (for instance, which "maintain steady states") and which is "reasonnable" for the computational point of view. This is achieved with a finite volumes scheme which take into account source terms on the interfaces on the mesh. The source terms appear in an approximate Riemann solver on each interface. Some examples are given in the case of shallow water with topography, including convergence (as time goes to infinity) towards steady states and vacuum occurence.

Numerical approximation of conservation laws with stiff source term for the modelling of detonation waves

Christiane Helzel

The approximation of detonation waves may lead to numerical difficulties, which are caused by different time and space scales that arise in the model equations. The chemical reactions are often very rapid compared to the gas transport. In a numerical simulation one usually has to take these different scales into consideration, even if one is only interested in the global solution structure and not in a detailed description of the processes inside the very thin reaction zone. In this talk, a modified fractional step scheme was described, which allows the approximation of detonation waves without resolving the reaction zone. This numerical scheme uses the structure of the Riemann problem, which arises in the discretization, in order to determine where burning should arise in each time step.

Furthermore, the resolved approximation of detonation waves was considered. On structured grids a crossflow instability can arise in numerical simulations. The mechanism which leads to this numerical instability in the approximation of detonation waves was described. The crossflow instability can be avoided by a simple change of the Riemann solver.

This is a joint work with Randall J. LeVeque, Derek S. Bale, and Gerald Warnecke.

Operator splitting - theory and applications

Helge Holden

In this talked we consider operator splitting, also known as the fractional steps method, for constructing physically relevant (entropy weak) solutions of the Cauchy problem for scalar and weakly coupled systems of nonlinear mixed hyperbolic-parabolic partial differential equations. The class of equations is rich, and contains, for instance, scalar conservation laws, heat equations, porous medium equations, two-phase reservoir flow equations, as well as some strongly degenerate convection-diffusion equations with applications to sedimentation. We first present an abstract 'Kružkov type' convergence theory for product formulas. This theory includes and improves previous convergence results for problem specific splitting methods. Applications to flow in porous media and sedimentation were presented. This is joint work with K. H. Karlsen (Bergen), K-A Lie (Oslo), and N. H. Risebro (Oslo)

The random projection method for hyperbolic conservation laws with stiff reaction terms

Shi Jin

Hyperbolic systems with source terms arise in the modeling of chemically reacting flows. In these problems, the chemical time scale may be orders of magnitude faster than the fluid dynamical time scales, making the problem numerically stiff. The numerical difficulty of this problem is classical – one always gets the wrong shock speed unless one fully resolves the small chemical scale numerically. We introduce a novel numerical method – the random projection method– that is able to capture the correct shock speed without resolving the small scale. The idea is to replace the ignition temperature by a uniformly distributed random variable in a suitable domain. The statistical average of this method corrects the spurious shock speed, as will be proved with a scalar model problem and demonstrated by a wide range of numerical examples in inviscid denotation waves in both one and two space dimensions, and for multi-species reactions.

Continuous dependence estimates for viscosity solutions of fully nonlinear degenerate parabolic equations

Kenneth Hvistendahl Karlsen

Using the maximum principle for semicontinuous functions (Crandall, Ishii, Lions), we establish here an explicit "continuous dependence on the nonlinearities" estimate for viscosity solutions of fully nonlinear degenerate parabolic equations with time and space dependent nonlinearities. Our result generalizes a result by Souganidis for first order Hamilton-Jacobi equations and a recent result by Cockburn, Gripenperg, and Londen for a class of degenerate parabolic second order equations. We apply our result to the Hamilton-Jacobi-Bellman partial differential equation associated with optimal control of a degenerate diffusion process over a finite horizon. Without appealing to probabilistic arguments, we then obtain the following two results: (i) An explicit rate of convergence for the corresponding vanishing viscosity method. (ii) An explicit estimate of the continuity of the value function (viscosity solution) with respect to the coefficients in the Hamilton-Jacobi-Bellman equation. We also use the basic result to derive an explicit rate of convergence for certain numerical approximations. This is joint work with Espen Jakobsen.

