

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 43/2000

RANDOM MATRICES

29.10. - 04.11. 2000

Organizers: Philippe Biane (Paris), Andreas Knauf (Erlangen), Peter C. Sarnak (Princeton).

Random matrices has been a popular topic among physicists for several decades, since the epoch making work of Wigner. Recently they have attracted more and more interest from mathematicians while relations to many different topics have emerged.

The organizers have tried to prepare a program which would bring together a great number of these aspects. This program reflected also the international character of the interest in random matrices, since people coming from universities of 12 different countries participated.

Talks were given on connections between random matrices and areas as diverse as integrable systems and Painlevé equations, growth models in random environment, the theory of the Riemann zeta and L -functions, quantum chaos and semi-classical limits, C^* -algebras and free probability theory, harmonic analysis on large groups, as well as more classical topics in the theory of random matrices like location of eigenvalues and convergence of empirical distributions. Most of the talks would describe recent results by the speakers, but some of them had more of a survey character, allowing participants to learn about topics they were not familiar with.

Abstracts of talks

- **Adler, Mark:**

- Symmetric Matrix Integrals,
Combinatorics and Zonal Polynomials**

- We compute Laplace transforms of functions in $Gr(m, \mathbb{F}^n)$, $\mathbb{F} = \mathbb{R}, \mathbb{C}$ and H suggested by statistical sampling theory for the sample canonical correlation coefficients in a normal population. These three cases lead to Painlevé transcendents for the case $\mathbb{F} = \mathbb{C}$ or Painlevé type recursion relations. We also compute probability ensembles on $Gr(m, \mathbb{F}^n)$, leading to partial differential equations which in the case in which they become ordinary differential relations lead to either Painlevé equations or Painlevé type recursion relations.

- **Baik, Jinho:**

- Limiting Fluctuations of Polynuclear Growth Models**

- We investigate the limiting distribution functions for various versions of PNG (polynuclear growth) models. PNG is a model for crystal growth in a simplified way. For the droplet initial condition, we map PNG model to random permutations following Spohn and Prähofer, while flat PNG model is mapped to involutions without fixed points. These combinatoric questions are investigated by Deift, Johansson and Baik, and also Rains and Baik, and the relationship to the largest eigenvalues of GUE, GOE random matrices is found. We also introduce external sources for droplet PNG and in a special case, obtained a new limiting distribution whose random matrix interpretation is yet to be found.

- **De Bièvre, Stephan:**

- Exponential Mixing and $|\ln \hbar|$ Time Scales**

- In quantum chaos the goal is to understand the signature of the chaoticity of the underlying classical dynamics on the behaviour of the corresponding quantum system. Typical systems in which those questions have been studied are hyperbolic billiards, negatively curved Riemannian manifolds and - much more simply - hyperbolic automorphisms of the torus. One aspect of the problem is to understand the degree of non-commutativity of the semiclassical ($\hbar \rightarrow 0$) and the $t \rightarrow +\infty$ limits. In a recent paper with F. Bonechi (Commun. Math. Phys. 211, 659-686, 2000) we studied the Wigner and Husimi distributions of coherent states evolved under quantized hyperbolic toral automorphisms and the quantized baker map. We show in particular that, due to the exponential mixing of those maps, those phase space distributions equidistribute on the torus as $\hbar \rightarrow 0$, for times between $\frac{1}{2\gamma} |\ln \hbar|$ and $\frac{1}{\gamma} |\ln \hbar|$.

- **Bohigas, Oriol:**

- Some New Properties of Spectral Fluctuations: Random Matrices, Semiclassics and Riemann Zeros**

- Some spectral properties of Wigner-Dyson random matrices, only marginally considered so far, are studied. Specifically we investigate (nearest-neighbor) spacing autocovariances and the distribution of sums of consecutive eigenvalues. Besides

random matrix theories, and for the sake of comparison, we also derive the corresponding expressions for chaotic systems using the Gutzwiller trace formula. The domain of validity of random matrix theories and of semiclassics is discussed.

The approach is applied to zeros of the Riemann ζ -function. The phenomenon of resurgences is exhibited. Long range spacing autocovariances previously found from the "data" by Odlyzko can be quantitatively explained. The distribution of the fluctuating part of the total energy of "Riemannium", namely the sum of the imaginary part of consecutive zeros, is also derived.

