MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Algebraische Gruppen

March 4th – March 10th, 2001

Die diesjährige Tagung über Algebraische Gruppen stand unter der Leitung von M. Brion (Grenoble), J.C. Jantzen (Aarhus) und P. Slodowy (Hamburg). An ihr nahmen 43 Mathematiker aus 12 Ländern teil.

Im Mittelpunkt der 19 ausgewählten Vorträge standen neuere Ergebnisse und Entwicklungen aus den folgenden Bereichen:

- Algebraische Transformationsgruppen, Invariantentheorie, Modulräume
- Darstellungstheorie algebraischer Gruppen
- Darstellungstheorie von affinen Gruppen und verwandte Gebiete
- Schubertvarietäten
- Nilpotente Bahnen

Für eine detailliertere Beschreibung ziehe man die folgenden Vortragsauszüge zu Rate.

Abstracts

On the *n*! conjecture C. PROCESI

This conjecture was formulated by A. Garsia and M. Haiman in order to prove a previous conjecture by Macdonald on the positivity of certain coefficients of special polynomials. The n! conjecture states the following: A partition λ of an integer n gives rise to n pairs of integers which represent the lattice points of λ , say $(i_1, j_1), ..., (i_n, j_n)$. Form a matrix with entry $x_h^{i_k} y_h^{j_k}$ at the (h, k) position and let $D_{\lambda}(x_1, ..., x_n; y_1, ..., y_n)$ be the determinant of this matrix. Let V_{λ} denote the space of polynomials spanned by all the derivatives of all orders of D_{λ} . Then dim $V_{\lambda} = n!$. This has been recently proved by M. Haiman by making a link with Hilbert schemes and a very complex geometric and combinatorial argument. The Macdonald conjecture follows.

Combinatorial aspects of large Schubert varities

T.A. Springer

Let G be an adjoint semi-simple group over an algebraically closed field. Denote by B a Borel subgroup of G. Let X be the "wonderful" compactification of G (introduced by De Concini-Procesi). It is a smooth, projective G-variety. The Borel subgroup $B \times B$ of $G \times G$ acts on X with finitely many orbits. An instance of such an orbit is a double coset BwBin G. Its closure is a large Schubert variety.

In the talk I discussed the description of the set V of $B \times B$ -orbits in X in terms of Weyl group data, and the "Bruhat order" on V (defined by inclusion of orbit closures). This leads to a cellular decomposition of certain orbit closures, in particular of large Schubert varieties.

Let \mathfrak{H} be the Hecke algebra associated to G. The intersection cohomology of the orbit closures can be studied, a la Kazhdan-Lusztig, as an $\mathfrak{H} \otimes \mathfrak{H}$ -module. The main result is that local and global intersection cohomology of the orbit closures are even. Apart from algebraic manipulations, the proof uses the local geometry of X.

Equivariant *K*-Theory and Standard Monomial Theory P. LITTELMANN

Consider the Grothendieck ring $K_T(G/B)$ of the category of T-equivariant coherent sheaves on G/B, where G is semisimple, simply connected algebraic group (defined over an algebraically closed field k), and, as usual, B denotes a Borel subgroup and T a maximal torus in B. By the results of Kostant and Kumar, $K_T(G/B)$ is a free module over $\mathbb{Z}[\Delta]$ with basis given by the classes $[\mathfrak{O}_w]$ of the structure sheaves of the Schubert varieties $X_w \subset G/B, w \in W$. Here W denotes the Weyl group, Δ the weight lattice and $\mathbb{Z}[\Delta]$ the group ring over Δ , or, equivalently, the representation ring of T.

For $\lambda \in \Delta$ let \mathfrak{L}_{λ} be the corresponding line bundle on G/B. The product $[\mathfrak{L}_{\lambda}]$. $[\mathfrak{D}_w]$ has an expression in terms of the basis, i.e. there exist virtual characters $a_{w,\tau}^{\lambda} \in \mathbb{Z}[\Delta]$ such that $[\mathfrak{L}_{\lambda}]$. $[\mathfrak{O}_w] = \sum a_{w,\tau}^{\lambda}[\mathfrak{O}_{\tau}]$, and it is natural to ask for combinatorial formulas as well as representation theoretic interpretations of these characters.

Using standard monomial theory (SMT), we provide, for a dominant weight λ , a filtration \mathfrak{F} of $\mathfrak{L}_{\lambda} \otimes_{\mathfrak{O}_{G/B}} \mathfrak{O}_{X(w)}$, where $\mathfrak{F} = \mathfrak{F}^{i}_{i \in I}$, such that the subquotients $\mathfrak{F}^{i}/\mathfrak{F}^{i-1}$ are isomorphic to some \mathfrak{O}_{τ} , twisted by a *B*-character μ . The filtration is indeed *B*-equivariant.

The \mathfrak{O}_{τ} and the corresponding characters μ can be combinatorially calculated using the path model of L-S-paths, so one obtains in this way an "effective" version of the combinatorial formula of Pittie and Ram. The same construction applies also to other cases, for example the diagonal $G/B \subset G/B \times G/B$. Some indication was given that these filtrations should provide a K-theoretic construction of SMT.

