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4-dimensional Manifolds

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The talks at the conference presented a broad view of current research on 4-manifolds. Particular emphasis was placed on the interrelation of three- and four-dimensional geometry.

One focal point thus consisted of various Floer-type invariants: In a series of three talks, P. Ozsvath and Z. Szabo presented 3- and 4-manifold invariants constructed by the use of holomorphic disks in configuration spaces. Other aspects, including Manolescu's stable homotopy picture, were outlined in various other research reports.

Low dimensional symplectic and contact geometry was another focal point of the conference with talks for example on open book decompositions and on tight contact structures.

Other talks addressed the interplay of 4-manifold theory with complex geometry, stable homotopy and L^2-homology theory.

Abstracts

Legendrian Knots
John Etnyre

In this talk I will discuss what is currently known concerning the classification of Legendrian knots in tight contact three manifolds. Specifically, I will describe the classification of Legendrian torus and figure eight knots and then explain some general "structure theorems." The first structure theorem describes how the classification of reducible Legendrian knots can be understood in terms of the classification of its prime summands. It is a little surprising that using this theorem one can construct many Legendrian knots that are not determined by their classical invariants. The second structure theorem says that under certain circumstances Legendrian knots in a cabled knot type can be classified if a Legendrian classification for the uncabled knot type is known. Finally, I will indicate how to use convex surface theory to prove these results.

Knotted curves in complex surfaces
Sergey Finashin

Theorem: For any $d \geq 5$, there exist infinitely many surfaces $F_i \subset \mathbb{C}P^2$ homoeomorphic to and realizing the same homology class as non-singular curves of degree d, with $\pi_1(\mathbb{C}P^2 \backslash F_i) \cong \mathbb{Z}/d$, but pairwise non-diffeomorphic as pairs $(\mathbb{C}P^2, F_i)$. This answers a long-standing problem.

The surfaces F_i are obtained from an algebraic curve A by a version of rim-surgery of Fintushel and Stern, applied along a 0-framed annulus membrane on A. Non-existence of a diffeomorphism is detected by Seiberg-Witten invariants in the covering spaces $X_i \to \mathbb{C}P^2$ branched along F_i.

This theorem can be generalized and applied to simply connected algebraic surfaces X with a curve A admitting a degeneration to an irreducible curve with a unique singularity of the type X_g (four non-singular pairwise transverse branches), with $A \cdot A > 16$.

Indeed, one can define rim-basic classes α_i, for an embedded surface $F \hookrightarrow X$, lying in $H_1(F) \cong H^1(F)$. These classes (1) distinguish diffeomorhpism types of embeddings $F \hookrightarrow X$, (2) provide adjunction inequalities for membranes M on F: $-\chi(M) \geq M \cdot M + |\partial M \cdot \alpha|$. The basic classes α_i can be viewed as differences of the relative basic classes in $X \backslash N(F)$, where $N(F)$ is a tubular neighborhood of F.

Seiberg-Witten Floer theory for rational homology 3-spheres
Kim Froyshov

We defined irreducible and equivariant Seiberg-Witten Floer homology groups of oriented rational homology 3-spheres with a spin-c structure. While the equivariant group is an invariant of the oriented spin-c manifold, the irreducible Floer group depends in addition on the chamber of the metric and perturbation form, which we label by a rational number. This additional parameter runs through a coset mod $\mathbb{Z}$.

The irreducible Floer groups come with some extra structure (u-map, and interaction with the reducible critical point), which we use to construct a group homomorphism from the spin-c rational homology cobordism group onto the rational numbers. This invariant, which has analogues both in instanton Floer theory (of integral homology 3-spheres) and in the Ozsvath-Szabo Floer groups, constrains the 4-manifolds that the homology 3-sphere in question can bound. As an application we showed that if a simple lens space $L(p,1)$, $p > 1$, bounds a smooth rational homology 4-ball then $p = 4$ (and $L(4,1)$ does bound). P. Kronheimer pointed out that the same argument proves that $L(p,1)$ is not rational homology cobordant to any integral homology 3-sphere unless $p = 4$.

