Mathematisches Forschungsinstitut Oberwolfach

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Nichtnegative Matrizen, M-Matrizen und deren Verallgemeinerungen

26. November – 02. Dezember 2000

The conference was organized by D. Hershkowitz (Haifa), V. Mehrmann (Berlin) and H. Schneider (Madison). The topics of the conference were the theory and applications of nonnegative matrices, M-matrices and their generalizations. There were 31 plenary talks and informal research sessions and an open problem session. The five days were organized along the themes general theory of nonnegative matrices, nonnegative matrices and M-matrices in Markov chains, combinatorial aspects of nonnegative matrices, convergence of iterative methods for nonnegative matrices and spectral theory of graphs. Informal sessions took place on nonnegative matrices in control, inverse eigenvalue problems, Markov chains, and Perron Frobenius theory. The list of abstracts includes those talks in plenary sessions as well as talks and problems given during informal sessions or the open problem session. A special issue of *Linear Algebra And Its Applications* will be devoted to the talks of the conference as well as papers based on problems posed during the conference and their solution.

Abstracts

Perron eigenvector of the Tsetlin matrix

Ravi Bapat

We consider the Markov chain on the set of permutations on n symbols arising from the operation of move-to-position k scheme. Explicit formulas are presented for the stationary distribution of the chain for $k = 1, 2, n-1, n$. Partial results for other values of k are obtained. The formulas and the techniques employ the concept of the Perron complement.

Upper bounds for graphs and matrices

Avi Berman

The following results were discussed

1. Of all the connected graphs with n vertices and k cut vertices, the maximal spectral radius is obtained uniquely at the graph obtained by adding paths of (almost) equal lengths to K_{n-k}.
2. Let G be a cubic graph on n vertices. Then an upper bound for the bipartite density of G, is $\frac{4}{7+\lambda_{\min}(G)}$, where $\lambda_{\min}(G)$ is the smallest eigenvalue of the adjacency matrix of G.
3. The maximum of CP-rank A, taken over all completely positive matrices A of rank r, is $\frac{r(r+1)}{2} - 1$.

The first two results were obtained with Xiao Dong Zhang. The third with Francesco Barioli.

Monomial and canonical subgraphs of reachable positive discrete-time systems

Rafael Bru

Based upon monomial paths and cycles properties of canonical subgraphs are studied in order to characterize whether or not a positive discrete–time linear control system (A, B) is reachable. The criteria is given in terms of the digraph of A, which must be a disjoint union of some specific canonical subgraphs as tree, flowers, palms and non-monomial cycles. The controllability property is studied in the same way. This is joint work with Louis Caccetta and Ventsi G. Rumchev.

Generalised M-matrices, rational matrix equations and Newton's method

Tobias Damm

We recall an extension of the concept of Z- and M-matrices to general spaces. Let X be a real Banach space ordered by a proper cone C. A linear operator $T : X \to X$ is called "positive" if $T(C) \subseteq \mathbb{C}_+$. It is called "resolvent positive" if for all large enough $\alpha \in \mathbb{R}$ their resolvent $(\alpha I - T)^{-1}$ is positive. We call T stable if $\sigma(T) \subseteq \mathbb{C}_-$ Resolvent positive operators (e.g.the Lyapunov operator) play an important role in stability theory. They also arise naturally as the linearizations of quadratic and rational matrix equations occurring in optimal control theory (e.g. Riccati equations). We show, how the properties of resolvent positive operators can be used to solve such equations. More specifically we prove a non-local convergence result for Newton's method, roughly speaking, the following holds: let $R : X \to X$ be concave with resolvent positive derivatives. If then exist $x_0, \hat{x} \in X$ such that $R(\hat{x}) \geq 0$ and the derivative of R at x_0 is stable, then the Newton iterations starting at x_0 converges to the largest solution x_+ of the equation $R(x_+) = 0$.

Spectral Structures of Totally Nonnegative matrices

Shaun Fallat

An n-by-n matrix A is called totally nonnegative if every minor of A is nonnegative. The problem of interest is to characterize all possible Jordan canonical forms (also called Jordan structures) of irreducible totally nonnegative matrices. In this talk we prove that the positive eigenvalues of an irreducible totally nonnegative matrix are always distinct, which can be viewed as a generalization of the classical spectral result for totally positive matrices by Gantmacher and Krein. We also demonstrate key relationships between the number and sizes of the Jordan blocks corresponding to zero. These notions initiate a characterization of all possible Jordan canonical forms for irreducible totally nonnegative matrices.