A single fluid algorithm for multifluids

Smadar Karni

The main difficulty in extending state-of-the-art single fluid algorithms to multifluid flows is building into the numerical method the ability to recognize and respect pressure equilibrium between fluid components. Failing to do so results in unphysical pressure oscillations near interfaces, and consequently lead to false interface dynamics. An added obstacle is the known fact that fully conservative schemes cannot preserve pressure equilibrium. Hence multifluid algorithms often give up strict conservation in favour of good control over the pressure field.

Recent years have seen a growing interest in developing suitable methods for computing multifluid dynamics. Among those are methods that have a single-fluid 'flavour', which capitalize on the fact that pressure oscillations do not arise in single-fluid flows, the internal energy correction algorithm (Jenny et al. JCP, 132:91-107, 1997) and the Ghost-Fluid-Method (Fedkiw et al., JCP, 152:457-492, 1999). Recognizing the advantage of single-fluid approaches, this talk presents an extremely simple algorithm for multifluids, based on computing two different flux functions across material fronts: one assuming that both fluids on either side are of type A, say, and the other assuming they were both of type B. The algorithm provides a general framework for discretization by any numerical method. It conserves total mass and momentum and essentially conserves total energy in the sense that conservation errors, while extremely small on standard grids (on the order of a fraction of a percent), further decay to zero with mesh refinement. Results are presented of shock-interface interactions, and interfacial instabilities involving ideal and stiff fluids.

Singular shocks in nonhyperbolic models for incompressible two-phase flow

Barbary Keyfitz

Many models for multi-fluid flow result in equations which fail to be hyperbolic. In the simplest model for incompressible flow (given in the text of Drew and Passman, for example), the principal part of the differential operator has characteristics with nonzero imaginary part for any state of the fluid which contains both phases. That is, the linearized equations are catastrophically unstable at every point.

This linear instability has caused great distrust of the model equations and concern about the modeling processes from which they are derived.

However, the nonlinear equations which form the model behave very differently from their linearizations. Although states which are linearly unstable are also unstable in the nonlinear equations, nonlinear theory predicts jump transitions, via stable shocks, from unstable to stable states. Furthermore, the nonlinear theory eliminates both infinite growth modes and high-frequency oscillations. The solution depends continuously on the data except at certain values where threshold or bifurcation phenomena occur.

Some of the shock transitions are of a novel type, *singular shocks*, first found in work of Keyfitz and Kranzer. Singular shocks can be described by means of approximations, using self-similar or regular viscosity. In the limit of zero viscosity, they are weighted measures; however, the sense in which they satisfy the equation, in the limit, is not well-understood.

The interpretation of the solutions in the incompressible flow equations, however, appears reasonable, and computations also appear consistent with these solutions.

Computing Euler flow at high Mach numbers

Christian Klingenberg

When doing hydrodynamic simulations there are situations (for example the hydrodynamic simulation of protostellar jets in astrophysics) where the velocities are extremely high and pressure very low. Experience shows that standard numerical schemes fail to compute internal variables (like temperature and pressure) with the desired accuracy. We propose a relaxation approach which remedies this problem. This scheme satisfies the discrete entropy inequality, and thus guarantees positive pressure.

This is joint work with Frederic Coquel.

Local error analysis and adaptive semi-discrete central schemes for hyperbolic conservation laws

Alexander Kurganov

We consider systems of one-dimensional hyperbolic conservation laws subject to compactly supported (or periodic) initial data. Since typical solutions of nonlinear conservation laws are nonsmooth, standard methods of truncation error analysis, based on the Taylor expansions, are invalid.

We propose a new method for practical measurement of the local Lip' truncation errors by using a basis of locally supported test-functions. Our particular choice of such test-functions is the localized quadratic B-splines. A global, compactly supported test-function may then be approximated by means of the local test-functions, and thus, in the scalar convex case the global Lip' truncation error can be obtained from the local ones.