Spin 4-manifolds with $b_1 > 0$

Mikio Furuta

In this talk I tried to explain that one should consider the cup product on H^1 to formulate the geography of 4-dimensional closed spin manifolds. In a joint work with Y. Kametani, H. Matsue and N. Minami, we showed that a spin 4-manifold with the same rational cohomology ring as $K3\#K3\#T^4$ does not have any $S^2 \times S^2$ summand. A proof of this statement is given by using a kind of Pin bordism group. An extension of the above statement to spin 4-manifolds with larger b_2 or b_1 was discussed, which is a joint work with Kametani.

Contact structures, linkings and fibrations

Emmanuel Giroux

The goal of the talk was to show that isotopy classes of contact structures on a closed three-manifold are in one-to-one correspondence with "stable isotopy" classes of open book decompositions. We also described some applications of this result, and in particular explained how it applies to answer a question of Harer on fibered links. Finally, we presented the high-dimensional generalization of the main theorem.

On the Finiteness of Tight Contact Structures

Ko Honda

In this joint work (in progress) with Vincent Colin and Emmanuel Giroux, we improve on finiteness results of Eliashberg and Kronheimer-Mrowka to prove the following theorems:

1. On every closed 3-manifold there exist finitely many homotopy classes of 2-plane fields which carry tight contact structures.
2. On a closed, atoroidal 3-manifold there exist finitely many isotopy classes of tight contact structures.

Bounded cohomology of mapping class groups

Dieter Kotschick

We discussed the comparison map from the second bounded cohomology in the sense of Gromov to ordinary cohomology. In the case of mapping class groups of closed oriented surfaces of genus at least three, this comparison map is surjective but not injective. The image is generated by Meyer's signature cocycle, and estimating its Gromov-Thurston norm is equivalent to proving signature bounds for surface bundles over surfaces. We proved such bounds using symplectic geometry, specifically the existence of symplectic submanifolds Poincaré dual to the canonical class of any symplectic structure, see [2].

In joint work with H. Endo [1], this argument was extended to Lefschetz fibrations, proving that the commutator lengths of powers of Dehn twists along separating simple closed curves have linear growth. This shows that mapping class groups are not uniformly perfect, and that the above comparison map is not injective.

At the end of the talk we discussed briefly some related work of Korkmaz, Bestvina-Fujiwara, Farb-Lubotzky-Minsky, and of Burger-Monod.

References

1. H. Endo and D. Kotschick, *Bounded cohomology and non-uniform perfection of mapping class groups*, Invent. math. **144** (2001), 169–175.
2. D. Kotschick, *Signatures, monopoles and mapping class groups*, Math. Research Letters **5** (1998), 227–234.

Floer homotopy type of rational homology 3-spheres, after Manolescu

Peter Kronheimer

This talk presented the work of Ciprian Manolescu, which provides a new viewpoint on Seiberg-Witten Floer homology of rational homology 3-spheres. For the sake of exposition, the talk dealt with the case of integral homology 3-spheres: in this case, for each Y^3, Manolescu constructs a space $\sigma(Y)$ (actually an S^1-equivariant suspension spectrum), whose ordinary equivariant cohomology groups should recover the Floer cohomology of Y. Two properties of σ are: (i) if $Y = S^3$, then $\sigma(Y)$ is the sphere spectrum; and (ii) if X is a 4-dimensional cobordism with spin-c structure and with $\partial X = Y_2 - Y_1$, then there is an associated map $\sigma(X)$ from $\sigma(Y_1)$ to $\sigma(Y_2)$. In the case that X is obtained from a closed manifold by removing two balls, $\sigma(X)$ recovers the Furuta-Bauer invariant of the 4-manifold.

The construction of $\sigma(Y)$ uses finite-dimensional approximation and Conley index theory, two ideas which were part of the early development of Floer homology and the related work of Conley and Zehnder.

