MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop:

Aging and Glassy Systems

August 19th – August 25th, 2001

This mini-workshop was organized by GÉRARD BEN AROUS (EPFL, Lausanne), ANTON BOVIER (WIAS, Berlin) and VÉRONIQUE GAYRARD (EPFL, Lausanne). The participants, 14 in number, came from Brazil, France, Germany, Italy, Switzerland and the USA.

A key concept that has emerged over the last few years in the study of dynamical properties of complex disordered systems, such as spin glasses, is that of "Aging". While it has by now become the focus of an intense research activity in theoretical physics, a very small group of people in probability theory have, only recently, begun working in this field. The aim of the workshop was to encourage and coordinate this emerging effort by bringing together four physicists, experts in aging dynamics, and the small group of people from probability theory that have just produced the first steps into this area.

10 talks were devoted to an exposition of the existing mathematical contributions, thereby providing a complete picture of the advances made in the field. On the other hand state of the art results in physics as well as conjectures and open problems were presented and discussed through a series of 5 talks and a mini-course on "glassy dynamics".

This workshop was a very fruitful experience as it allowed to clarify the immediate and medium terms objectives in the mathematical investigation of aging, and encouraged contacts between the physics and the mathematics community.

Abstracts

Glauber dynamics of the random energy model

Gérard Ben Arous, Anton Bovier, Véronique Gayrard

I. Metastable motion on the extreme states.

We investigate the long-time behavior of the Glauber dynamics for the random energy model below the critical temperature. We give very precise estimates on the motion of the process to and between the states of extremal energies. We show that when disregarding time, the consecutive steps of the process on these states are governed by a Markov chain that jumps uniformly on all possible states. The mean times of these jumps are also computed very precisely and are seen to be asymptotically independent of the terminal point. A first indicator of aging is the observation that the mean time of arrival in the set of states that have waiting times of order T is itself of order T. The estimates presented here furnish the crucial input for a detailed analysis of the aging phenomenon.

II. Aging below the critical temperature

We establish that for a suitably chosen timescale that diverges with the size of the system, one can prove that a natural autocorrelation function exhibits aging. Moreover, we show that the long-time asymptotics of this function coincide with those of the so-called "REM-like trap model" proposed by Bouchaud and Dean. Our results rely on very precise estimates on the distribution of transition times of the process between different states of extremely low energy.

Simple models of aging and open problems

Jean-Philippe Bouchaud

I discussed several extensions of the trap model of aging. One is the case where the traps are organized in a one dimensional chain. This model leads to subaging for some observables, and to the very interesting phenomenon of dynamical localisation and its possible connection with the replica theory with a negative number of replicas. The second extension deals with the case where the hopping rate depends both on the starting site and the final site. In this case, one can observe both subaging and two scaling regimes for the aging correlation function. The third extension is the multi-level trap model, built to describe the so called rejuvenation and memory experiments in glassy systems. Some remarks on the similarities and differences between this model and Sinai's model were given – the latter containing some of the features of the former in embryo. Finally, I introduced the notion of temperature chaos and discussed its presence in several simple disordered systems, such as the Random Energy Model and the directed polymer on a hierarchical lattice.

Glassy dynamics

Leticia Cugliandolo, Jorge Kurchan

We consider Langevin dynamics (gradient descent + noise) restricted to the sphere $\mathcal{S}$ defined by $\sum_{i=1}^{N} x_i^2 = N$, with energy function

$$E(x) = \sum_p A_p \sum_{i_1,\ldots,i_p} J_{i_1,\ldots,i_p} x_{i_1} \ldots x_{i_p}$$

where the $J_{i_1,\ldots,i_p}$ are random variables with Gaussian distribution.
This family of models is known to contain the full phenomenology of mean-field spin glasses, plus avoiding inessential complications of, say, the more standard Sherrington-Kirkpatrick model.

An equation for the two-time correlation and response functions starting from a random configuration, exact in the large N limit for finite times, is known in the physics literature. An ansatz for the asymptotic solution to these equations ($t \to \infty$ *after* $N \to \infty$) has been obtained: it is believed to be the exact solution in that regime.

Both the rigorous justification of the equations themselves, as well as their asymptotic solutions seem an interesting and feasible program for the future.

Aging properties of Sinai's model of random walk in random environment

Amir Dembo

This talk, based on a joint work with Alice Guionnet and Ofer Zeitouni, is about aging properties of Sinai's nearest neighbour Random Walk in Random Environment (RWRE). With $\mathbb{P}^o$ denoting the annealed law of the RWRE X_n, our main result is a full proof of the following statement due to P. Le Doussal, C. Monthus and D. S. Fisher:

$$\lim_{\eta \to 0} \lim_{s \to \infty} \mathbb{P}^o \left(\frac{|X_{s^\alpha} - X_s|}{(\log s)^2} < \eta \right) = \frac{1}{\alpha^2} \left[\frac{5}{3} - \frac{2}{3} e^{-(\alpha - 1)} \right].$$

Aging in some one dimensional systems

Luiz Renato Fontes, Marco Isopi,
Charles Newman, Daniel Stein

I. Aging in the 1D voter model

We derive exact expressions for a number of aging functions that are scaling limits of non-equilibrium correlations, $R(t_w, t_w + t)$ as $t_w \to \infty$, $t/t_w \to \theta$, in the $1D$ homogenous q-state Potts model for all q with $T = 0$ dynamics following a quench from $T = \infty$. One such quantity is $\langle \vec{\sigma}_0(t_w) \cdot \vec{\sigma}_n(t_w + t) \rangle$ when $n/\sqrt{t_w} \to z$. Exact, closed-form expressions are also obtained when an interlude of $T = \infty$ dynamics occurs. Our derivations express the scaling limit via coalescing Brownian paths and a "Brownian space-time spanning tree," which also yields other aging functions, such as the persistence probability of no spin flip at 0 between t_w and $t_w + t$.