- **Boutet de Monvel, Anne:**

- **On Mesoscopic Spectral Universality in the Wishart Ensemble**

We describe a method to study the eigenvalue distribution of random hermitean (or real symmetric) $N \times N$ matrices in the limit $N \rightarrow \infty$. We consider the Wishart random matrices. Basing on the calculus developed, we study the eigenvalue density $\rho_N = \rho(\Delta_N)$ with $\Delta_N = (\lambda - \delta_N, \lambda + \delta_N)$ in two different limiting transitions.

- When $\delta_N = \mathcal{O}(N^{-1})$ we show that in the limit $N \rightarrow \infty$ the moments of the random variable $\rho(\Delta_N)$ are bounded by a factorial-type expression.
- When $\delta_N = \mathcal{O}(N^{-\alpha})$ $0 < \alpha < 1$, $N \rightarrow \infty$, called mesoscopic regime, we show that the average value $E\rho_N$ converges to the limiting integral eigenvalue distribution of the ensemble. We also prove that the random variables $\gamma_N = (\rho_N - E\rho_N)N^{1-\alpha}$ converge to a Gaussian random variable γ .

We find an explicit expression for the corresponding correlation function. It coincides with that obtained for other random matrix ensembles. This proves the universality of the spectral fluctuations of large random matrices in the mesoscopic regime previously observed by physicists.

- **Degli Esposti, Mirko:**

- **Non Random Spin Models and Random Matrix Theory**

We discuss and investigate the statistical properties of a class of Ising spin model with no-disorder (the so called "sine" model, introduced by Parisi and coll. in '94-'95). We focus our attention to the connection with number theory and its use in the understanding of the Glauber dynamics. We finally discuss the distribution and the structure of metastable (1-flip stable) in this model and also in the corresponding annealed one (random orthogonal model) by using an approximate formula for the average over the ensemble due to Itzykson and Zuber.

- **Deift, Percy:**

- **Fredholm Determinant Identities and the Convergence of Moments for Random Young Tableaux**

We (J. Baik, P.D. and E. Rains) obtain identities between Fredholm determinants of two different kinds of operators, the first kind acting on functions on the unit circle and the second acting on functions on the positive integers. The identities are generalizations of an identity between a Toeplitz determinant and a Fredholm determinant that appeared in the random permutation context. Using these identities, we prove, in particular, the convergence of moments for arbitrary rows of a random Young diagram under Plancherel measure.

- **Dueñez, Eduardo:**

- **Compact Symmetric Spaces and associated Matrix Ensembles**

We describe how to fit the classical compact ensembles into the framework of the theory of Riemannian Symmetric Spaces, as well as how to use this viewpoint to obtain families of matrix ensembles associated to each irreducible compact symmetric space.

Besides the classical compact ensembles, we get others largely falling within the Jacobi family. As a bonus, we sketch a simple proof of the universality of correlations in the bulk and at the edge of the spectrum for classical Jacobi ensembles.

- **Edelman, Alan:**

- **Counting Real Eigenvalues and Zonal Polynomial Formulas**

There is a simple proof that $P(A \text{ has all real eigenvalues}) = 2^{-n(n-1)/4}$ if A is n^2 i.d.d. standard Gaussians. If A is non-symmetric and $A = QRQ^T$ then $(dA) = \Pi(\lambda_i - \lambda_j)(d\Lambda)(dR)(Q^T dQ)$. If A is symmetric and $A = Q\Lambda Q^T$ then $(dA) = \Pi(\lambda_i - \lambda_j)(d\Lambda)(Q^T dQ)$.

Multiplying by the densities $\left(\frac{1}{\sqrt{2\pi}}\right)^{n^2} e^{-\frac{1}{2}\text{tr}(A^T A)}$ and $\left(\frac{1}{\sqrt{\pi}}\right)^{\frac{n(n-1)}{2}} \left(\frac{1}{\sqrt{2N}}\right)^n e^{-\frac{1}{2}\text{tr}(A^2)}$ respectively, integrating, and taking the ratio we get $2^{-n(n-1)/4}$.

This formula has been generalized but still the cleanest formula is waiting to be found. We discuss some progress including new zonal polynomial formulas.