S^1-valued Morse Theory and Seiberg-Witten Invariants of 3-Manifolds

Thomas Mark

We describe a technique introduced by Donaldson for computing the Seiberg-Witten invariants of a closed 3-manifold Y having $b_1 > 0$ by using a circle-valued Morse function ϕ and a topological field theory arising from that situation. We show that the Seiberg-Witten invariant is equal to the intersection number of a pair of submanifolds in a product of symmetric powers of a slice for ϕ, and use this fact to interpret the Seiberg-Witten invariant as a count of certain gradient flow lines for ϕ. In particular we verify a conjecture of Hutchings and Lee, leading to an alternative proof of a theorem of Meng and Taubes relating Seiberg-Witten and torsion invariants of 3-manifolds. Finally, this result suggests the possibility of defining a Floer-type homology theory based on data given by the Morse function, along the lines of recent work of Ozsváth and Szabó.

Amoebas in dimension 4: complex curves and surfaces

Grigory Mikhalkin

Given an algebraic variety in the complex torus $(\mathbb{C}^*)^n$ one may consider its amoeba which is the image of the variety under the map $(z_1, \ldots, z_n) \mapsto (\log|z_1|, \ldots, \log|z_n|)$. It turns out that the shape of the amoeba carries a lot of information about the topology of the variety. In the talk I reviewed the current state of knowledge on amoebas as well as their applications to topology of real and complex varieties, in particular to complex surfaces.

Holomorphic disks and invariants for three- and four-manifolds

Peter Ozsvath

(joint work with Zoltan Szabo)

I discussed joint work with Zoltan Szabo in which we defined invariants for three- and four-manifolds. In my talk I focused mainly on the three-dimensional case. Start with a Heegaard diagram for the three-manifold Y, and with splitting surface Σ and attaching circles $\{\alpha_1, ..., \alpha_g\}$ and $\{\beta_1, ..., \beta_g\}$ for the two handlebodies (where g is the genus of the surface). The invariants are then defined by using a suitable variant of Lagrangian Floer homology in the g-fold symmetric product of Σ, relative to the totally real subspaces $\alpha_1 \times ... \times \alpha_g$ and $\beta_1 \times ... \times \beta_g$.

Dual decompositions of 4-manifolds

Frank Quinn

A *dual decompositon* is a description as a union of two submanifolds, each of which is a handlebody with handles of index ≤ 2. This is somewhat like a Heegard decomposition of a 3-manifold, but the purposes and flavor of the theory are very different than in 3 dimensions, so we avoid use of the same term. Three areas of application have been identified so far:

1) As a general setting for certain types of knot and link invariants. The links are the attaching maps of 2-handles in one part of the decomposition, and invariants come from algebraic topology of the other part. A perturbative version of the Cochran-Teichner-Orr invariants arises this way.

2) As a setting for topological field theories in 3+1 dimensions. For some of these, state spaces of 3-manifolds are defined in terms of Heegard decompositions. Dual decompositions of 4-manifolds seem to be the right notion of bordism for Heegard decompositions. Specifically, half of a Heegard decomposition is a 3d thickening of a graph. Consider joining two of these by a relative 4d thickening of a 2-complex. Map this to the interval, with the two 3d thickenings mapping to the endpoints. Then time slices can be arranged to generically be thickenings of graphs (slices through the 2-complex spine). This presents the bordism as a 1-parameter family (with singularities) of thickenings of 1-complexes.

3) As a setting for a weak subsitute for standard handlebody theory. Generally handles can be manipulated to follow algebra, particularly the chain complex of the universal cover, but this fails for 2-handlebodies. This means we cannot manipulate handle structures on half of a dual decomposition if the decomposition is fixed. The weak subsitute is to get the

desired sort of handle structure by controlled changes of the decomposition. The ambient manifold is still held fixed.

The lecture described implementations of the third item.

An invariant of homology cobordism

Nikolai Saveliev

We prove the following theorem. Let M be an integral homology 3-sphere which is homology cobordant to a Seifert fibered one. If the Rohlin invariant of M is non-trivial then no finite multiple of M can bound a smooth homology ball.

This is proved by defining an integral lift of the Rohlin invariant on the class of plumbed homology spheres and proving its homology cobordism invariance for (at least) Seifert fibered homology spheres and their multiples. Such a lift is defined as the Fukumoto-Furuta invariant for a special class of plumbed 4-orbifolds which are introduced in the talk.