M-matrices and related classes

Miroslav Fiedler

1. Ultrametric matrices ($a_{ij} \geq \min_k(a_{ik}, a_{kj})\ \forall i, j, k,\ a_{ii} > \max_{k \neq i} a_{ik},\ a_{ij} \geq 0$).
   Every symmetric u.m. is a sum of a nonnegative diagonal matrix and a ,,special" s.u.m. which satisfies $a_{ii} = \max_{h \neq i} a_{ik}$ and can be obtained as follows from a positively edge-weighted undirected connected graph G with vertices $1, \ldots, n$: If i, j distinct and P_{ij} is a path in G joining them, let $\gamma(P_{ij})$ be the minimum of the weights of edges in P_{ij}. Then $a_{ij} = \max \gamma(P_{ij})$, over all such paths. Also, a_{ii} is the maximum of weights on edges incident with i.
2. We study inverse M-matrices with constant row- and column sums in the neighborhood of the matrix J with all ones.
3. We present inequalities completely characterizing the relationship between the diagonal entries of an M-matrix and its inverse and partly generalize them to relationship between square diagonal blocks of M-matrices.

4. We study equilibrated anti-Monge matrices, *i.e.* real $m \times n$ matrices (e_{ij}) satisfying $e_{ik} + e_{jl} \geq e_{il} + e_{jk}\ \forall i, j, k, l,\ i < j, k < l$, as well as $\sum_i e_{ik} = 0, \sum_k e_{ik} = 0$. Main results: Every square non-zero e.a.-Monge matrix has a monotone eigenvector which corresponds to the positive eigenvalue of maximum modulus. The product of e.a.-Monge matrices which can be multiplied is again an e.a.-Monge matrix.

Spectra of operator polynomials and of graph expansions

Karl-Heinz Förster

Let G be the directed weighted graph of a matrix S. If $S = A_0 + \cdots + A_l$, then the expanded graph G_{exp} is defined as follows: The vertices of G are vertices of G_{exp} and of $A_k(i,j) \neq 0$, replace the edge (i,j) in G by $l-k+1$ edges (i.e. add $l-k$ vertices). S.Friedland and H. Schneider (ELA 6 (1999), 2-10) considered the case of $0-1$ matrices S and A_k.

There exist some relations between the nonzero eigenvalues of the adjacency matrix A of G_{exp} and those of the matrix polynomial $L(\lambda) = \lambda^{l+1} - \lambda^l A_l - \cdots - A_0$. Further, A is irreducible iff S is irreducible; therefore it is possible to extend results of the combinatorial spectral theory of nonnegative matrices to matrix polynomials with nonnegative coefficients.

On nonnegative matrix splittings

Andreas Frommer

(Almost) all comparison theorems for two splittings $A = M_1 - N_1 = M_2 - N_2$ with $M_1^{-1} \geq M_2^{-1}$ can be viewed as special cases of the following proposition: let $M_1^{-1}N_1 \geq 0$ and assume that $M_2^{-1}N_2$ admits a nonnegative eigenvector x such that $M_2^{-1}N_2 x = \rho(M_2^{-1}N_2)x$ and $Ax \geq 0$. The talk proves this proposition and shows how the other comparison theorems appear as special cases. We then discuss applications in comparison results for additive and multiplicative Schwarz methods for M-matrices where the amount of overlap is increased.

Intervals of totally nonnegative matrices

Jürgen Garloff

We consider the class of the totally nonnegative (t.n.n.) matrices, i.e., the matrices having all minors nonnegative, and intervals of matrices with respect to the chequerbord ordering. For these intervals we had stated in 1982 the following conjecture: If the left and the right endpoints of the interval are nonsingular and t.n.n. then all matrices taken from the interval are nonsingular and t.n.n. In our talk we survey previous results on settling this conjecture and present a new class of t.n.n. matrices for which the conjecture holds true.

Quasipositive elements in ordered Banach algebras

Gerd Herzog

A matrix $A \in \mathbb{R}^{n\times n}$ is an M-matrix if and only if the mapping $x \mapsto -Ax$ is quasimonotone increasing (qmi) and if the right spectral bound of $-A$ is negative. Here qmi is meant with respect to the natural cone $K = \{x \in \mathbb{R}^n : x_k \geq 0\}$. One possibility of generalizing M-matrices is to consider qmi linear mappings on $\mathbb{R}^n$ with respect to other cones $K \subseteq \mathbb{R}^n$. We will present results on such matrices in the Banach algebra setting and discuss some special cones. Moreover, by means of one-sided estimates it is possible to get informations on the right spectral bound of such matrices.

Bounded invertibility of special collections of matrices

Olga Holtz

The basic question addressed in the talk is the following. Given a collection $\mathcal{A}$ of matrices bounded in some matrix norm and such that the spectrum of each $A \in \mathcal{A}$ lies outside a disk of fixed radius centered at zero, determine whether the collection $\mathcal{A}^{-1}$ is bounded in the same norm. For any norm, the answer is yes for matrices of bounded order, which is an easy consequence of the compactness of the unit ball in a finite-dimensional space. The talk is devoted to collections of matrices of unbounded order and the 'simplest' ∞-norm, the choice motivated by applications. It turns out that the answer is still yes for totally nonnegative Hermitian matrices, but, in general, no for positive definite Hermitian matrices. Two counterexamples are presented, one based on the Hilbert matrix and the other with Hermitian Toeplitz matrices. Finally, an interesting problem of the same type is discussed that arises in spline theory.