De Rham complex of formal loop spaces E. VASSEROT

For any smooth k-scheme X (chark = 0) we define an ind-scheme of ind-infinite type L(X) representing the functor

 $SCH \longrightarrow SET, S \mapsto \operatorname{Hom}((S, \mathfrak{O}_S((t))^N), X),$

where, for any ring R,

$$R((t))^{N} = \{a(t) = \sum a_{i}t^{i} \in R((t)) | a_{i} \in rad(R) \text{ for all } i < 0\}.$$

L(X) is a nilpotent extension of the jet scheme $L^0(X)$. We prove that the Schechtmann-Malikov-Vaintraub chiral De Rham complex on X is isomorphic, as a sheaf of complexes of vertex algebras, to the De Rham complex of the $D_{L(X)}$ -module of distributions on L(X) supported on $L^0(X)$.

Central, tilting, and anti-spherical sheaves on affine flag varieties R. BEZRUKAVNIKOV

Let G be a simple algebraic group over an algebraically closed field k and \hat{G} the formal loop group (so $\hat{G}(k) = G(k((t)))$). We study the category P of perverse sheaves on the affine flag variety $Fl = \hat{G}/I$ (where I is the Iwahori subgroup), constant along I-orbits (Schubert cells), and its derived category $\mathfrak{D} = D^b(P)$. We also study certain quotient categories $\mathfrak{D}^{asp}, \mathfrak{D}^{asp,l}, \mathfrak{D}^{asp,r}$ (of anti-spherical, resp. left anti-spherical, resp. right antispherical) sheaves as an obvious categorical analogue of the anti-spherical subalgebra \mathfrak{H}_{asp} in the affine Hecke algebra and bimodules $\mathfrak{H}_{asp}, \mathfrak{H}, \mathfrak{H}, \mathfrak{H}_{asp}$ respectively (here $\mathfrak{H}_{asp} = \delta_{asp}$. \mathfrak{H} . δ_{asp} where $\delta_{asp} = \sum_{w \in W_f} (-1)^{l(w)} q^{l(w)} T_w$ is the q-Weyl antisymmetrizer). We describe $\mathfrak{D}^{asp}, \mathfrak{D}^{asp,l}, \mathfrak{D}^{asp,r}, \mathfrak{D}$ in terms of the Langlands dual group, more precisely, we construct equivalences of categories

$$\begin{array}{lll} \mathfrak{D}^{asp} &\simeq \ \mathfrak{D}^{b}(Coh^{G^{L}}(\mathfrak{N} \times \mathfrak{h}^{0})),\\ \mathfrak{D}^{asp,l} &\simeq \ \mathfrak{D}^{b}(Coh^{G^{L}}(\tilde{\mathfrak{N}} \times \mathfrak{h}^{0})),\\ \mathfrak{D}^{asp,r} &\simeq \ \mathfrak{D}^{b}(Coh^{G^{L}}(\tilde{\mathfrak{N}}))\\ \mathfrak{D} &\simeq \ \mathfrak{D}^{b}(Coh^{G^{L}}(St')), \end{array}$$

where G^L is the Langlands dual dual group, $\mathfrak{N} \subset G^L$ is the set of unipotent elements, $\mathfrak{h}^0 = \mathfrak{h} \times_{\mathfrak{h}/W} \{0\}$ where \mathfrak{h} is the Cartan algebra of G^L , $\tilde{\mathfrak{N}} \longrightarrow \mathfrak{N}$, $\tilde{\mathfrak{g}} \longrightarrow \mathfrak{g}^L$ are Grothendieck-Springer maps, and $\tilde{\mathfrak{N}}' = \tilde{\mathfrak{g}} \times_{\mathfrak{g}^L} \mathfrak{N}$, $St' = \tilde{\mathfrak{g}} \times_{\mathfrak{g}^L} \tilde{\mathfrak{N}}$. These results are direct geometric analogues of a realisation of the affine Hecke algebra \mathfrak{H} as the Grothendieck group of $Coh^G(St)$ due to Kazhdan-Lusztig and Ginzburg; they are also a manifestation of the geometric Langlands conjecture in the Beilinson–Drinfeld formulation. The proof is based on the construction of "central sheaves" due to Gaitsgory, Beilinson and Kottwitz.

The intersection cohomology of Drinfeld's compactification A. BRAVERMAN

Let G be a reductive group such that [G, G] is simply connected. Let $P \subset G$ be a parabolic subgroup. For a smooth projective curve X we can consider the moduli spaces Bun_G and Bun_P of G (resp. P) bundles on X. We have the natural map $q_P : Bun_P \longrightarrow Bun_G$. This map is not proper. Drinfeld introduced a relative compactification of this map which we denote by $\overline{Bun_P}$. This stack is usually singular.

In joint work with M. Finkelberg, D. Gaitsgory and I. Mirkovic we compute the intersection cohomology sheaf on \overline{Bun}_P . This allows us to give a geometric interpretation of the periodic parabolic polynomials introduced by Lusztig in his paper "Periodic W–graphs".

