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Singularitäten

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The conference was organized by G.-M. Greuel (Kaiserslautern), J.H.M. Steenbrink (Nijmegen) and V.A. Vassiliev (Moscow) and attended by 44 participants from Europe, North America and Israel. As the conference showed, singularity theory is still a very active area with many new exciting interactions with other areas in mathematics. This was represented in 24 talks which were devoted, besides to singularity theory itself, to knot theory, motivic integration, Frobenius manifolds and theoretical physics, global algebraic geometry, hyperplane arrangements, combinatorics and toric geometry, characteristic classes and elliptic genera, and differential equations. One of the talks was devoted to algorithms and computational aspects of singularity theory together with a demonstration of the computer algebra system Singular.

The lectures, in particular the talks with relations to different mathematical areas, were very inspiring and created a lively exchange of mathematical ideas.

Abstracts

Indices of 1–forms on an isolated complete intersection singularity

Sabir Gusein-Zade (Moscow)
(joint work with Wolfgang Ebeling)

The problem to generalize the Eisenbud–Levine–Khimshinshvili formula to vector fields on singular varieties led to a number of papers devoted to indices of analytic vector fields on hypersurface, resp. complete intersection, singularities. We offer an alternative approach, which considers 1-forms instead of vector fields. One can define the index of a 1-form on an isolated real singularity, and also the index of a holomorphic 1-form on a complete intersection singularity. The last one can be expressed as the dimension of an appropriate algebra. There is a family of quadratic forms defined on the (complex) linear space of dimension equal to the complex index, which depends on a point in the target, is non-degenerate when this point is outside of the bifurcation set, is real if the point is real, and its signature is equal to the (real) index plus $\chi(\text{fibre}) - 1$.

2–dimensional McKay-correspondence

Oswald Riemenschneider (Hamburg)

Let $\Gamma \subset \mathrm{GL}(2,\mathbb{C})$ be a finite small subgroup, and define by $Y := Y_\Gamma = \Gamma{-}\mathrm{Hilb}^n(\mathbb{C}^2)$ the scheme of all Γ-invariant ideals $I \subset \mathcal{O}_{\mathbb{C}^2}$ with colength $\dim_{\mathbb{C}} \mathcal{O}_{\mathbb{C}^2}/I = n := \mathrm{ord}\,\Gamma$ such that Γ acts on $\mathcal{O}_{\mathbb{C}^2}/I$ as the regular representation. The Hilbert-Chow morphism

$$Y_\Gamma \longrightarrow \left((\mathbb{C}^2)^n/\mathfrak{S}_n\right)^\Gamma \cong \mathbb{C}^2/\Gamma =: X_\Gamma$$

is a resolution of the singularities of the quotient singularity X_Γ. It was shown that this resolution is minimal by

- Yukari Ito and Iku Nakamura in case $\Gamma \subset \mathrm{SL}(2,\mathbb{C})$,
- Rie Kidoh for cyclic $\Gamma \subset \mathrm{GL}(2,\mathbb{C})$,
- Akina Ishii in the general case.

The exceptional set $E = \bigcup_\rho E_\rho \subset Y_\Gamma$ consists of all ideals $I \in Y_\Gamma$ with support at $0 \in \mathbb{C}^2$. The following theorem is due to Ito and Nakamura for subgroups of $\mathrm{SL}(2,\mathbb{C})$ (where all representations are "special"), a remark by the speaker in the cyclic case, and proved by A. Ishii by means of an equivalence of derived categories.

Theorem: For a non-trivial "special" representation ρ of Γ,

$$E_\rho := \{I \in E \mid V(I) \text{ contains } V(\rho)\}$$

is isomorphic to $\mathbb{P}_1$. For $\rho \neq \rho'$, the intersection $E_\rho \cap E_{\rho'}$ is either empty or consists of exactly one "point", and

$$E = \bigcup_{\substack{\rho \text{ non-trivial} \\ \rho \text{ special}}} E_\rho\,.$$

More precisely, we have $V(I) = V(\rho)$ for $I \in E_\rho$ corresponding to smooth points of E, and $I \in E_\rho \cap E_{\rho'}$ if and only if $V(I) = V(\rho) \oplus V(\rho')$.

Here, $V(I) := I/(\mathfrak{m}I + \mathfrak{n})$, where $\mathfrak{m} = \mathfrak{m}(\mathcal{O}_{\mathbb{C}^2,0})$, $\mathfrak{n} = \mathfrak{m}_x\mathcal{O}_{\mathbb{C}^2,0}$, $\mathfrak{m}_x = \mathfrak{m}(\mathcal{O}^\Gamma_{\mathbb{C}^2,0})$.