On 0–1 and Stochastic Matrices Satisfying a Certain Condition, Preliminary Report

Surrender Jain

We characterize nonnegative matrices A satisfying $(A^T)^p = \sum_{i=1}^k a_i A^{m_i},\ p < m_1 < \cdots < m_k$, and in particular, describe stochastic matrices and 0–1 matrices satisfying the above constraint. The interpretation of $(A^T)^p = A^m$ may be of interest in probability theory, and of the equation, $(A^T)^p = \sum_{i=1}^k A^{m_i}$, may be of interest in the work dealing with incidence matrices of graphs. Our approach provides also a shorter proof of the main theorem in our earlier paper (Bapat - Jain - Prasad, Generalized Power Symmetric Stochastic Matrices, Proc AMS(1999), 1987-94). Our main technique relies on showing that the group inverse $A^\#$ is nonnegative in case index $A = 1$ (or $A^p)^\# \geq 0$, in general), and then invoking the characterization of nonnegative group inverses (see Jain - Kwak - Goel, Trans AMS (1980), 371-385).

A Smorgasboard of Nonnegativity

CHARLES R. JOHNSON

Entry–wise nonnegative matrices have been a a central and, perhaps, the most enduring theme of matrix analysis and its applications for 100 years. The feast will include several different recent results on nonnegative matrices and closely related topics. The Hors d'oeurves (brief reports on several different topics) are

(1) those matrices that occur as arbitrary products of M-matrices and inverse M-matrices
(2) resolution of the nonnegative inverse eigenvalue problem for symmetric nonnegative matrices subordinate to a given bipartite graph
(3) characterization of those totally nonnegative (TN) matrices whose Hadamard product with any TN matrix is TN
(4) precise relations among zero patterns of inverses of principal submatrices of inverse M-matrices
(5) the completion problem for partial TN matrices whose graph is a standardly labelled cycle.

The Hauptspeise is a more detailed discussion of recent progress on determinantal inequalities for M-matrices, inverse M-matrices, positive definite matrices, totally positive matrices and tridiagonal sign-symmetric P-matrices. In three of these cases a complete description is given.

For Nachspeise a brief report is given on the status of a long term project to determine all possible lists of multiplicities for the eigenvalues of real symmetric matrices with a given graph and unrestricted diagonal.

Question: How to compute the operator norm $\|M\|_{\infty,1}$?

MICHAEL KAROW

The operator norm $\|M\|_{\infty,1} := \max\{\|Mx\|_1 \mid x \in \mathbb{C}^n,\ \|x\|_\infty = 1\}$, $M \in \mathbb{C}^{n\times n}$, occurs in the following way in the theory of spectral value sets (Pseudospectra). Let $A \in \mathbb{C}^{n\times n}$, $\rho > 0$, and let $\|\cdot\|$ be an arbitrary norm on $\mathbb{C}^{n\times n}$. The spectral value set $\sigma(A,\rho)$ is defined as the union of all spectra of the matrices $A+\Delta$, where $\Delta \in \mathbb{C}^{n\times n}$ and $\|\Delta\| < \rho$. Let $\|\cdot\|$ be the max-norm, i.e. $\|\Delta\| = \|(\Delta_{jk})_{1\le j,k\le n}\| = \max_{j,k} |\Delta_{j,k}|$. Then the boundary of $\sigma(A,\rho)$ is given by

$$\partial\,\sigma(A,\rho) = \{\, s \in \mathbb{C} \mid \|(s-A)^{-1}\|_{\infty,1} = \rho^{-1} \,\}$$

Thus spectral value sets with respect to the max-norm can be visualized by plotting the level sets of the function $s \mapsto \|(s-A)^{-1}\|_{\infty,1}$.
It is easily seen that the computation of $\|M\|_{\infty,1}$ is a maximization problem on the n-torus, i.e.

$$\|M\|_{\infty,1} = \max\{\, \|Mx\|_1 \mid x \in \mathbb{C}^n,\ |x_1| = |x_2| = \ldots = |x_n| = 1\,\}.$$

If M is a real matrix then the maximum is attained at a real vector $x \in \mathbb{R}^n$. Thus

$$\|M\|_{\infty,1} = \max\{\, \|Mx\|_1 \mid x \in \{-1,1\}^n \,\} \quad \text{for all } M \in \mathbb{R}^{n\times n}.$$

Recently, Jiri Rohn has shown that the latter maximization problem is NP-hard.

Minimizing algebraic connectivity over the class of connected graphs on n vertices with girth g

Steve Kirkland

For a graph G, its Laplacian matrix is $L = D - A$, where A is the adjacency matrix and D is the diagonal matrix of vertex degrees. The algebraic connectivity of G is the second smallest eigenvalue of the singular M-matrix L.

In this talk, we use nonnegative matrix techniques to describe the graph which minimizes algebraic connectivity over the class of connected graphs on n vertices with girth g, under the hypothesis that $n \geq 3g - 1$.