In the case of a system of conservation laws, no rigorous error estimates can be obtained. However, one may still compute the local Lip' truncation error. Moreover, our numerical experiments demonstrate a remarkably similar behavior of the local truncation error and the actual error. This suggests that even in the system case, the local truncation error may serve as a reliable error indicator.

This is the key idea in developing adaptive semi-discrete schemes. The difference of several orders of magnitude in the local truncation errors between smooth and nonsmooth regions provides an effective tool for identifying nonsmooth parts of the solution. This is utilized in the scheme- and mesh-adaption algorithms.

L^1-continuous dependence property for systems of conservation laws

Philippe G. LeFloch

We are concerned with the uniqueness and L^1 continuous dependence of entropy solutions for nonlinear hyperbolic systems of conservation laws. On one hand, we study a class of linear hyperbolic systems with discontinuous coefficients: Each propagating shock wave may be a Lax shock, or a slow or fast undercompressive shock, or else a rarefaction shock. We establish the L^1 continuous dependence of solutions upon their initial data in the case that the system does not contain rarefaction shocks. In the general case our estimate takes into account the total strength of rarefaction shocks. In the proof, a new time-decreasing, weighted L^1 functional is obtained via a step-by-step algorithm.

To treat nonlinear systems, we introduce the concept of admissible averaging matrices which are proven to exist for solutions with small amplitude of genuinely nonlinear systems. Interestingly, for many systems of continuum mechanics, they also exist for solutions with arbitrary large amplitude. The key point is that an admissible averaging matrix does not exhibit rarefaction shocks. As a consequence, the L^1 continuous dependence estimate for linear systems can be extended to nonlinear hyperbolic systems.

Quasi-steady methods for hyperbolic equations with source terms

Randall J. LeVeque

Conservation laws with source terms often have steady states in which the flux gradients are nonzero but exactly balanced by source terms. Many numerical methods (e.g., fractional step methods) have difficulty preserving such steady states and cannot accurately calculate small perturbations of such states. I discussed an approach to this problem based on introducing a Riemann problem in the center of each grid cell whose flux difference exactly cancels the source term. This leads to modified Riemann problems at the cell edges in which the jump now corresponds to perturbations from the steady state. A similar idea may also be useful for quasi-steady problems with spatially-varying flux functions.

Kinetic formulation of entropic schemes for scalar conservations laws

Charalampos Makridakis

A kinetic formulation of the wider known class of entropic schemes, the E-schemes, is considered. Based on this formulation one is able to derive many properties of the E schemes and to prove convergence using the Kinetic formulation of the conservation law. In particular, in the one dimensional case, using this formulation one can give a new proof of the local entropy inequalities but under improved and more natural CFL conditions. In addition several properties, e.g., as the weak BV bound for finite volume schemes are now easily proved.
This is joint work with B. Perthame, ENS, Paris.

Conservation laws with source terms and applications

Dan Marchesin

Systems of conservation laws can be generalized to deal with mass and energy transfer in thin zones where there are chemical reactions or phase changes. In such thin zones (which are shocks), diffusion and source terms become important: they are $O(1)$ in L^1_{loc} and their balance determines the internal structure of the shock.

We show applications of such concepts to multiphase flow in porous media in one spatial dimension, for the case of steam injection and fireflood, where there are condensation fronts and combustion waves.