- **Forrester, Peter:**

- **Distributions and Spacing Probabilities in Classical Random Matrix Ensembles**

A definition of matrix ensembles $OE_n(g)$, $UE_n(g)$ and $SE_n(g)$ (ensembles with orthogonal, unitary and symplectic symmetry) is given, with the theme of the talk being relationships between these ensembles. The strongest results reported apply to the situation of superimposing two matrix ensembles of orthogonal symmetry, integrating over every second eigenvalue, and classifying the precise cases for which a matrix ensemble with unitary symmetry results. This and similar results are used, in the spirit of Dyson, to obtain formulas relating the gap probability for ensembles with different symmetry. These formulas are illustrated on the exact formulas for the soft edge gap probability of Tracy and Widom, and used in the derivation of the corresponding result at the band edge. At the end, brief mention is made of the application of the Okamoto τ -function to the calculation of averages generalizing the gap probability in the GUE .

- **Gamburd, Alexander:**

- **Expander Graphs, Random Matrices, and Quantum Chaos**

One of the basic conjectures in Quantum Chaos, formulated by Bohigas, Giannoni, and Schmit, asserts that the eigenvalues of a quantized chaotic Hamiltonian behave like the spectrum of a typical member of the appropriate ensemble of random matrices. The basic conjecture in the theory of expander graphs, formulated by A. Lubotzky, is whether being an expander family for a family of Cayley graphs is a property of the groups alone, independent of the choice of generators (Independence

Conjecture); this conjecture has a number of striking consequences, implying, for example, that a group with the property (T) has the congruence subgroup property. Both conjectures can be viewed as asserting that a deterministically constructed spectrum "generically" behaves like the spectrum of a large random matrix: in the bulk (Quantum Chaos Conjecture) and at the edge of the spectrum (Independence Conjecture). After explaining this approach in the context of the spectra of elements in group rings, we report on our recent work related to these conjectures (A. Gamburd, Spectral gap for infinite index "congruence" subgroups, to appear in Israel Journal of Mathematics), joint work with D. Jakobson and P. Sarnak (Spectra of elements in the group ring of $SU(2)$, Journal of the European Mathematical Society, 1, 1999), and work in progress with D. Rockmore (Level spacings for quantized cats maps).

- **Ge, Liming:**

- **Noncommunicative Geometry and Riemann Zeta-Function**

- We construct functions on $\mathbb{R}_+/\mathbb{N}^*$ as averagings of functions on \mathbb{R}_+ by \mathbb{N}^* . The orthogonal complement of these functions in $L^2(\mathbb{R}_+, d\mu)$ (e.g. $d\mu = \frac{1}{x(1+x^2)}dx$) becomes our Hilbert-Polya-space and the infinitesimal generator $D (= L_x \frac{d}{dx})$ of the regular representation of \mathbb{R}_+ has zeros of Riemann zeta function as its eigenvalues. Certain trace formula is also obtained.

- **Guhr, Thomas:**

- **Gelfand-Tsetlin Coordinates, Harmonic Analysis and Supersymmetric Dyson's Brownian Motion**

- A certain type of harmonic analysis arises naturally in Random Matrix Theory. It leads to spherical functions and to matrix Bessel functions. Using group theory, we find that these functions can be constructed recursively. The recursion is built upon a new type of Gelfand-Tsetlin coordinates. We show that the recursion is also valid for arbitrary Dyson index $\beta > 0$. Thus, it defines a more general class of radial functions which includes the matrix Bessel functions as special cases $\beta = 1, 2, 4$. We also generalize the recursion to superspace and derive explicit results for some kernels of the supersymmetric analogue of Dyson's Brownian Motion. Finally, we comment on various new aspects of representation theory for supergroups.

- **Guionnet, Alice:**

- **Large Deviations for the Spectral Measure of Large Random Matrices and Asymptotics of Itzykson-Zuber Integrals**

- After a brief summary of the previous large deviations principles obtained by Ben Arous and myself, Ben Arous, Zeitouni and Hiai and Petz, I described a concentration of measure type of results obtained by Zeitouni and myself. Then, to investigate the large deviations of the spectral measure of the generalized Wishart matrices XTX^* , I showed how one needs to estimate the asymptotics of Itzykson Zuber integrals. We could obtain such estimates with O. Zeitouni by using large deviation techniques developed by T. Cabanal-Duvillard and myself to prove LDP for the spectral measure of the Hermitian Brownian motion $(T + H_N(t))_{t \in [0,1]}$.