Whose Transition Matrix has large Exponent

Steve Kirkland

Let T be an $n \times n$ primitive stochastic matrix, and suppose that its exponent is at least $\lfloor \frac{(n-1)^2+1}{2} \rfloor + 2$. Using a combinatorial approach, we give a formula for $(I-T)^{\#}$, the group inverse of $(I-T)$. The formula is then used to discuss the stability of the stationary distribution vector corresponding to T. (Joint work with M. Neumann)

Inverse eigenvalue problem for nonnegative matrices

Thomas J. Laffey

Let $\sigma = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ be a list of complex numbers. The NIEP (nonnegative inverse eigenvalue problem) asks for necessary and sufficient conditions for the existence of an (entrywise) nonnegative $n \times n$ matrix A with spectrum σ.

A number of necessary and sufficient conditions are known from which the solution of the problem for $n = 2$ and $n = 3$ have been deduced. The $n = 3$ result is due to Loewy and London and they have also solved the problem in the case $n = 4$ on the assumption that σ is a list of real numbers. The case $n = 4$ and $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0$ has been solved by Reams. In this talk we present joint work with Eleanor Meehan which completes the solution when $n = 4$. An algorithm will be described which implements the solution.

A Pair of Matrices Sharing Common Lyapunov Solutions — A Closer View

Izchak Lewkowicz

Let A, B be a pair of matrices with the same regular inertia. If A and B share common solutions to Lyapunov matrix inclusion, then all matrices in $conv(A, A^{-1}, B, B^{-1})$ have identical regular inertia. This, in turn, implies that both $conv(A, B)$ and $conv(A, B^{-1})$ are non-singular.

In general, neither of the converse implications holds. If however A and B are real 2×2 matrices, both converse implications do hold.

These and additional aspects of the above statements will be discussed. Joint work with Nir Cohen.

CP Rank of Completely Positive Matrices

Raphael Loewy

An $n \times n$ symmetric matrix A is called completely positive (CP) if A can be written as $A = BB^t$ for some $n \times m$ nonnegative (entrywise) matrix B. The smallest m in a factorization of A of this type is called the CP rank of A, and is denoted by $\#(A)$.

Let CP_n denote the set of all $n \times n$ completely positive matrices. It is natural to ask what is the maximum value of $\#(A)$ as A ranges over CP_n. An obvious upper bound for $\#(A)$ is $n(n+1)/2$, and it has been conjectured that this bound can be replaced by $[n^2/4]$, provided that $n \geq 4$. This conjecture is still open, although it has been solved in some special cases. For example, if the graph of A is triangle free; if the graph of A contains no odd cycle of length 5 or more; if the comparison matrix of A is an M-matrix.

It is our purpose to consider $\#(A)$. We state some general remarks about $\#(A)$ for an arbitrary n, and then prove the conjecture for any $A \in CP_5$ which has at least one zero entry.

This is a joint work with Bit-Shun Tam.

Markov chains and aggregation techniques

Ivo Marek

There are very many examples of mathematical models in which central state variables are represented by stationary probability vectors of Markov chains. Our motivation comes from biology [1, 2, 3, 4] and from reliable safety systems engineering [6]. In the latter area of applications the problem consists of computing probabilities of some events with extremely high accuracy: e.g. the computed probability should not exceed 1.10^{-12}. To this purpose very efficient methods of computing stationary probability vectors of stochastic matrices are required.

We propose and analyze aggregation/disaggregation iteration algorithms. We introduce a new concept of Y–convergent iteration process and show its usefulness in our analysis of some classes of aggregation/disaggregation iteration algorithms based on quite general splittings of the original stochastic matrices. We show that the speed of convergence of such processes may be very high: We identify some special cases in which just one or two iterations return the exact solutions. It should also be mentioned that the splittings defining the basic iterations may generally be divergent. This possibility not only does not exclude convergence of the appropriate aggregation/disaggregation algorithms but may offer an optimal rate of convergence.

One the most important tools in our investigations is introducing new inner products that alow us to treat the appropriate Perron eigenprojections as orthogonal projections. Some optimal error estimates are then established.

Our analysis does not exploit any particular features of special stochastic matrices under consideration such as the concept of near complete decomposability of Markov chains etc. Our theory is valid for any stochastic matrix with no restrictions on its elements and/or blocks.

References

[1] Bohl E., Marek I. A model of amplification. J Comp. Appl. Math. 63, 27-47 (1995). as Proc. Modelling '94, Editor J. Nedoma.
[2] Bohl E., Marek I. A nonlinear model involving M–operators. An amplification effect measured in the cascade of vision. J. Appl. Comp. Math 60 (1995).
[3] Bohl E., Marek I. A stability theorem for a class of linear evolution systems. Integral equations and Operator Theory 34,251-269 (1999).
[4] Bohl E., Marek I. Existence and uniqueness results for nonlinear cooperative systems. Submitted to Integral equations and Operator Theory 1999.
[5] Hoerova T., Marek I. Aggregation/disaggregation in analysis and numerics of some Markovian models. Proc. SANM'99, University of West Bohemia, Nečtiny September 1999, 95-112.
[6] Klapka S., Mayer P. Some aspects of railway safety. Proc. SANM'99, University of West Bohemia, Nečtiny September 1999, 95-112.