Conservative versus non conservative models for multimaterial compressible flows: real gas flow applications

Antonio Marquina

We formulate two models: The mass fraction model and the level set model, as prototypes of conservative and non conservative models, respectively. We use a Riemann solver introduced by the author as the more consistent solver for the two-component fluid flow. We will explain the “Ghost Fluid Method”, recently developed to improve the level set model. Concerning the boundary/interface condition, we consider the general class of material interface problems where numerical methods can predict pressure and velocity adequately, but fail in their predictions of density and temperature. Motivated by total variation considerations and physical assumptions, we have developed a simple but general boundary condition for this class of problems. This new boundary condition does not change the pressure or the velocity, but does change the density and the temperature in a fashion consistent with the equation of state resulting in new values that minimize a specific measure of variation at the boundary. We perform different 1D and 2D experiments using high order accurate shock capturing schemes. We compare the behavior of both models, remarking the advantages and disadvantages.

Large asymptotic expansions and numerical methods for low Mach number Euler- and Navier-Stokes-equations

Andreas Meister

The results of the asymptotic analysis of the Euler and Navier-Stokes equations are employed to extend the validity of compressible flow solvers to the low mach numbe regime. In particular, we consider two different approaches. One possibility is the use of a flux-correction-approach which means that the fluxes computed by a standard or slightly modified Riemann-solver are corrected by means of the results of the asymptotic analysis. Furthermore, we present a preconditioning technique, whereby the preconditioner is only introduced within the numerical dissipation of a compressible Riemann-solver in order to enable the use of the numerical method for the simulation of steady as well as unsteady flow fields. A discrete asymptotic analysis is employed in order to prove the validity of the developed numerical method in the low mach number regime. Numerical results are presented for both approaches.

Adaptive finite volume methods based on local multiscale decompositions

Siegfried Müller

A new approach is presented by which any standard Finite Volume Method (FVM) can be accelerated. The basic idea is to incorporate data compression strategies based on wavelet techniques which has been originally suggested by A. Harten.

Starting point is a so-called multiscale decomposition corresponding to a sequence of nested grids which is determined by the discrete flow field at hand. To this end, a sequence of mean values corresponding to a finest resolution level is decomposed into to an equivalent sequence of mean values on a coarsest resolution level and details describing the difference of the solution on two successive resolution levels. Since the details may become negligible small in regions where the flow field exhibits a moderate variation in the data, the complexity of the data can be reduced applying hard thresholding techniques to the multiscale decomposition. By means of the truncated sequence of multiscale coefficients a locally adapted grid with hanging nodes is predicted on which the time evolution is performed.

In collaboration with A. Cohen and his team it has been recently verified analytically that the threshold error introduced in each time step can be controlled, i.e., the error does not blow up over all time steps, provided the threshold value is judiciously chosen. This result implies that the accuracy of the reference FVM is preserved.

Moreover, numerical computations show that the resulting adaptive FVM is much more efficient than the reference FVM. In particular, the gain in computational time as well as the reduction of memory requirements are improved with an increasing number of refinement levels. This is different to Harten's original concept which is only aiming at the reduction of expensive numerical flux computations, i.e., the complexity of the scheme still corresponds to that of the finest resolution level. This is different for the new approach where the complexity is proportional to the number of significant details.

Convergence of diffusive BGK approximations for nonlinear strongly parabolic systems

Roberto Natalini

We study a class of BGK approximations of parabolic systems in one space dimension. We prove stability and existence of global solutions for these models. Moreover, under certain conditions, we prove a rigorous result of convergence toward the formal limit, by using compensated compactness techniques. Starting from these results it is possible to design numerical schemes for nonlinear degenerate parabolic systems. General stability conditions are derived, and for scalar equations convergence is proved. These methods may be also adapted to unstructured meshes. This is joint work with Corrado Lattanzio.

The MoT-ICE: a new multi-dimensional wave-propagation-algorithm based on Fey's Method of Transport

Sebastian Noelle

The numerical solution of systems of hyperbolic conservation laws is dominated by Riemann-solver based schemes, which are usually extended to several space-dimensions either by using dimensional splitting on cartesian grids or by the finite-volume approach on unstructured grids. One disadvantage of these schemes is that the Riemann-solver is applied in the grid- rather than the flow-direction, which may lead to grid orientation effects and cross-flow instabilities.