- **Haagerup, Uffe:**

- **Application of Random Matrices to C^* -Algebra Theory**

Voiculescu's random matrix model [Inventiones Math. 1991] has been a very important tool in our understanding of the von Neumann algebra's associated to free groups. Motivated by this random matrix model, Steen Thorbjørnsen and I proved the following:

Theorem: [Documenta Math. 4 (1999), 341-350]

Let $a_1 \dots a_r$ be bounded operators on a Hilbert space H , such that $C^*(a_1 \dots a_r)$ is an exact C^* -algebra, and

$$\sum_{i=1}^r a_i^* a_i = c \mathbb{1}_H \quad , \quad \sum_{i=1}^r a_i a_i^* \leq d \mathbb{1}_H$$

where $d \leq c$. Let moreover $Y_1^{(n)}, \dots, Y_r^{(n)}$ be r random matrices, whose entries are $r \cdot n^2$ independent complex Gaussian random variables each with density $\frac{n}{\pi} \exp(-n|z|^2)$. Put $S_n = \sum_{i=1}^r a_i Y_i^{(n)}$. Then with probability 1, one has

$$\limsup_{n \rightarrow \infty} (\max(\text{spectrum}(S_n^* S_n))) \leq (\sqrt{c} + \sqrt{d})^2$$

$$\liminf_{n \rightarrow \infty} (\min(\text{spectrum}(S_n^* S_n))) \geq (\sqrt{c} - \sqrt{d})^2$$

□

The result (which has applications to K -theory for exact C^* -algebras) can be viewed as a generalization of results of Geman (1980) and Silverstein (1985) about the largest and smallest eigenvalue of a Wishart matrix T^*T .

- **Jakobson, Dmitry:**

- **Some New and Old Results on Eigenfunctions**

We give a survey of some new and old results on eigenfunctions of Laplacians on Riemannian manifolds. We concentrate on results concerning critical points of eigenfunctions, the rate of growth of their L^∞ norms, the relationship between positive and negative parts of (nonconstant) eigenfunctions, and on limits of eigenfunctions on arithmetic hyperbolic manifolds.

- **Johansson, Kurt:**

- **Universality in Certain Hermitian Wigner Matrices**

A hermitian Wigner matrix is a random hermitian matrix where the elements are independent random variables with mean zero, and all have the same variance. It is conjectured that the local statistical properties of such a matrix, as the size of the matrix goes to infinity, e.g. the level spacing distribution, should, under some assumption on the moments, be the same as those of the Gaussian Unitary Ensemble (GUE). This should hold independently of the particular distribution of the matrix elements. To prove this rigorously is an open problem. In the talk I describe how this can be proved for a certain subclass of hermitian Wigner matrices. More precisely it is true for any matrix M of the form $M = W + aV$, $a > 0$ fixed, where W is a Wigner matrix and V an independent GUE matrix. The assumption on W is that the $(6 + \varepsilon)$ -moments of the elements of W are uniformly bounded.

- **Mehta, Madan Lal:**

- **Probability Density of Random Determinants**

- Probability density of an $n \times n$ real, complex or quaternion matrix, whose elements are independent identically distributed Gaussian random variables, can be calculated by using singular values. Results agree with the known ones. They are certain Meijer G -functions.

- **Mezzadri, Francesco:**

- **Quantum Anosov Maps, Symmetries and Random Matrix Ensembles**

- It has been conjectured that, in the semiclassical limit, the spectral statistics of classically chaotic systems are generically determined solely by the symmetries of their dynamics, and that, in that limit, they coincide with those of random matrix theory (RMT). Cat maps are an exception in this respect. The origin of their anomalous spectral statistics is related to the arithmetic properties of the classical and quantum dynamics. When a small non-linear perturbation is introduced, their arithmetic nature is lost and RMT eigenvalue distributions are recovered.

- We show that there exist certain families of perturbations whose spectral statistics (even though still consistent with RMT distribution) do not belong to the universality classes that one would expect from the symmetries of the classical dynamics the maps generate. These anomalies are caused by arithmetical quantum symmetries which do not have a classical limit. They are related to the dynamics generated by associated unperturbed linear maps on a particular rational lattices that form the support of the quantum Wigner functions.