An application of the Krein-Schaefer spectral Radius Theorem

Ivo Marek

Motivating Problem. To find necessary and sufficient conditions guaranteeing that a difference equation

$$x_{k+N+1} = a_1 x_{k+1} + a_2 x_{k+2} + \cdots + a_N x_{k+N},$$

where $a_j \in \mathbb{R}^1,\ j = 1, \ldots, N$, posesses a strictly monotone solution $\{x_k\}$,i.e. either $x_{k+1} - x_k > 0$, or $x_{k+1} - x_k < 0,\ k \geq M$.

Such and similar problems appear in estimating the topological entropy of some classes of functional spaces. The required necessary and sufficient conditions are derived by applying the Krein-Schaefer spectral radius theorem. Some generalisations of the above problem to abstract operator equations are discussed.

On the fixed points of the interval function $[\boldsymbol{f}]([\boldsymbol{x}]) = [\boldsymbol{A}][\boldsymbol{x}] + [\boldsymbol{b}]$

Günter Mayer

Let $[A]$ be a real $n \times n$ interval matrix and let $[b]$ be a real interval vector with n components. We consider the fixed points $[x]^*$ of the interval function $[f]$ defined by $[f]([x]) = [A][x] + [b]$ under the following points of view: Existence, uniqueness, shape, degeneracy, connection with the solution set $S = \{\, x \mid (I - A)x = b,\ A \in [A],\ b \in [b] \,\}$. The results are based on nonnegative matrices of auxiliary character such as the absolute value $|[A]|$ and the diameter $d([A])$ of $[A]$. In addition we apply parts of the Perron and Frobenius theory for irreducible matrices. We extend a well–known theorem of O. Mayer which assumes $\rho(|[A]|) < 1$ where $\rho(\cdot)$ denotes the spectral radius of a matrix. For $|[A]|$ being irreducible with $\rho(|[A]|) \geq 1$ we were able to study the above–mentioned items exhaustively. For $\rho(|[A]|) < 1$ we present some results on the shape of $[x]^*$ for selected classes of $[A]$ and $[b]$.

Eventually Nonnegative Matrices

Judith J. McDonald

An $n \times n$ matrix A is said to be eventually nonnegative if there exists an integer N such that $A^m \geq 0$ for all $m \geq N$. In this talk we will draw some comparisons between the properties of eventually nonnegative matrices with those of nonnegative matrices. We will look at some results by Zaslavsky and Tam on the Jordan forms of irreducible eventually nonnegative matrices. Lastly, we will examine necessary and sufficient conditions for a matrix in Jordan canonical form to be similar to an eventually nonnegative matrix whose irreducible diagonal blocks satisfy the cyclicity conditions identified by Zaslavsky and Tam, and whose subdiagonal blocks are nonnegative. Joint work with B. Zaslavsky.

The recursive inverse eigenvalue problem for nonnegative matrices

Volker Mehrmann

Given a sequence of nonnegative vectors l_i, r_i of increasing size and nonnegative numbers ρ_i we discuss the problem of finding a matrix A, such that l_i and r_i are left and right eigenvectors of the i–th principal submatrix associated with the eigenvalue ρ_i. We give necessary and sufficient conditions for the existence of a solution as well as an explicit solution formula.

Joint work with M. Arav, D. Hershkowitz and H. Schneider.

Algebraic Theory of Multiplicative and Additive Schwarz Methods

Reinhard Nabben

The convergence of multiplicative and additive Schwarz-type methods for solving linear systems when the coefficient matrix is either a nonsingular M-matrix or a symmetric positive definite matrix is studied using classical and new results from the theory of splittings. The algebraic analysis presented complements the analysis usually done on these methods using Sobolev spaces. The effect on convergence of algorithmic parameters such as the number of subdomains, the amount of overlap, the result of inexact local solves and of coarse grid corrections (global coarse solves) is analyzed in an algebraic setting. Joint work with *Michele Benzi*, *Andreas Frommer* and *Daniel B. Szyld*

A Divide And Conquer Approach To Computing The Mean First Passage Matrix For Markov Chains Via Perron Complement Reductions

Michael Neumann

Let M_T be the mean first passage matrix for an n–state ergodic Markov chain with a transition matrix T. We partition T as a 2×2 block matrix and show how to reconstruct M_T efficiently by using the blocks of T and the mean first passage matrices associated with the nonoverlapping Perron complements of T. We present a schematic diagram showing how this method for computing M_T can be implemented in parallel. We analyze the asymptotic number of multiplication operations necessary to compute M_T by our method, and show that for large size problems, the number of multiplications is reduced by about 1/8, even if the algorithm is implemented in serial. We present five examples of moderate size (orders 20 to 200) and give the reduction in the total number of flops (as opposed to multiplications) in the computation of M_T. The examples show that when the diagonal blocks in the partitioning of T are of equal size, the reduction in the number of flops can be much better than 1/8. Joint work with Stephen J. Kirkland and Jianhong Xu.