In this contribution we focus on an alternative, genuinely multi-dimensional approach, Fey's Method of Transport (**MoT**) [1], which belongs to the family of flux-vector-splitting schemes. The starting point of such schemes is a multi-dimensional wave-model, which leads to a reformulation of the system of conservation laws as a finite set of coupled nonlinear advection equations. This decoupling may be justified from gas kinetic theory.

Many upwind schemes are inconsistent at sonic points, and so is the first-order version of Fey's method. Here we develop a new version of the MoT based on **I**nterface-**C**entered-**E**volution, the **MoT-ICE** [2]. The new method draws ideas from the flux-vector-splitting and the flux-difference-splitting approaches: the multi-dimensional wave-models are inherited from Fey's Method of Transport or other flux-vector-splitting schemes, while a predictor-step which provides auxiliary transport-velocities on the cell-interfaces uses flux-difference-splitting techniques.

For the new method, we prove uniform first- resp. second-order consistency, including at sonic points. Numerical experiments confirm second-order-accuracy for smooth solutions and high-resolution nonoscillatory shock-capturing properties for discontinuous solutions. The second-order version of the new MoT-ICE is several times faster than Fey's second-order scheme and seems to be as fast as standard second-order algorithms.

In [3], the MoT-ICE has been extended to adaptive cartesian grids and the equations of magneto-hydrodynamics (MHD).

Besides reporting this progress, we also discuss open questions, drawbacks and possible improvements of the MoT, among them the cumbersome computation of the correction coefficients for the decomposition error, the large dissipation in the linear fields, and instabilities for some multi-d MHD calculations.

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A posteriori error estimates for implicit vertex centered finite volume approximations of nonlinear convection-diffusion-reaction equations

Mario Ohlberger

This talk is devoted to the study of *a posteriori* error estimates for a scalar nonlinear convection-diffusion-reaction equation in two space dimensions. The estimates for the error between the exact solution and an implicite vertex centered upwind finite volume approximation to the solution are derived in the L^1-norm, independent of the diffusion coefficient. The resulting *a posteriori* error estimate is used to define an grid adaptive solution algorithm for the finite volume scheme. Finally numerical experiments underline the applicability of the theoretical results.

A survey of the kinetic approach to conservation laws

Benoit Perthame

The shallow water description through the Saint-Venant system is usual for many applications (rivers flow, tidal waves, but also narrow tubes). This is a hyperbolic system, relatively simple, that, however, contains a source term describing the bottom topography. Classical finite volumes schemes give a very low accuracy on such a system, especially because they do not preserve the steady states. This question has been considered by many authors who modified the Roe or Godunov solvers (Leroux et al, Gallouet et al, Jin, Leveque).

In this talk we will give two progresses on the Saint-Venant system. First, we will show how a derivation from Navier-Stokes (and not Euler) equations allow to justify the friction term, the viscosity terms and the Bousinesq coefficients. Second, we will show how the kinetic approach allows a sipmle understanding of stiff topography and to derive a kinetic solver for finite volumes methods.

Weakly non-oscillatory schemes for scalar conservation laws

Bojan Popov

A new class of semi-discrete Godunov-type numerical methods for solving nonlinear scalar conservation laws is introduced. This new class of methods, called here weakly non-oscillatory (WNO), is a generalization of the classical non-oscillatory schemes. Under certain conditions, convergence and error estimates of the methods are proved. The main new idea is that we can violate the entropy inequalities if we have a WNO numerical method and initial data. Examples of such WNO schemes include modified versions of Min-Mod and UNO.

Zero diffusion–dispersion limits for self-similar Riemann problems

Christian Rohde

We study the Riemann problem for nonlinear hyperbolic systems of conservation laws regularized with vanishing diffusion and dispersion terms. We prove the existence of a smooth self-similar solution to this problem and we derive a uniform estimate on its total variation. A generalization of the zero-viscosity wave-fan criterium introduced by Dafermos is used. Our proof relies on work of Tzavaras on purely diffusive regularizations for general $(m \times m)$-systems.