- **van Moerbeke, Pierre:**

- **Survey on Random Matrices and Permutations, in Connection with Integrable Lattices**

- Three matrix integrals, over $U(n)$, $O(n)$ and S_n (the space of symmetric matrices) were discussed. I showed their connection with random permutations, random involutions (longest increasing sequences) and random matrices. Letting these three integrals flow in time in an appropriate way, each of those integrals is very naturally associated with an integrable system, namely

$$U(n) \implies \text{Toeplitz lattice}$$

$$O(n) \implies \text{Toda lattice}$$

$$S_n \implies \text{Pfaff lattice.}$$

- This point of view leads to PDE 's and ODE 's describing the probabilities of the length of the longest increasing sequence and also the probabilities of the spectrum of symmetric matrices, by combining the integrable equations with the Virasoro constraints for the corresponding tau-functions. This is part of a general set-up.

- **Olshanski, Grigori:**

- **Determinantal Point Processes Arising in Harmonic Analysis on Big Groups**

- The problem of harmonic analysis on groups with infinite-dimensional dual space leads to certain point processes on the real line (in other words, to infinite random

point configurations). The correlation functions of these processes have determinantal form. The corresponding correlation kernels (which are expressed through the Gauss hypergeometric function in the case of the group $U(\infty)$) are similar to the kernels arising from spectra of random matrices. The talk is based on a recent joint work with Alexei Borodin.

- **Pastur, Leonid:**

- **On $1/n$ -Expansions in Random Matrix Theory**

- We present results on asymptotic expansion of certain spectral and related characteristics of random real-symmetric and hermitian matrices in the case when their order is large.

- **Robert, Didier:**

- **Large Time Behaviour for Solutions of Schrödinger Equations**

- For smooth Hamiltonians (C^∞ or analytic), we consider approximate solutions for the times dependent Schrödinger equation $i\hbar \frac{\partial \psi}{\partial t} = \hat{H}(t)\psi$, $\psi_{t=0} = f$ in the semi-classical regime, i.e. when the Planck constant \hbar is small, starting at time $t = t_0$ with a Gaussian function picked at a point ζ in the phase space $\mathbb{R}_q^n \times \mathbb{R}_p^n$. These approximate solutions are defined by power series in $\sqrt{\hbar}$. We give in this talk accurate estimates for the coefficients of the expansion and for the remainder terms with control in the Planck constant \hbar , the time t and the order N of truncation of the series. The Ehrenfest time $\text{const.} \times \log \frac{1}{\hbar}$ plays an important role in the formulation of the result.

- **Shlyakhtenko, Dimitri:**

- **Random Band Matrices and Free Probability Theory**

- We show that random matrices $A(N)$ of the form $D(N) + B(N)$, when D is a deterministic diagonal matrix and $B(N) = (g_{ij})$ is a Gaussian random matrix, with $E(|g_{ij}|^2) = \sigma_{ij}$ converge in a certain sense to operator-valued semicircular variables. This establishes the link between such matrices and free probability theory; more precisely, with free independence with amalgamation over a subalgebra.

- **Silverstein, Jack W.:**

- **On the Rate of Convergence of the Empirical Spectral Distribution of Large Dimensional Sample Covariance Matrices**

- The talk consists of a review of the latest information on the eigenvalues of matrices of the form $B_n = (1/N)S_n X_n X_n^* S_n^*$, where X_n is an $n \times N$ matrix containing i.i.d. standardized entries, and S_n is $n \times n$ independent of X_n . This matrix can be viewed as the sample covariance matrix of N samples of the random vector $S_n X_{\cdot 1}$ having population matrix $T_n = S_n S_n^*$. It is assumed that $n \rightarrow \infty$ with $n/N \rightarrow c$ and the empirical distribution function (d.f.) of the eigenvalues of T_n converges to a nonrandom d.f. H . Strong (almost sure) limiting behavior of the eigenvalues of B_n is known. The limiting d.f. F is nonrandom, depends on c and H , and has a continuous derivative. Moreover, for n large, individual eigenvalues behave exactly as one would intuitively expect from the shape of F^l . An example of this can be seen in the two graphs on my web site located at

<http://www.math.ncsu.edu/~jack/>

Here $n = 200$, $N = 4000$, and T_n has 3 distinct eigenvalues: 1, 3, and 10, with respective multiplicities 40, 80, and 80. The limit theorem on the empirical d.f. explains the shape of the histogram. The scatter plot of the eigenvalues of B_n reveals what has also been mathematically verified: the exact number of eigenvalues of B_n appearing in each interval in the support of F , 40 in the leftmost interval and 80 in each of the other two.