On the Roots of Certain Polynomials Arising From Analysis of the Nelder–Mead Simplex Method

Michael Neumann

The analysis of the convergence of the Nelder–Mead method for the unconstrained minimization of the function $f(x_1, \dots, x_n) = x_1^2 + x_2^2 + \cdots + x_n^2$ leads us to study the two parametric family of polynomials of the form $p_n(z) = b - az - \cdots - az^{n-1} + z^n$. We show that provided that $a(\bar{b} - 1)$ is real, it is possible to use the Schur–Cohn criterion in order to determine the configuration of the roots of $p_n(z)$ with respect to the unit circle. Joint work with Lixing Han and Jianhong Xu.

Digraphs With Large Exponent

Dale Olesky

Primitive digraphs on n vertices with exponents at least $\lfloor w_n/2 \rfloor + 2$, where $w_n = (n-1)^2 + 1$, are considered. For $n \geq 3$, all such digraphs containing a Hamilton cycle an characterized; and for $n \geq 6$, all such digraphs containing a cycle of length $n - 1$ are characterized. Each eigenvalue of any stochastic matrix having a digraph in one of these two classes is proved to be geometrically simple.

On some classes of matrices related to P-matrices

Juan Manuel Peña

We study the closure properties of the following class of P-matrices. Matrices with positive row sums and whose off–diagonal elements are bounded above by the row means. These matrices have been called B-matrices and applied to the localization of the eigenvalues of a real matrix. We consider some properties common to B-matrices and other classes of P-matrices, such as symmetric positive definite matrices, nonsingular totally positive matrices, nonsingular M-matrices and strictly diagonally dominant by rows with positive diagonal elements. Classes of matrices containing the mentioned classes of P-matrices are also considered.

Combinatorial vis a vis Analytic Analysis of Generalized Eigenspaces of Nonnegative Matrices Corresponding to the Spectral Radius

Uri Rothblum

The classic approach to extend the Perron Frobenius (P-F) Theorem from irreducible to arbitrary nonnegative matrices is to perturb the zero entries of a nonnegative matric P, compute a normalized P-F eigenvector of the perturbed matrix and let the perturbation parameter approach zero, yielding a (nonunique) semipositive eigenvector corresponding to the spectral radius of P. The specialization of this conclusion to irreducible matrices yield a weaker result than the one available from the P-F Theorem. A combinatorial approach was developed in the to's to overcome this difficulty by constructing a (preferred) basis of the generalized eigenspace of nonnegative matrices corresponding to the spectral. For a matrices with special structure (the basic classes forming a chain) such a basis was recently constructed from coefficients of truncations of fractional power series expansions of the P-F eigenvector of perturbation of the underlying matrix. We conjecture that this approach can be extended to arbitrary nonnegative matrices. (Joint work with Hans Schneider)

Partition polytopes

Uri Rothblum

Consider the problem of partitioning a finite set $N = \{1, \ldots, n\}$ into p parts, where each element in N is associated with a vector $A^i \in R^R$. The objective function associated with a partition $\pi = (\pi_1, \ldots, \pi_p)$ is assumed to depend on the sums of the vectors in each part of π, that is, on the matrix $A_\pi = (\sum_{j \in \pi_1} A^j, \cdots, \sum_{j \in \pi_p} A^j)$. One approach to analyze the problem of optimizing $f(A_\pi)$ over a set of partitions π is to study the extension of f on the partition polytope P^π defined as the convex hull of the A_π's. When $k = 1, A^1 \leq \cdots \leq A^n$, and π consists of all partitions π with prescribed part–sizes. We show that the vertices of the partition polytope correspond to portions whose parts consist of consecutive integers and whose edges have directions in $\{e^i - e^j : i, j, = 1, \ldots, p\}$ (here e^t is the t-unit vector in R^p). The latter ensures that if f is asymmetric Schur convex, f attains a maximum over P^π at a vertex. It follows that a consecutive partition is optimal. Some of this result have been extended to situations where the set of partitions π is determined by lower and upper bounds on the part–sizes.

3 open problems — 1 solution

Siegfried M. Rump

The 3 open problems are

i) an extension of Perron-Frobenius theory to general real and complex matrices,
ii) the conjecture, that the componentwise distance to the nearest singular matrix is inverse proportional to the (componentwise) condition number, and
iii) the conservatism of the circle criterion in control theory.