The class of the structure sheaf of a Schubert variety in equivariant K-theory W. GRAHAM

Let G be a complex semisimple group, $B \,\subset\, G$ a Borel, and $T \,\subset\, B$ a maximal torus, $X = G/B, X_w = \overline{BwB}$. There are classes $[X_w]$ in equivariant cohomology $H_T^*(X)$ and $[\mathfrak{O}_w]$ in equivariant K-theory $K_T(X)$. The torus T acts with fixed points $x.B_{x\in W}$ on X (W =Weyl group) and the pull-back maps $\bigoplus_{x\in W} i_x^* : H_T^*(X) \longrightarrow \bigoplus H_T^*(xB) = \bigoplus H_T^*(pt)$ and $\bigoplus_{x\in W} i_x^* : K_T(X) \longrightarrow \bigoplus K_T(pt) = \bigoplus R(T)$ (the representation ring of T) are injective. Because of this, some problems on X can be reduced to questions about the finite set $\{xB\}$. In this talk I discuss combinatorial formulas (due to various authors) for the pullbacks $i_x^*[X_w] \in H_T^*(pt)$ and $i_x^*[\mathfrak{O}_w] \in R(T)$. I also discuss a proof of a character formula of Kumar for multiplicities on the tangent cone (a proof previously obtained by Bressler) and a generalization of the K-theory injectivity to the case where X is any complex filtrable nonsingular T-variety.

Gröbner bases, Hilbert polynomial and multiplicity for Schubert varieties V. LAKSHMIBAI

(This is a report on joint work with V. Kreiman.) For Schubert varieties in the Grassmannian, we give a closed formula for the multiplicity at singular points; further, this formula is described in terms of the root system. Let $G = SL_n(k)$, $T = \{$ diagonal matrices in $G\}$, $B = \{$ upper triangular matrices in $G\}$, $W = S_n$, the Weyl group of G. Let R be the root system, $R^+ = \{$ positive roots relative to $B\}$, and $R^- = \{$ negative roots relative to $B\}$. For a parabolic subgroup P, let R_P denote the associated system of roots. Identifying the Grassmann variety with G/P, for a suitable maximal parabolic subgroup, we have that the *T*-fixed points (for the canonical *G*-action given by left multiplication) in *G/P* are $C_{\tau} := \tau P, \tau \in W/W_P$. Let $X(\tau) (= \overline{B.C_{\tau}})$ be the Schubert variety associated to τ . Consider a subset *J* of $R^- \backslash R_P^-$ with the following property: if $\alpha_1, ..., \alpha_r$ is a sequence in *J* such that $s_{\alpha_1} > ... > s_{\alpha_r}$, then $\tau \geq s_{\alpha_1}...s_{\alpha_r} \pmod{W_P}$. Let Z_{τ} denote the collection of subsets of $R^- \backslash R_P^-$ as above. Then $mult_{C_{id}}X(\tau)$, the multiplicity of $X(\tau)$ at C_{id} , equals card $\{J \in Z_{\tau} | \text{card } J \text{ is maximum } \}$. We have similar formulas for the multiplicity at other *T*-fixed points C_{θ} in $X(\tau)$. Also, the Hilbert polynomial to $X(\tau)$ at C_{id} is given by $P(m) = \sum_{\{J, \text{as above}\}} \binom{(m-1)}{\#J-1}$. We also give a Gröbner basis for the tangent cone to $X(\tau)$ at C_{id} .

Some representations of affine Hecke algebras at roots of unity G. LEHRER

Let G be a connected reductive group over \mathbb{C} , with simply connected derived group. Let $\mathfrak{H}^{a}(q), q \in \mathbb{C}$, be the (affine) Hecke algebra associated with the affine Weyl group $W^{a} =$ $W \ltimes R$ (W the Weyl group of G, R the root lattice). We normalise so that for generators. $(T_i - q)(T_i + q^{-1}) = 0$. The (extended) affine Hecke algebra $\tilde{\mathfrak{H}}^a(q)$ associated with the extended affine Weyl group $\tilde{W^a} = W \ltimes X$ (X = X(T), the weight lattice) satisfies $\tilde{\mathfrak{H}^a}(q) \cong$ $\mathbb{C}[X/R] \otimes_{twist} \mathfrak{H}^{a}(q)$. Its "generic" representation theory was determined by Kazhdan and Lusztig. Identifying $\tilde{\mathfrak{H}}^{a}(q)$ as a specialization of a convolution algebra of coherent sheaves on the Steinberg variety $Z = \{(N, B_1, B_2) \in \mathfrak{g}_{nil} \times \mathfrak{B} \times \mathfrak{B}, N \in \text{Lie } B_1 \cap \text{Lie } B_2\}$ (\mathfrak{g}_{nil} the nilpotent cone in Lie $G = \mathfrak{g}, \mathfrak{B}$ the flag variety), yields "standard modules" $M_{s,N,\rho} =$ $H_*(\mathfrak{B}^s_N)_{\rho}$, where $()_{\rho}$ denotes ρ -isotypic part, $\mathfrak{B}^s_N = \{B \in \mathfrak{B} | s \in B, N \in \text{Lie } B\}, \rho \in \mathbb{N}$ $\{Z_G(s) \cap Z_G(N)/(Z_G(s) \cap Z_G(N))^\circ\}$ and $Ads(N) = q^2N$. Kazhdan and Lusztig show that when q is not a root of unity, $M_{s,N,\rho}$ has an irrducible "top quotient" $L_{s,N,\rho}$ and that these provide a complete list of distinct irreducible $\mathfrak{H}^{a}(q)$ -modules (finite-dimensional). When q is a root of unity Leclerc, Lascoux, Thibon and Ariki have proved "Kazhdan-Lusztig"type results in type A. Grojnowski has sketched similar results in general. We give an explicit analysis of some representations for all q. All work is joint with J.J. Graham. Let $G = SL_n(\mathbb{C})$. Write $\tilde{\mathfrak{H}}^a(q) = \tilde{\mathfrak{H}}^a_n$; we may take $\rho = 1$ above. We deal with pairs (s, N)with N 2-step nilpotent,