References: cond-mat/0103494 - Phys. Rev Lett., to appear.

II. Aging in the random walk with random rates

Let $\tau = (\tau_i : i \in \mathbf{Z})$ denote i.i.d. positive random variables with common distribution F and (conditional on τ) let $X = (X_t : t \geq 0, X_0 = 0)$, be a continuous-time simple symmetric random walk on $\mathbf{Z}$ with inhomogeneous rates $(\tau_i^{-1} : i \in \mathbf{Z})$. When F is in the domain of attraction of a stable law of exponent $\alpha < 1$ (so that $\mathbf{E}(\tau_i) = \infty$ and X is subdiffusive), we prove that (X, τ), suitably rescaled (in space and time), converges to a natural (singular) diffusion $Z = (Z_t : t \geq 0, Z_0 = 0)$ with a random (discrete) speed measure ρ. The convergence is such that the "amount of localization", $\mathbf{E}\sum_{i\in\mathbf{Z}}[\mathbf{P}(X_t = i|\tau)]^2$ converges as $t \to \infty$ to $\mathbf{E}\sum_{z\in\mathbf{R}}[\mathbf{P}(Z_s = z|\rho)]^2 > 0$, which is independent of $s > 0$ because of scaling/self-similarity properties of (Z, ρ). A similar limit is valid for $Y_k(t) = \mathbf{E}\sum_{i\in\mathbf{Z}}[\mathbf{P}(X_t = i|\tau)]^k$ as $t \to \infty$ for general k. For the Random Walk with Random Rates on $\mathbf{Z}^d$, with $d \neq 1, Y_k(t) \to 0$ as $t \to \infty$. The scaling properties of (Z, ρ) are also closely related to the aging of (X, τ); e.g. $\mathbf{E}\,\mathbf{P}(X_{t_w+t} = X_{t_w}|\tau)$ converges as $t_w \to \infty$ and $t/t_w \to \theta$, to the aging function $\mathbf{E}\,\mathbf{P}(Z_{(1+\theta)s} = Z_s|\rho)$. Our main technical result is a general convergence criterion for localization and aging functionals of diffusions/walks $W^{(\epsilon)}$ with (nonrandom) speed measures $\mu^{(\epsilon)} \to \mu$ (in a sufficiently strong sense).

References: Probab. Theory Related Fields 115 (1999) 417-443; math. PR/0009098 - Ann. Probab., to appear.

Aging phenomenon in SSK

Alice Guionnet

Sompolinski and Zippelius (1981) propose the study of dynamical systems whose invariant measures are the Gibbs measures for (hard to analyze) statistical physics models of interest. In the course of doing so, physicists often report of an "aging" phenomenon. For example, aging is expected to happen for the Sherrington-Kirkpatrick model, a disordered mean-field model with a very complex phase transition in equilibrium at low temperature. We shall study the Langevin dynamics for a simplified spherical version of this model. The induced rotational symmetry of the spherical model reduces the dynamics in question to an N-dimensional coupled system of Ornstein-Uhlenbeck processes whose random drift parameters are the eigenvalues of certain random matrices. We obtain the limiting dynamics for N approaching infinity and by analyzing its long time behavior, explain what is aging (mathematically speaking), what causes this phenomenon, and what is its relationship with the phase transition of the corresponding equilibrium invariant measures.

This is a joint work with G. Ben Arous and A. Dembo

Spectral theory and metastability for markov chains

Marcus Klein

For a large class of Markov chains with discrete state space and transition matrix P_N we study the relation between the low-lying spectrum of the discrete generator $1 - P_N$ and the metastable behaviour of the chain in the limit where the system size N of the chain tends to infinity. We define the notion of a *metastable set* as a subset of the state space Γ_N such that (i) this set is reached from any point $x \in \Gamma_N$ without return to x with probability at least b_N, while (ii) for any two point x, y in the metastable set, the probability $T_{x,y}^{-1}$ to reach y from x without return to x is smaller than $a_N^{-1} \ll b_N$; finally, the invariant mass of each of the metastable points is required not to be too small. Under an additional non-degeneracy assumption, we show that in such a situation:

(i) To each metastable point corresponds a metastable state, whose mean exit time can be computed precisely.
(ii) To each metastable point corresponds one simple eigenvalue of $1 - P_N$ which is essentially equal to the inverse mean exit time from this state.

Moreover, these results imply very sharp uniform control of the deviation of the probability distribution of metastable exit times from the exponential distribution.

The error estimates are best in the case of reversible chains. Using symmetrization of a non-reversible chain with respect to the quadratic form induced by its invariant measure one obtains somewhat weaker estimates for non-reversible chains as well.

Our methods use a combination of potential and spectral theory with probability theory.

The basic capacitary estimates and the corresponding variational principle is applicable to the case of continuous diffusions in $\mathbf{R}^n$.

Edited by Jiří Černý

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