The first problem is solved by the sign-real and sign-complex spectral radius. The striking similarities of the Perron root in the nonnegative case to these quantities is elaborated. We show that lower bounds are easily obtained by max-min bounds, almost identical to Perron-Frobenius, but upper bounds are NP-hard to calculate. Especially one lower bound depending on the maximum geometric mean of cycles turns out to solve the second problem. The obtained bounds are sharp up to a constant factor. The third problem is No. 30 in "Open Problems in Mathematical Systems and Control Theory" edited by Blondel et al. According to the author of the problem, Alexander Megretski, the solution establishes an unexpected link between control theory, harmonic analysis and combinatorics, and it determines the conservatism of the so-called circle criterion in control theory. This problem is solved by a characterization of the sign-real spectral radius that the Cayley transform of a certain matrix is a P-matrix. We show that our bounds obtained for problem iii) are sharp up to a constant factor. Using this and triggered by the nice algorithm by Michael Tsatsomeros for checking whether a matrix is a P-matrix, also an NP-hard problem, we found during the enjoyable meeting in Oberwolfach new necessary and sufficient criterions for this problem, and also an algorithm for checking P-property which is not a priori exponential.

Conditions for strict inequality in comparisons of spectral radii of nonnegative matrices

Hans Schneider

Let F, T and T' be nonnegative matrices such that $F + T$ is irreducible. Suppose that $F \neq 0$, $T' \leq T$, $T' \neq T$ and $\rho(T) < 1$. Let $Q = F(I - T)^{-1}$ amd $Q' = F(I - T')^{-1}$. At the 1982 Oberwolfach meeting on Linear Algebra we presented a result that implied that $\rho(Q) > 0$. We now amplify this result to prove that $Q' \leq Q$, $Q' \neq Q$ and that $\rho(Q') < \rho(Q)$. We reformulate our in terms of M-splittings of M-matrices and we pose the question whether it extends to regular splittings.

Exponents of nonnegative matrix pairs

Bryan Shader

The notion of the exponent of a nonnegative matrix is generalized to a pair of nonnegative matrices. It is shown that the largest exponent of a pair of n by n nonnegative matrices lies in the interval $[\frac{(n^3-5n^2)}{2}, \frac{3n^3+2n^2-2n}{2}]$. In addition, the exponent of a pair of nonnegative matrices is related to properties of an associated 2-dimensional dynamical system.

Comparison Theorems for the convergence factor of iterative methods for singular Matrices

Daniel B. Szyld

We consider the solution of $Ax = b$ with iterative methods based on splittings $A = M - N$. These are of the form $X_{k+1} = M^{-1}N_{X_k} + M^{-1}b,\ k = 1, \ldots$. As is well known, when A is non-singular, the convergence rate of the iterative method is governed by the spectral radius of the iteration matrix, *i.e.*, by $\rho(M^{-1}N)$. There are several comparison theorems in the literature of the following form: If $A = M_1 - N_1 = M_2 - N_2$ are regular splittings and $A^{-1} \geq 0$, with

$(*)$ either $N_1 \leq N_2$ or $M_1^{-1} \geq M_2^{-1}$

then $\rho(M_1^{-1}N_1) \leq \rho(M_2^{-1}N_2)$.
When A is singular, the convergence rate of the iterative method is governed by $\gamma(M^{-1}N) = \max(\lambda), \lambda \neq 1, \lambda \in \sigma(M^{-1}N)$, where $\sigma(T)$ is the spectrum of T. Since the early 1980s there have been examples is the literature showing that conditions such as (*) do not imply $\gamma(M_1^{-1}N_1) \leq \gamma(M_2^{-1}N_2)$. In this talk, we present comparison theorems for splittings of singular matrices. We use a partial order different than the used one given by $\mathbb{R}^n_+$. With this new partial order, if either $N_1 \leq N_2$ or $M_1^{-1} \geq M_2^{-1}$ and the splittings are regular with respect to the new partial order, then $\gamma(M_1^{-1}N_1) \leq \gamma(M_2^{-1}N_2)$.

Joint work with Ivo Marek.

On a Class of D_k-symmetrizable Matrices

Tomasz Szulc

It is known that for every real square matrix A there exists a nonsingular real symmetric matrix S such that $SA = A'S$, where A' is the transpose of A.

Using the notion of an M-matrix we derive a criterion for A to satisfy the above equality with a diagonal S having signature k. Such A will be called D_k-symmetrizable and the work presents some results on this concept. In particular, we show that a D_k-symmetrizable matrix shares many properties with a real symmetric matrix and that any A is, up to an orthogonal similarity, D_k-symmetrizable. (Joint work with Stawonir Jonek, and Frank Uhlig)

Linear Equations over Cones. Collatz-Wielandt Numbers and Local Perron–Schaefer Conditions

Bit–Shun Tam

Let K be a proper cone in $\mathbb{R}^n$, let A be an $n \times n$ real matrix that satisfies $AK \subseteq K$, let b be a given vector of K, and let λ be a given positive real number. The following two linear equations are considered in this talk: (i) $(\lambda I_n - A)x = b,\ x \in K$, and (ii) $(A - \lambda I_n)x = b,\ x \in K$. We obtain several equivalent conditions for the solvability of the first equation. For the second equation, we give an equivalent condition for its solvability in case when $\lambda > \rho_b(A)$, where $\rho_b(A)$ denotes the local spectral radius of A at b, and we also find a necessary condition when $\lambda = \rho(A)$, where $\rho(A)$ is the spectral radius of A. Then we derive some new results about local spectral radii and Collatz–Wielandt sets (or numbers) associated with a cone-preserving map, and extend a known characterization of M-matrices among Z-matrices in terms of alternating sequences. In the last part of my talk I introduce the local Perron-Schaefer condition, prove several equivalent conditions, and deduce some known intrinsic Perron-Frobenius theorem, discovered by Hans Schneider in the early 80's.