$$N_k = \operatorname{diag}(J_{n-k}, J_k)$$
 (J_k Jordan block of size k)

and $s = \text{diag}(a_1, a_1q^{-2}, ..., a_2, a_2q^{-2}, ...), a_1^{n-k}a_2^k = q^{n(n-1)-2k(n-k)}, k \le n-k$. Write s (above) as $s(a_1, a_2)$; $\{\mathfrak{P}\}^+ = \text{all such pairs modulo } (\sim)$, where $(s(a_1, a_2), N_{n/2})(\sim)(s(a_2, a_1), N_{n/2})$. Theorem 1:

For all $(s, N) \in \mathfrak{P}^+$ there exists a cell module $W_{s,N}$ for $\tilde{\mathfrak{H}}^a{}_n$ such that

(i) $W_{s,N} = M_{s,N}$ in the Grothendieck group.

(ii) For all (s, N) there exists a unique (s', N') and an invariant pairing $\phi_{s,N} : W_{s,N} \times W_{s',N'} \longrightarrow \mathbb{C}$ such that if $R_{s,N} \subset W_{s,N}$ is the radical of $\phi_{s,N}$, the module $W_{s,N}/R_{s,N}$ is irreducible or 0.

(iii)

$$\{(s,N)|L_{s,N} \neq 0\} = \begin{cases} \mathfrak{P}^+ & \text{if } q^2 \neq -1\\ \mathfrak{P}^+ \setminus \{(s(\xi,(-1)^{n/2-1}\xi),N_{n/2})\}(\xi^n = 1) & \text{if } q^2 = -1. \end{cases}$$

Define a partial order \preccurlyeq on \mathfrak{P}^+ by: $(s(a_1, a_2), N_k) \prec^{\circ} (s(a'_1, a'_2), N_{k'})$ if (a) k' < k, (b) there exists a sign $\epsilon = +, -$ such that $a_1 a_2^{-1} = q^{t+\epsilon t'} (t = n - 2k, t' = n - 2k')$,

(c) $a'_1 = a_1 q^{(1-\epsilon)(k-k')}$,

(d) $a'_2 = a_2 q^{-(1-\epsilon)(k-k')}$. Then \preccurlyeq is the partial order generated by \prec° .

Theorem 2:

In the Grothendieck ring

(i)
$$W_{s,N} = \sum_{(s,N) \preccurlyeq (s',N')} L_{s',N'},$$

(ii) $L_{s,N} = \sum_{(s,N) \preccurlyeq (s',N')} (-1)^{l(s,N,s',N')} W_{s',N'}$ (character formula).

Corollary:

The $W_{s,N}$ may have arbitrarily long composition series.

The method is to translate results concerning affine Temperley–Lieb algebras.

Kazhdan-Lusztig polynomials and indecomposable bimodules W. SOERGEL

Let (W, S) be a Coxeter system and \mathfrak{H} its Hecke algebra,

$$\mathfrak{H} = \bigoplus_{x \in W} \mathbb{Z}[v, v^{-1}]T_x,$$
$$T_s^2 = v^{-2}T_e + (v^{-2} - 1)T_s \text{ for all } s \in S,$$
$$T_x T_y = T_{xy} \text{ if } l(x) + l(y) = l(xy).$$

Let V be a complex reflection representation, R = R(V) the ring of regular functions on V. We consider the category $R - \operatorname{mod}_{\mathbb{Z}}^{bf} - R$ of all \mathbb{Z} -graded R-bimodules, finitely generated as left modules and as right modules, with the same action of \mathbb{C} from both sides. Let $< R - \operatorname{mod}_{\mathbb{Z}}^{bf} - R >$ be its split Grothendieck group, so just split short exact sequences give a relation and the classes of indecomposables (as opposed to irreducibles) form a basis. We consider the usual grading shift [1] and $R^s \subset R$ the invariants of $s \in S$.

Theorem:

1) There exists a ring homomorphism

$$\mathfrak{E}:\mathfrak{H} o < R ext{-mod}_{\mathbb{Z}}^{bf} - R >, \otimes_R \ v\mapsto < R[1]>$$

such that $(T_s + 1)$ maps to $\langle R \otimes_{R^s} R \rangle$.

2) A left inverse is given by $B \mapsto \sum \overline{rk}_{\mathbb{Z}} \operatorname{Hom}_{R-R}(B, R_x) T_x$ for R_x the regular functions on $\{(xv, v) | v \in V\} \subset V \times V$ and $\overline{rk}_{\mathbb{Z}} \in \mathbb{Z}[v, v^{-1}]$ a suitable notion of rank for a free *R*-module, \mathbb{Z} -graded.

We also give a classification of indecomposables in the image of \mathfrak{E} by $W \times \mathbb{Z}$ and conjecture, that they should correspond to the KL-selfdual elements under \mathfrak{E} . This would give positivity of KL-polynomials.