Equivalently, this lift can be defined as the Neumann-Siebenmann invariant, and also as an equivariant Casson invariant (joint with Olivier Collin) and as a particular linear combination of the Floer Betti numbers.

Symplectic homotopy projective planes

Ivan Smith

It is a standard conjecture that a symplectic four-manifold homotopy equivalent to the complex projective plane is in fact symplectomorphic to $\mathbb{CP}^2$. A partial result was proved by Taubes: if $X \simeq \mathbb{CP}^2$ and satisfies $K_X \cdot [\omega] < 0$ then X is standard. The crux of Taubes' proof is the construction of a symplectic sphere in X, which he deduces from "Seiberg-Witten = Gromov". Following joint work with Simon Donaldson, we explained a new proof of Taubes' theorem, building on the existence of Lefschetz pencils. A pencil gives rise to a family of symmetric products over $\mathbb{CP}^1$ and there is an associated Gromov invariant counting J-holomorphic sections of this new fibration. We proved (i) that the non-vanishing of this invariant is enough to construct symplectic surfaces and (ii) the invariant is often computable - for instance in the case at hand - by using the Abel-Jacobi map to reduce to families of tori and projective spaces. At the end we described a (speculative!) parallel programme to prove that if X satisfies $c_1^2 = 3c_2$ and $K_X \cdot [\omega] > 0$ then it cannot be a homotopy $\mathbb{CP}^2$. The strategy is to construct flat connexions on X from ones on the fibres of a Lefschetz fibration.

Holomorphic disks and invariants for smooth 4-manifolds

Zoltan Szabo

(joint work with Peter Ozsvath)

This talk presented a smooth four-manifold invariant $F : Spin^c(X) \longrightarrow \mathbb{Z}$, where X is a smooth, closed oriented 4-manifold, with $b_2^+(X) > 1$. F is contructed by using an appropriate handle decomposition of X together with a pairing on the recently defined

Floer-homology groups $HF^+(Y,s)$, $HF^-(Y,s)$ where s is a spin-c structure over the three-manifold Y. While HF^+ and HF^- are defined by using a version of Lagrangian Floer-homology in the g-fold symmetric product $Sym^g(\Sigma_g)$ and counting holomorphic disks with totally real boundary conditions, the pairing is given by counting holomorphic triangles.

As one of the application of this theory we proved that if Y is a three-manifold that corresponds to a plumbing tree G, where the coefficient k_i of any vertex v_i satisfies that $k_i \geq d(v_i)$ (where $d(v_i)$ denotes the degree of v_i in G), then it is impossible to embed Y to a symplectic four-manifold $X = X_1 \cup_Y X_2$ so that $b_2^+(X_1) > 0$, $b_2^+(X_2) > 0$. As another application by using surgery exact sequences for the Floer homologies, we gave a combinatorial formula for $HF^-(Y,s)$ for a certain class of plumbed three-manifolds that included for example all Seifert fibered three-manifolds with $b_1 = 0$. These are all joint work with Peter Ozsvath.

Knot concordance and von Neumann η-invariants

Peter Teichner

(joint work with Tim Cochran and Kent Orr)

Using gropes or Whitney towers in the 4-ball, we have defined a filtration of the knot concordance group given by so called n-solvable knots. The previously known concordance invariants, including those by Casson and Gordon, only detect whether the knot bounds a symmetric grope of height 4, i.e. whether the knot is 4-solvable. We have previously constructed infinitely many examples of 4-solvable knots which are not 5-solvable, showing that the previously known concordance invariants cannot detect the concordance group.

Using the Cheeger-Gromov estimate for the von Neumann η-invariant, we can now show that there are n-solvable knots which are not $(n+1)$-solvable for all n. The idea is to start with a seed fibred ribbon knot of genus 2, and then infect it with a lot of trefoil knots which are tied into curves in the n-th term of the derived series of the ribbon knot group.