A Recursive Test for P-Matrices and Methods for Constructing P-Matrices

Michael J. Tsatsomeros

P-matrices, i.e., matrices all of whose principal minors are positive, are associated with nonnegative matrix theory in many ways, most notably via the unification theory of various subclasses of P-matrices (e.g., M-matrices, totally nonnegative matrices and positive definite matrices).

How would you test whether a given 'large' matrix is a P-matrix or not?

If no other information is known about the matrix, this is a co-NP-complete problem. So how would you test a 25×25 matrix? An answer is provided based on a new algorithm that reduces the time complexity of such a test from $O(2^n n^3)$ to $O(2^n)$. This is achieved by applying recursively a criterion for P-matrices based on Schur complementation. A Matlab program implementing this algorithm (for complex matrices) is provided, as well as information on how to exploit its parallel nature.

We also discuss two methods for generating P-matrices borrowed from mathematical programming. One of them involves the principal pivot transform and the other is a little bit of a mystery.

Maximal graphs with maximal spectral radius

Pauline van den Driessche

A maximal graph is a connected graph with degree sequence not majorized by the degree sequence of any other graph. Relationships between different sequences of integers that describe maximal graphs are given. A correspondence between maximal graphs and connected graphs with stepwise adjacency matrices shows that, among all connected graphs with n vertices and e edges, the graph with maximal spectral radius is a maximal graph. Such maximal graphs are identified for certain values of n and e.

Joint work with Dale D.Olesky and Aidan Roy.

A Sign Pattern Inertia and Eigenvalue Problem

Pauline van den Driessche

For $n \geq 2$, let $T_n = [t_{ij}]$ be the fixed n-by-n tridiagonal sign pattern with $t_{11} = -$, $t_{nn} = +$, $t_{i,i+1} = +$, $t_{i+1,i} = -$ for $i = 1, \ldots, n-1$, and all other entries equal to zero. A real matrix $A = [a_{ij}] \in T_n$ if $\text{sign}(a_{ij}) = t_{ij}$ for all i, j. In [2], the conjecture was made that T_n allows any inertia (i.e., given any triple of nonnegative integers (n_1, n_2, n_3) that sum to n, there exists $A \in T_n$ so that the inertia of A is (n_1, n_2, n_3)). A stronger conjecture was also made, namely that T_n allows any spectrum (i.e., for any set of n complex numbers with nonreals occurring as complex conjugates, there exists $A \in T_n$ that has this set as spectrum). If true, this second conjecture would imply the truth of the first. By constructing matrices, the inertia result is proved for $n_3 \in \{0, 1, 2, n-1, n\}$ for all values of $n \geq 2$, see [2, section 3]. In [2, Section 4], the spectral conjecture (and thus the inertia conjecture) is proved for each $n \in \{2, \ldots, 7\}$ by constructing a nilpotent matrix and showing that a certain Jacobian is nonzero so that the implicit function theorem applies. More recently [1] this spectral result has been extended to $n = 8$ by this same method and the use of MAPLE. For the remaining values of n_3 (for the inertia) and $n \geq 9$ (for the spectrum), this real inverse eigenvalue problem for the sign pattern T_n remains open. It appears that some new tools are needed, as the methods used above do not easily extend.

References

[1] J.H. Drew and C.R. Johnson, Personal communication.

[2] J.H. Drew, C.R. Johnson, D.D. Olesky and P. van den Driessche, Spectrally arbitrary patterns, Linear Algebra Appl. 308 (2000): 121-137.

Blockwise perturbation theory for nearly uncoupled Markov chains and its application

JUNGONG XUE

Let P be the transition matrix of a nearly uncoupled Markov chain. The states can be grouped into aggregates such that P has the block form $P = (P_{ij})_{i,j=1}^{k}$ where P_{ii} is square and P_{ij} is small for $i \neq j$. Let π^T be the stationary distribution partitioned conformally as $\pi^T = (\pi_1^T, \cdots, \pi_k^t)$. In this paper we bound the relative error in each aggregate distribution π_i^T caused by small relative perturbations in P_{ij}. The error bounds demonstrate that nearly uncoupled Markov chains usually lead to well conditioned problems in the sense of blockwise relative error. As an application, we show that with appropriate stopping criteria, iterative aggregation/disaggregation algorithms will achieve such structured backward errors and compute each aggregate distribution with high relative accuracy.

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