Essentially, the only question remaining is how fast F_n , the empirical spectral d.f. of B_n , converges to F . Evidence will be given to suggest that the rate of convergence is $1/n$. (joint work with Z.D. Bai at National University of Singapore).

- **Smilansky, Uzy:**

- **Quantum Graphs and Random Matrix Theory**

- The spectral fluctuations for compact quantum graphs, and the S -matrix fluctuations for scattering problems on graphs were discussed and compared with the predictions of Random Matrix Theory. Using exact trace formulae, a combinatorial link was established.

- **Snaith, Nina:**

- **The Riemann Zeta Function, L-Functions and Random Matrix Theory**

- Motivated by the evidence that the zeros of the Riemann zeta function and families of L -functions possess zeros distributed like the eigenvalues of random matrices, we study the characteristic polynomials $Z(U, \Theta)$ of $N \times N$ unitary matrices, U , in various ensembles of random matrices. Exact expressions for the moments of $|Z|$ are determined, as well as their $N \rightarrow \infty$ asymptotics. These asymptotics are then compared with known results for moments of the Riemann zeta function and L -functions.

- **Soshnikov, Alexander:**

- **Determinantal Random Point Fields**

- Determinantal random point fields (d.r.p.f.) appeared recently in random matrix theory, quantum and statistical mechanics, random growth models, representations of infinite symmetric and unitary groups, combinatorics, etc. In the talk we give an introduction to the theory of determinantal random point fields, prove necessary and sufficient conditions for the existence of such fields with Hermitian correlation kernel, describe all d.r.p.f. with i.i.d. spacings, and study Gaussian fluctuation for linear statistics and ergodic properties of (translation-invariant) d.r.p.f.

- **Speicher, Roland:**

- **Free Probability Theory and Random Matrices**

- I give a general introduction into free probability theory and illuminate certain aspects of the theory by looking closer at free diffusion (the latter is a joint project with P. Biane). In particular, I outline the motivation of Voiculescu to introduce the concept of freeness, define the latter and show how it gives a rule for calculating mixed moments in free random variables. I present two concrete realizations of the free Brownian motion - one by creation and annihilation operators on the full Fock space, the other by Gaussian random matrices - and present the free Fokker-Planck equation.

- **Weidenmüller, Hans A.:**

- **Spectral Properties of the k -Body Embedded Gaussian Ensembles of Random Matrices**

We consider m spinless Fermions in $l > m$ degenerate single-particle levels interacting via a k -body random interaction with Gaussian probability distribution and $k \leq m$ in the limit $l \rightarrow \infty$ (the embedded k -body random ensembles). We address the cases of orthogonal and unitary symmetry. We derive a novel eigenvalue expansion for the second moment of the Hilbert-space matrix elements of these ensembles. Using properties of the expansion and the supersymmetry technique, we show that for $2k > m$ the average spectrum has the shape of a semicircle, and the spectral fluctuations are of Wigner-Dyson type. Using the binary correlation approximation and explicit results for $k = 1$, we show that for $k \ll m \ll l$, the spectral fluctuations are Poissonian. We construct limiting ensembles which are either fully integrable or fully chaotic and show that the k -body random ensembles lie between these two extremes. Combining all these results we find that in the regime $2k \lesssim m$, the embedded ensembles do not possess Wigner-Dyson spectral fluctuation properties.

- **Widom, Harold:**

- **A Growth Model in a Random Environment**

Joint work with J. Gravner and C.A. Tracy. We consider the interface growth given by a height function on the sites of a one-dimensional integer lattice. The update rule is that the height above the site x increases to the height above $x-1$ if the latter height is larger; otherwise the height above x increases by 1 with probability P_x . We assume that the P_x are chosen independently with a common distribution, and that the initial state is such that the origin is far above the other sites. We explicitly identify the limiting shape and prove that, in the pure regime, the fluctuations about that shape are normal. The crucial step in the proof establishes uniform convergence to the F_2 distribution (which arises in the theory of random matrices) in the inhomogeneous version of these dynamics.

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