Homotopy K3's with several symplectic structures

Stefano Vidussi

Let M be a smooth 4-manifold. Whenever M admits a symplectic form, it is interesting to discuss the uniqueness of this form up to homotopies in the space of symplectic forms and diffeomorphisms of M. The canonical class of the symplectic form, lying in $H^2(M,\mathbb{Z})$, can help detect inequivalent structures. McMullen and Taubes provided the first example of a manifold (a homotopy $E(4)$) admitting 2 inequivalent structures, following the idea (that can be implicitely found in some Thurston's papers) of using a link exterior admitting inequivalent fibrations, and constructing from this, via Fintushel-Stern link surgery, a smooth, simply connected 4-manifold admitting inequivalent symplectic structures, distinguished by their canonical class.

In my talk I have shown how, building a suitable family of 2-component fibered links, it is possible to produce a family of homotopy $E(2)$'s whose n-th member admits n inequivalent symplectic structures. It is necessary, in this approach, to exploit some accidental diffeomorphism of the link surgery construction, which allow to present the same smooth manifold as result of different symplectic fiber sums, inducing different symplectic structures. These are finally distinguished by their canonical classes, that can be studied by analysing the Seiberg-Witten polynomial of the 4-manifold.

Edited by Birgit Schmidt and Markus Szymik

Participants

Prof. Dr. Selman Akbulut
akbulut@math.msu.edu
Department of Mathematics
Michigan State University
East Lansing, MI 48824-1027
USA

Prof. Dr. Stefan Alois Bauer
bauer@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Prof. Dr. Hans U. Boden
boden@icarus.mcmaster.ca
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West
Hamilton, Ont. L8S 4K1
CANADA

Dr. Christian Bohr
bohr@rz.mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
80333 München

Prof. Dr. Tim D. Cochran
cochran@rice.edu
Dept. of Mathematics
Rice University
P.O. Box 1892
Houston, TX 77005-1892
USA

Prof. Dr. Olivier Collin
oc210@dpmms.cam.ac.uk
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane
GB-Cambridge, CB2 1SB

Prof. Dr. John Etnyre
etnyre@math.stanford.edu
Department of Mathematics
Stanford University
Stanford, CA 94305-2125
USA

Prof. Dr. Paul M. N. Feehan
feehan@maths.tcd.ie
Dept. of Mathematics
Trinity College
University of Dublin
Dublin 2
IRELAND

Prof. Dr. Sergey Finashin
serge@metu.edu.tr
Dept. of Mathematics
Middle East Technical University
06531 Ankara
TURKEY

Prof. Dr. Ronald A. Fintushel
ronfint@math.msu.edu
Department of Mathematics
Michigan State University
East Lansing, MI 48824-1027
USA

Dr. Kim A. Froyshov
froyshov@math.harvard.edu
Dept. of Mathematics
Harvard University
1 Oxford Street
Cambridge, MA 02138
USA

Prof. Dr. Mikio Furuta
furuta@ms.u-tokyo.ac.jp
Graduate School of
Mathematical Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Prof. Dr. Emmanuel Giroux
giroux@umpa.ens-lyon.fr
Mathematiques
Ecole Normale Superieure de Lyon
46, Allee d'Italie
F-69364 Lyon Cedex 07

Prof. Dr. Ian Hambleton
ian@icarus.math.mcmaster.ca
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West
Hamilton, Ont. L8S 4K1
CANADA

Prof. Dr. Chris Herald
herald@unr.edu
Department of Mathematics
University of Nevada
Reno, NV 89557
USA

Prof. Dr. Ko Honda
khonda@math.usc.edu
Mathematics Department
University of Southern California
Los Angeles, CA 94306
USA

Prof. Dr. Robion C. Kirby
kirby@math.berkeley.edu
Department of Mathematics
University of California
at Berkeley
815 Evans Hall
Berkeley, CA 94720-3840
USA

Prof. Dr. Dieter Kotschick
dieter@rz.mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
80333 München

Prof. Dr. Matthias Kreck
kreck@mathi.uni-heidelberg.de
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288
69120 Heidelberg

Dr. Peter B. Kronheimer
kronheim@math.harvard.edu
Dept. of Mathematics
Harvard University
1 Oxford Street
Cambridge, MA 02138
USA