Simple singularities and subregular representations D. RUMYNIN

Let $\mathbb{K} = \overline{\mathbb{K}}$ a field of char p > n, $v(x) \in \mathbb{K}[x]$ a polynomial of degree n, $t(v) = \mathbb{K} < a, b, h > /(ha - ah - a, hb - bh + b, ba - v(h), ab - v(h^{-1}), a^p, b^p, h^p - h >$ the corresponding reduced Hodges algebra. Let $\mathfrak{g} = sl_n(\mathbb{K}) = \text{Lie } G$, Λ the weight lattice, $Z(U(\mathfrak{g})) = Z_p \otimes_{Z_p \cap Z_{\mathbb{K}}} Z_{\mathbb{K}}$ where $Z_p = \mathbb{K}[x_i^p - x_i^{[p]}](x_i \text{ a basis of } \mathfrak{g}) \cong \mathfrak{O}(\mathfrak{g}^{*(1)}),$ $Z_{HC} = U(\mathfrak{g})^G \cong S(\mathfrak{g})^G \cong S(\mathfrak{h})^W \cong \mathfrak{O}(\mathfrak{h}^*/W)$. Let $\chi \in \mathfrak{g}^*$ be nilpotent, $\lambda \in \Lambda$. Define $U_{\chi,\lambda} = U(\mathfrak{g}) \otimes_{\mathbb{Z}(U(\mathfrak{g}))} \mathbb{K}(\chi,\lambda)$, and let Γ_n be the double A_{n-1}^1 -quiver, i.e. the quiver constructed from the Dynkin diagram of A_{n-1}^1 type replacing each edge by two arrows pointing in opposite directions. Define the no-cycle algebra $NC_{\mathbb{K}}(n) = \mathbb{K}\Gamma_n/$ (oriented cycles).

Theorem (Gordon, Rumynin):

Let v(x) have all roots in GF(p) and no multiple ones. Let χ be subregular nilpotent. Let $\lambda + \rho = \sum r_i \omega_i, r_i > 0, \sum r_i < p, \overline{r_i} = r_i \mod p$. Then

1) We have Morita equivalences (~): $NC_{\mathbb{K}}(n) \sim t(v) \sim U_{\chi,\lambda}$.

2) Let $v_{\lambda}(-x) = \prod_{i=0}^{n-1} (x - (\overline{r}_1 + ... + \overline{r}_i))$. Then $U_{\chi,\lambda} \cong M_{p^{(n^2 - n - 2)/2}}(t(v))$.

Corollary:

 $U_{\chi,\lambda}, t(v)$ are tame algebras.

New facts about baby Verma modules are derived.

Quivers, reflexive polytopes and torus actions L. HILLE

Quivers and torus actions: Let Q be a finite connected quiver, without oriented cycles. We consider representations M of Q with dim $M_q = 1$ for all $q \in Q_0$. The isomorphism classes are in natural bijection with the orbits of $H = \prod_{q \in Q_0} k^*$ on $R(Q) = \bigoplus_{a \in Q_1} k$, where Q_0 are the vertices, Q_1 are the arrows and an arrow $a \in Q_1$ starts in s(a) and terminates in t(a). A linearisation of the action of H on R(Q) corresponds to a character $\chi_{\Theta}(g) = \prod_{q \in Q_0} g_q^{-\Theta(q)}$ with $\sum_{q \in Q_0} \Theta(q) = 0$. The function $\Theta : Q_0 \longrightarrow \mathbb{R}$ with $\sum_{q \in Q_0} \Theta(q) = 0$ is called a weight. We are interested in the quotients $R/\!\!/H$, χ_{Θ} in the sense of Mumford's Geometric Invariant Theory.

Quivers and Polytopes (joint work with K. Altmann): Let $\Delta(\Theta)$ be the polytope consisting of all flows $\epsilon : Q_1 \longrightarrow \mathbb{R}$ with $\epsilon(\alpha) \ge 0$ and $\Theta(q) = \sum_{s(\alpha)=q} \epsilon(\alpha) - \sum_{t(\alpha)=q} \epsilon(\alpha)$. It turns out that the toric variety $\mathbb{X}(\Delta(\Theta))$ coincides with the quotient $R/\!\!/H$, χ_{Θ} . For the particular weight Θ^c defined by $\Theta^c(q) = \operatorname{card}\{\alpha|s(\alpha) = q\} - \operatorname{card}\{\beta|t(\beta) = q\}$, the polytope $\Delta(\Theta^c)$ is reflexive in the sense of Batyrev.

Applications:

a) Let G/P be a Grassmannian, $G = GL_n, P \subset G$ a parabolic subgroup. We identify Lie $P_u = \mathfrak{p}_u$ with a big Schubert cell in G/P. We define a quiver $Q(\mathfrak{p}_u)$ so that $\mathfrak{p}_u/T, \chi = R(Q(\mathfrak{p}_u))/\!\!/H, \chi_{\Theta}$ for $T \subset GL_n$ the maximal torus.

b) It is known by Gonciulea /Lakshmibai that G/P degenerates to a toric variety. Using results of Batyrev/Cioncau-Fontanine /Kim/van Straten and Altmann/van Straten we construct a quiver Q(G/P) so that G/P degenerates to $\mathbb{X}(\Delta(\Theta^c))$ for the quiver Q(G/P).