In the limit, when the diffusion and the dispersion coefficients vanish, the regularized solution converges in a strong topology to a discontinuous solution of the hyperbolic system of conservation laws. Our result provides a new existence theorem for the Riemann problem, in which the characteristic fields need not be genuinely nonlinear and dispersive effects are taken into account.

This is joint work with P.G. LeFloch.

Stability stabilizes blow-up in quasilinear parabolic equations with balanced nonlinearity

Steve Schochet

Let L be a self-adjoint uniformly strongly elliptic operator with smooth, time-independent coeficients. Then the initial-value problem for the degenerate higher-order quasilinear parabolic PDE $u_t = -L(|u|^{m-1}u)$ has a weak solution locally in time. The minimal time of existence can be estimated from the L^{m+1} norm of the initial data. A sufficient condition for the solution to blow up in finite time is that $\int(|u|^{m-1}u)L(|u|^{m-1}u)$ is negative at time zero. When finite-time blow-up occurs then an appropriate rescaling of the solution tends at the blow-up time to a solution of $L(|u|^{m-1}u) + u = 0$. For the case $L = (-\partial_x^2)^r - 1$ with $r > 1$, this limit equation has no non-negative compactly supported solutions.

Relaxed ENO-schemes

Achim Schroll

Numerical experiments with relaxed ENO schemes were presented. Methods of up to formally 4th order accuracy were applied to the Euler equations of gas dynamics and the planar MHD system. In a test example involving a Mach 3 shock, a convergence rate of approximately 1.8 in L^1-norm was observed numerically.

Higher order schemes based on relaxation are due to Jin and Xin (1995). Following their terminology, relaxed schemes are obtained by sending the

relaxation parameter to zero in the well known Jin-Xin-relaxation-scheme. The resulting methods are related to central schemes by Tadmor et al. They differ however in the choice of variables which are reconstructed in order to obtain the higher order approximation.

Boundary layer stability in real vanishing viscosity limit

Denis Serre

This is a joint work with K. Zumbrun (Bloomington, Indiana)

In a previous work, one developed an Evans function machinery for the study of boundary layer stability. There, the analysis was restricted to strongly parabolic perturbations, that is to an approximation of the form $u_t + (F(u))_x = \nu(B(u)u_x)_x$ ($\nu << 1$) with an "elliptic" matrix B. However, real models, like the Navier-Stokes approximation of the Euler equation for a gas flow, involve incompletely parabolic perturbations : B is not invertible in general.

We first adapt the Evans function to this realistic framework, assuming that the boundary is not characteristic, neither for the hyperbolic first order system $u_t + (F(u))_x = 0$, nor for the perturbed system. We then apply it to the various kinds of boundary layers for a gas flow. We exhibit some examples of unstable boundary layers for a perfect gas, when the the viscosity dominates heat conductivity and the adiabatic constant γ is larger than two.

Remarks on the Chapman-Enskog Expansion

Marshall Slemrod

This talk discussed the Chapman-Enskog expansion used in approximating solutions of the Boltzmann equation. The idea of the expansion is to asymptotically represent the macroscopic fluid equations derived from expanding the solution of f(x,v,t) of the Boltzmann equation. The problem is truncations of this expansion beyond Navier- Stokes order are unstable. The talk focusses on a new idea of Jin and Slemrod eliminate this instability and still retain the usefulness of the approximation.

Critical threshold phenomena in Euler-Poisson equations

Eitan Tadmor

In this work we study the system of Euler-Poisson equations

encountered in various applications of Fluid Dynamics and Plasma physics. We show that if the initial configuration exceeds an intrinsic critical threshold then the solution of these equations develop shock discontinuities in a finite time, whereas initial configurations below critical threshold lead to globally smooth solutions. We also describe the large time behavior of such solutions, for the various cases of charged and uncharged particles, with or without viscosity, relaxation, ... in one- and multi-dimensional problems. The phenomena of critical threshold is shown to characterize these various cases.