Prof. Dr. Tian-Jun Li
tli@math.princeton.edu
111 Park Street
Apt. 3R
New Haven, CT 06511
USA

Prof. Dr. Yuhan Lim
ylim@math.ucsc.edu
Dept. of Mathematics
University of California
Santa Cruz, CA 95064
USA

Prof. Dr. Paolo Lisca
lisca@dm.unipi.it
Dipartimento di Matematica
Universita di Pisa
Via Buonarroti, 2
I-56127 Pisa

Dr. Matilde Marcolli
marcolli@mpim-bonn.mpg.de
Max-Planck-Institut für Mathematik
Vivatsgasse 7
53111 Bonn

Prof. Dr. Tom Mark
mark@math.berkeley.edu
Department of Mathematics
University of California
at Berkeley
Berkeley, CA 94720-3840
USA

Prof. Dr. Gordana Matic
gordona@math.uga.edu
Dept. of Mathematics
University of Georgia
Athens, GA 30602
USA

Prof. Dr. Grigory Mikhalkin
mikhalkin@math.harvard.edu
mikhalkin@math.utah.edu
Dept. of Mathematics
University of Utah
155 S. 1400 East
Salt Lake City, UT 84112-0090
USA

Prof. Dr. Tomasz S. Mrowka
mrowka@math.mit.edu
Department of Mathematics
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
USA

Dr. Vicente Munoz
vicente.munoz@uam.es
Departamento de Matematicas, C-XV
Universidad Autonoma de Madrid
Ciudad Universitaria de Cantoblanco
E-28049 Madrid

Prof. Dr. Peter Ozsvath
petero@math.princeton.edu
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton, NJ 08544-1000
USA

Prof. Dr. Victor Pidstrigach
pidstrig@uni-math.gwdg.de
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-7
37073 Göttingen

Prof. Dr. Frank S. Quinn
quinn@math.vt.edu
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall
Blacksburg, VA 24061-0123
USA

Prof. Dr. Daniel Ruberman
ruberman@brandeis.edu
Department of Mathematics
Brandeis University
Waltham, MA 02254-9110
USA

Prof. Dr. Dietmar Salamon
salamon@math.ethz.ch
Mathematik Departement
ETH Zürich
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Nikolai Saveliev
saveliev@math.tulane.edu
Dept. of Mathematics
Tulane University
New Orleans, LA 70118
USA

Birgit Schmidt
bschmidt@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
33615 Bielefeld

Dr. Ivan Smith
smithi@maths.ox.ac.uk
New College
University of Oxford
GB-Oxford OX1 3BN

Prof. Dr. Ronald J. Stern
rstern@math.uci.edu
Dept. of Mathematics
University of California at Irvine
Irvine, CA 92697-3875
USA

Prof. Dr. Andras Stipsicz
stipsicz@cs.elte.hu
Department of Analysis
L. Eötvös University
Muzeum krt. 6 - 8
H-1088 Budapest

Prof. Dr. Zoltan Szabo
szabo@math.princeton.edu
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton, NJ 08544-1000
USA

Markus Szymik
szymik@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
33615 Bielefeld

Prof. Dr. Peter Teichner
teichner@math.ucsd.edu
Dept. of Mathematics
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0112
USA

Prof. Dr. Andrei Teleman
teleman@cmi.univ-mrs.fr
Centre de Mathematiques et
d'Informatique
Universite de Provence
39, Rue Joliot Curie
F-13453 Marseille Cedex 13

Prof. Dr. Stefano Vidussi
svidussi@math.uci.edu
Department of Mathematics
Michigan State University
East Lansing, MI 48824-1027
USA

Prof. Dr. Bai Ling Wang
bwang@maths.warwick.ac.uk
Department of Pure Mathematics
The University of Adelaide
Adelaide, SA 5005
AUSTRALIA

Prof. Dr. Shuguang Wang
sw@wang.math.missouri.edu
Dept. of Mathematics
University of Missouri-Columbia
Columbia, MO 65211-0001
USA