Semiinvariants of quiver representations

H. Derksen

(joint work with J. Weyman and Weyman/Schofield) Let $Q = (Q_0, Q_1)$ be a quiver where Q_0 is the set of vertices and Q_1 is the set of arrows. $Rep(Q, \alpha) = \bigoplus_{a \in Q_1} \operatorname{Hom}(K^{\alpha(ta)}, K^{\alpha(ha)})$ is the representation space on which $GL(\alpha) = \prod GL(\alpha(x))$ acts. We study the space $SI(Q; \alpha)_{\sigma}$ of semiinvariants where $\alpha : Q_0 \longrightarrow \mathbb{N}$ is a dimension vector and $\sigma : Q_0 \longrightarrow \mathbb{Z}$ is a weight. For a particular quiver $T_{p,q}^r$ and choices of α and σ we have dim $SI(Q, \alpha) = c_{\lambda,\mu}^{\nu}$, the Littlewood-Richardson coefficients. Some consequences are:

1) If $j(\lambda)$ is the number of jumps in a partition λ , then the condition $1/(j(\lambda) + 1) + 1/(j(\mu) + 1) + 1/(j(\nu) + 1) > 1$ implies $c_{\lambda,\mu}^{\nu} \leq 1$.

2) Explicit inequalities can be found for the set $\{(\lambda, \mu, \nu) | c_{\lambda,\mu}^{\nu} > 0, \lambda, \mu, \nu \leq n \text{ parts}\}$. In particular, we have saturation: if $c_{M\lambda,M\mu}^{M\nu} > 0$ for some M, then $c_{\lambda,\mu}^{\nu} > 0$.

3) We have descriptions of faces of arbitrary codimension of the cone $\{(\lambda, \mu, \nu) | c_{\lambda,\mu}^{\nu} > 0, \lambda, \mu, \nu \leq n \text{ parts}\}.$

Twisted Verma modules

N. LAURITZEN

I reported on joint work with H.H. Andersen. Let G be a complex semisimple group along with the usual data $T \subset B, B^-, R = R(T, G), W, \mathfrak{g}, \mathfrak{b}, \dots$

Kempf realized in the mid–70's the BGG–resolution of a finite dimensional \mathfrak{g} –representation as the dual of the global Grothendieck–Cousin complex

$$0 \longrightarrow \Gamma(X, \mathfrak{L}(\lambda)) \longrightarrow H^{\circ}_{C(e)}(X, \mathfrak{L}(\lambda)) \longrightarrow \bigoplus_{l(w)=1} H^{1}_{C(w)}(X, \mathfrak{L}(\lambda)) \longrightarrow \dots, C(w) = BwB^{-}/B^{-},$$

where λ is a dominant weight on $X = G/B^-$. The crux is here that $DH_{C(w)}^{l(w)}(X, \mathfrak{L}(\lambda)) \cong M(w,\lambda)$, where $M(\mu) = \mathfrak{U}(\mathfrak{g}) \otimes_{\mathfrak{U}(\mathfrak{b})} \mu$ is the Verma module and D the duality on the BGGcategory \mathfrak{O} . Lesser known are the facts that the local cohomology group $H_{C(e)}^{\circ}(X, \mathfrak{L}(\lambda))$ with support in the big cell is the dual Verma module $DM(\lambda)$ and the local cohomology group $H_{C(w_0)}^{l(w_0)}(X, \mathfrak{L}(\lambda))$ with support in the point is the Verma module $M(w_0, \lambda)$ for arbitrary weight λ .

The main theorem of the talk was the fact that the \mathfrak{g} -structure of arbitrary local cohomology groups $H^i_{C(w)}(X, \mathfrak{L}(\lambda))$ can be described in terms of principal series Harish-Chandra modules in the appropriate block \mathfrak{O}_{λ} . More precisely, let λ be antidominant (and integral !), $y \in W$, then

$$M(x,y) \cong H^{l(x)}_{C(x)}(X, \mathfrak{L}(y,\lambda))$$

where M(x, y) denotes the principal series module in \mathfrak{O}_{λ} (coming from the Bernstein, Gelfand, Joseph, Enright equivalence) given by $x, y \in W$.