Joint work with Shlomo Engelberg (JCT) and Hailiang Liu (UCLA)

Well-posedness of systems of conservation laws near solutions containing two large shocks

Konstantina Trivisa

We consider the Cauchy problem for the strictly hyperbolic system of n conservation laws in one space dimension. Each characteristic field is assumed to be either linearly degenerate or genuinely nonlinear.

Major progress in the theory of hyperbolic systems of conservation laws has been the proof of the stability of solutions to the Cauchy problem with initial data of *small* total variation [1], [2], [5].

A significant problem in the field, which remains open, is the establishment of the well-posedness of solutions with initial data $\bar{U}$ being bounded but possibly *large*.

As a first step in that direction, we consider as initial data $\bar{U}$ a small BV perturbation of a fixed Riemann Problem (U_0^l, U_0^r) whose solution contains two *large* stable, Lax compressive shocks traveling with different characteristic speeds Λ^i and Λ^j.

We prove:

1 The (global) existence of entropy solution to the Cauchy problem with initial data $\bar{U}$ suitably close to the Riemann data (U_0^l, U_0^r).

2 The stability of the solution U under small BV perturbations of its initial data.

The principal tools in the analysis are the wave front tracking algorithm and the notion of an *entropy* functional.

Joint work with Marta Lewicka

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A variational approximation scheme for three dimensional elastodynamics with polyconvex energy

Athanasios Tzavaras

The topic of this talk is the construction of a variational approximation scheme for the equations of three dimensional elastodynamics with polyconvex stored energy. The assumption of polyconvexity is instrumental in the existence theory for the equations of elastostatics, and the purpose is to investigate its role for the equations of elastodynamics. The scheme is motivated by embedding the equations of elastodynamics into a larger system consisting of the equation of motion and some geometric evolutions of the null Lagrangians (the determinant and cofactor matrix). The scheme decreases the mechanical energy, and its solvability is reduced to the solution of a constrained convex minimisation problem. We will survey certain results on stability and convergence of such approximations of the equations of elastodynamics in the 3-d and in the 1-d setting. (joint work with S. Demoulini (Oxford) and D. Stuart (Cambridge)).

Challenges in (astrophysical) MHD simulations: transparent boundary conditions

Matthias Wesenberg

Our non–reflecting boundary conditions at artificial boundaries are based on an analytically exact boundary condition for the hyperbolic equation which describes the evolution of the pressure perturbation. This equation is derived by a linearization of the MHD equations about a background atmosphere, thus assuming that the perturbations at the boundary are sufficiently small and smooth. The boundary condition necessarily includes a *non–local term in time*. However, by using a special approximation this non–local term can be evaluated in a time–stepping manner. Therefore the numerical method stays *local in time*.

The numerical examples illustrate how strongly the structure of the solution is influenced by the choice of the boundary conditions. Moreover, we find that — up to a certain extent — even large perturbations are hardly reflected at the artificial boundaries.

The examples indicate that our transparent boundary conditions give good results and are very cheap with respect to their computational costs.
(Joint work with Andreas Dedner, Dietmar Kröner, Ivan L. Sofronov)

Admissible boundary conditions and stability of boundary-layers for a hyperbolic relaxation system

Wen-An Yong

This talk is concerned with boundary conditions for hyperbolic relaxation systems to have time-asymptotically stable boundary-layers. A new requirement is proposed to characterize a class of boundary conditions for a typical relaxation system. For the corresponding initial-boundary value problems, we prove the global (in time) existence and asymptotic decay of solutions with initial data close to the steady solutions or relaxation boundary-layers.

This is a joint work with Hailiang Liu (UCLA).

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