Non-commutative deformations of special transverse slices

A. Premet

Let \mathfrak{g} be a complex simple Lie algebra, e a nilpotent element in \mathfrak{g} , and (e, h, f) an sl_2 -triple in \mathfrak{g} containing e. Let $\mathfrak{g} = \bigoplus_{i \in \mathbb{Z}} \mathfrak{g}(i)$ be the \mathbb{Z} -grading associated with $\operatorname{ad}(h)$. Define $\psi_e : \mathfrak{g}(-1) \times \mathfrak{g}(-1) \longrightarrow \mathbb{C}$ by setting $\psi_e(x, y) = \kappa(e, [x, y])$, where κ is the Killing form. Let $z'_1, ..., z'_s, z_1, ..., z_s$ be a Witt basis of $\mathfrak{g}(-1)$ relative to ψ_e . Define $\mathfrak{m}_e = \mathfrak{g}(-1)^\circ \oplus \sum_{i \leq -2} \mathfrak{g}(i)$ where $\mathfrak{g}(-1)^\circ$ is the span of $z'_1, ..., z'_s$. By sl_2 -theory, \mathfrak{m}_e is a Lie subalgebra of \mathfrak{g} of dimension $1/2(\dim\Omega_e)$, where Ω_e is the adjoint orbit of e. Define $\chi \in \mathfrak{g}^*$ by $\chi(x) = \kappa(e, x)$, for all x. Let N_{χ} be the ideal of codimension 1 in $\mathfrak{U}(\mathfrak{m}_{\chi})$ generated by all $x - \chi(x)$ with $x \in \mathfrak{m}_{\chi}$. Define $\tilde{Q}_{\chi} := \mathfrak{U}(\mathfrak{g}) \otimes_{\mathfrak{U}(\mathfrak{m}_{\chi})} \mathbb{C}_{\chi}$, an induced \mathfrak{g} -module (here $\mathbb{C}_{\chi} = \mathfrak{U}(\mathfrak{m}_{\chi})/N_{\chi}$)). Let $\tilde{H}_{\chi} = (End_{\mathfrak{g}}\tilde{Q}_{\chi})^{op}$, an associative algebra over \mathbb{C} . Let Z denote the centre of $\mathfrak{U}(\mathfrak{g})$, and $\eta : Z \longrightarrow \mathbb{C}$ an algebra homomorphism. Define $\tilde{H}_{\chi,\eta} := \tilde{H}_{\chi} \otimes_Z \mathbb{C}_{\eta}$ where $\mathbb{C}_{\eta} = Z/\operatorname{Ker} \eta$. Let $f_1, ..., f_\ell$ be basic invariants in $S(\mathfrak{g}^*)^G$. Let $S_e = e + \operatorname{Ker} \operatorname{ad}(f)$, the special transverse slice to Ω_e . Let $\psi_e : S_e \longrightarrow \mathbb{A}^\ell$ denote the restriction of the adjoint quotient $x \in \mathfrak{g} \mapsto (f_1(x), ..., f_\ell(x)) \in \mathbb{A}^\ell$ to S_e . Let $\psi_e^{-1}(0)$ denote the zero-fibre of ψ_e .

Theorem:

(1) The variety $\psi_e^{-1}(0)$ is an irreducible, normal complete intersection of dimension $r - \ell$ where $r = \dim(\operatorname{Ker} \operatorname{ad}(e))$.

(2) The associative algebra $\tilde{H}_{\chi,\eta}$ has a natural filtration such that $\operatorname{gr}(\tilde{H}_{\chi,\eta}) \cong \mathbb{C}[\psi_e^{-1}(0)]$ as graded algebras (the algebra $\mathbb{C}[\psi_e^{-1}(0)]$ is viewed with its grading defined by Slodowy).

On the equivariant symplectic geometry of cotangent bundles

E. VINBERG

Let a reductive algebraic group G act on an irreducible quasiaffine algebraic variety X. A horosphere in X is an orbit of a maximal unipotent subgroup of G or, equivalently, an image under the action of G of a fixed maximal unipotent subgroup. A set of horospheres "in general position" has a natural structure of an irreducible algebraic variety. Denote it by Hor(X).

Main Theorem:

There is a canonical G-equivariant symplectic rational Galois covering $f: T^*Hor(X) \longrightarrow T^*X$. (A morphism $\phi: M \longrightarrow N$ of irreducible algebraic varieties is a rational (Galois) covering if it is dominant and the field $\mathbb{C}(M)$ is a finite (Galois) extension of $\phi^*\mathbb{C}(N)$.)

The Galois group of the above covering coincides with the "little Weyl group" introduced by F. Knop in another way. In particular, in the case of a symmetric space X, it is nothing else than the usual Weyl group of a symmetric space. The G-variety Hor(X) is in some respects simpler than X. In particular, if X is a spherical affine homogeneous space of G, there is a canonical G-equivariant isomorphism between Hor(X) and the contraction of X in the sense of V. Popov.

Special functions for multiplicity free spaces F. KNOP

Starting from a multiplicity free space (i.e. a finite dimensional representation of a reductive group such that a Borel subgroup has a dense orbit) we considered a combinatorial structure consisting of a finite dimensional \mathbb{C} -vector space \mathfrak{A} , a reflection group $W \subset GL(\mathfrak{A})$, a basis $\eta_1, \ldots, \eta_r \in \mathfrak{A}$ and a linear function $l : \mathfrak{A} \longrightarrow \mathbb{C}$. These are subject to a certain set of axioms. Then we defined an operator $L \in \operatorname{End}_{\mathbb{C}}\mathbb{C}[\mathfrak{A}]^W$ as follows:

For $\tau \in <\Lambda >_{\mathbb{Z}}$ put

$$f_{\tau}(z) := \left(\prod_{\omega \in \Phi} [\omega(z) - k_{\omega} || \omega(\tau)]\right) / \left(\prod_{\alpha \in \Delta} [\alpha(z) || \alpha(\tau)]\right)$$

where $\omega_i(\eta_j) = \delta_{i,j}$ (dual basis), $\Phi = \bigcup_i W \omega_i$, $\Delta = \text{roots for } W$,

$$[z||a] := \begin{cases} z(z-1)...(z-a+1) & if \quad a \in \mathbb{N}, \\ = 1 & \text{otherwise} \end{cases}$$

Then

$$L := \sum_{\eta \in \Phi, l(\eta) = 1} f_{\eta}(z) T_{\eta}, \ L^{-} := \sum f_{\eta}(-z) T_{-\eta},$$

where $T_{\eta}(z) := f(z - \eta)$.

Theorem:

We have $(\text{ad } L)^n(h) = 0$ whenever $\deg h < n, h \in \mathbb{C}[\mathfrak{A}]^W$.

Now, let $\mathfrak{A} \subset \operatorname{End}_{\mathbb{C}}\mathbb{C}[\mathfrak{A}]^W$ be the subalgebra generated by $L, \mathbb{C}[\mathfrak{A}]^W, L^-$.

Theorem:

 $S = \langle L, 2l, L^{-} \rangle$ is as a Lie algebra isomorphic to sl_2 .

Theorem:

For every line $s \in \mathbb{P}(S)$ there is a subalgebra $C_s \subset \mathfrak{A}$ with $C_s \cap S = s$ and $C_s \cong \mathbb{C}[\mathfrak{A}]^W$ (non-canonical).

The algebra \mathfrak{A} has the module $M = \mathbb{C}[\mathfrak{A}]^W$. Then one can show that $C_{\langle l-L \rangle}$ acts semisimply on M. For general $s \in \mathbb{P}(S)$ one has $M = C_s.1$. Thus, one can define polynomials $p^{(s)} \in C_s$ such that $p^{(s)}.1$ is an eigenvector of C_{l-L} . This construction generalizes Laguerre and Meixner polynomials. As an application we proved new integral identities of Mehta-Macdonald type.

Index of seaweed subalgebras and centralisers of nilpotent elements D. PANYUSHEV

Let \mathfrak{g} be any Lie algebra. The index of \mathfrak{g} , ind \mathfrak{g} , is the minimal dimension of the centralisers \mathfrak{g}_{ξ} , where ξ runs over \mathfrak{g}^* and \mathfrak{g} acts on \mathfrak{g}^* via the coadjoint representation. The Lie algebras of index zero are said to be Frobenius. The notion of index has interesting applications to constructing solutions of Yang-Baxter equations and to "characteristic p" business.

In my talk, I've considered the index for seaweed subalgebras and for centralisers of nilpotent elements. Seaweed algebras were introduced, for $\mathfrak{g} = sl_n$, by Dergachev and Kirillov(s.). It can be adapted to arbitrary semisimple, or even reductive Lie algebras as follows. Let $\mathfrak{p}, \mathfrak{p}'$ be two parabolic subalgebras of \mathfrak{g} . Suppose $\mathfrak{p} + \mathfrak{p}' = \mathfrak{g}$. Then $\mathfrak{p} \cap \mathfrak{p}'$ is called a seaweed algebra in \mathfrak{g} .

Conjecture: 1) $\operatorname{ind}(\mathfrak{p} \cap \mathfrak{p}') \leq \operatorname{rk} \mathfrak{g}$, 2) $\operatorname{ind}(\mathfrak{p} \cap \mathfrak{p}') = \operatorname{rk} \mathfrak{g}$ iff $\mathfrak{p} \cap \mathfrak{p}'$ is a Levi in both \mathfrak{p} and \mathfrak{p}' .

This conjecture is true for $\mathfrak{g} = A_n, C_n$ and G_2 . There are also some evidences in favour of this conjecture in case \mathfrak{g} is of types B, D.

There is a conjecture that $\operatorname{ind} \mathfrak{g}(x) = \operatorname{ind} \mathfrak{g}$ for any $x \in \mathfrak{g}$. As usual, it is sufficient to prove this for nilpotent elements.

Theorem 1: 1) Suppose $(\operatorname{ad}(x))^4 = 0$. Then $\operatorname{rk} \mathfrak{z}(x) = \operatorname{rk} \mathfrak{g}$. 2) $\mathfrak{z}(x)$ is abelian, iff x is regular.

Theorem 2:

Let x be a regular nilpotent element. Then $\mathfrak{n}_{\mathfrak{g}}(\mathfrak{z}(x))$ is Frobenius.

The Harder-Narasimhan filtration for non-semistable principal G-bundles V.B. MEHTA

(This is a report on joint work with S. Subramanian.) If V is a non-semistable vector bundle on a variety X, then one can construct the H.-N.-filtration of V which is unique and hence descends under separable extensions. It is also infinitesimally unique, so it descends under purely inseparable field extensions. In this work we discuss the H.-N.-filtration, or the H.-N.-parabolic, for non-semistable G-bundles, where G is any semisimple group. Two definitions had been given, one by Ramanathan and one by Atiyah–Bott, but without any proofs. Kai Behrend proved the existence and uniqueness for the canonical parabolic, in any characteristic. He conjectured that this reduction is also infinitesimally unique. In this talk we prove Behrend's conjecture in two cases, first by restricting the prime and, in the second case, by putting some restrictions on the variety. We also prove that all three definitions of the canonical parabolic given by Ramanathan, Atiyah–Bott and Behrend coincide under the above restrictions. The main idea of the proof is that the tensor product of two semistable vector bundles in char p is again semistable when the sum of their ranks is less than p. We also identify the "elementary vector bundles" of Behrend with subbundles of the adjoint representation of the Levi of the parabolic.

Edited by (Stephan Mohrdieck)

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