Remarks on equisingular deformations

THEO DE JONG (SAARBRÜCKEN)

In this talk the relation between equisingular deformations of plane curve singularities and sandwiched singularities was studied. To describe this, assume we have (for simplicity) an irreducible plane curve singularity C. Let $\ell \in \mathbb{N}$ be such that $\ell \geq \sum_i m_i$, where $m_1, m_2, \ldots$ is the multiplicity sequence of C. Associated to the pair (C, ℓ) is a sandwiched singularity $X(C, \ell)$ which is a rational surface singularity, obtained from some resolution of C (depending on ℓ) by blowing down non-(-1)-curves. The isomorphism class of C is, in general, not determined by the isomorphism class of $X(C, \ell)$, but it is if $\ell \gg 0$. In particular, we deduce that the dimension of the equisingular stratum of C and of $X(C, \ell)$ are equal if $\ell \gg 0$. Using two different formulas for the dimension of the Artin component we are able to reprove a formula of WALL on the (co-)dimension of the equisingular stratum of a plane curve singularity.

Frobenius manifolds and tt*-equations for singularities

CLAUS HERTLING (BONN, MPI)

The base space of a semiuniversal unfolding of an isolated hypersurface singularity can be equipped with the structure of a Frobenius manifold, by work of K. SAITO AND M. SAITO (1983). They used the Gauß-Manin connection and the mixed Hodge structure (MHS) of the singularity. One can give a slightly different construction, using the Fourier transform of the Gauß-Manin connection, that is, oscillating integrals. In this version, K. SAITO's higher residue pairing gets a simple topological meaning, in terms of an intersection form for Lefschetz thimbles.

Now, the MHS (and, thus, the real structure on the cohomology of the Milnor fibre) is used in the construction of the Frobenius manifold, but it is not visible in the Frobenius manifold. It is desirable to enrich the Frobenius manifold in some way with a "shadow" of the real structure or of the MHS.

It happens that there is a candidate coming from physics. In 1991, S. CECOTTI and C. VAFA wrote a paper "Topological and anti-topological fusion" (Nucl. Phys. B 367 (1991), 359–461). There they study the ground state metric for $N = 2$ supersymmetric conformal field theories. Special cases of these are related to quasihomogeneous singularities. The results of CECOTTI and VAFA yield for quasihomogeneous singularities a hermitian metric on the base space of a semiuniversal unfolding, which satisfies certain compatibility conditions with the Frobenius manifold structure. These conditions are called tt*-equations. We can give an independent construction of this hermitian metric, without any arguments from physics, again using oscillating integrals.

Motivic zeta-function associated with a family of two variables series

Michel Merle (Nice)

This talk is a report of a work by Gil Guibert, a summary of which is published in the C.R.A.S. (2001).

Given p analytic functions $f_1, \ldots, f_p$ on a complex manifold X of dimension d, one can associate a motivic zeta function depending on p variables $T_1, \ldots, T_p$ (after Denef–Loeser, Looijenga); namely,

$$Z(T) := \sum_{n \in (\mathbb{N}^*)^p} \big[X_n/(X_0 \times (\mathbb{C}^*)^p)\big] \mathbb{L}^{d|n|} T^n ,$$

where $[\quad]$ stands for the class in the Grothendieck group of varieties over $X_0 \times (\mathbb{C}^*)^p$; $X_0 = f_1^{-1}(0) \cap \ldots \cap f_p^{-1}(0)$ and X_n is the set (constructible and stable at order $|n| = \sum_i n_i$) of the truncated arcs φ at order $|n|$ for which $\operatorname{ord}(f_j \circ \varphi) = n_j$.

Gil Guibert proves that one can associate to the limit $-\lim_{T \to \infty} Z(T^{\alpha_1}, \ldots, T^{\alpha_p})$, which does not exist when $\alpha_1 > 0, \ldots, \alpha_p > 0$, a cycle on $\operatorname{Spec} \mathbb{C}[T_1, T_1^{-1}, \ldots, T_p, T_p^{-1}]$ which coincides with the Alexander zeta function defined by Claude Sabbah for the constant sheaf on X and the p functions $f_1, \ldots, f_p$.

As a corollary, when $X = \mathbb{C}^2$, he can compute the (motivic) Alexander zeta function associated to $f_1, \ldots, f_p$ in terms of their tree of contacts. For one function $f : \mathbb{C}^2 \to \mathbb{C}$, he gives also the Hodge–Steenbrink spectrum in terms of the same tree of contacts (which reduces to the data of Puiseux pairs for f irreducible), a result obtained by M. Saito for f irreducible. Of course, the Alexander polynomial associated to a reducible plane curve can also be computed in this way (cf. Eisenbud–Neumann).

Structure of discriminants, Cohen-Macaulay reduction, and free* divisors

James N. Damon (Chapel Hill)

There have been many directions in which properties of isolated singularities (especially ICIS) have generalized, such as e.g. completing the ADE classification of functions on manifolds with (singular) boundaries by Arnol'd, Lyashko, Goryunov and Zakalyukin. Each extension has its own equivalence group $\mathcal{G}$ in the Thom–Mather sense and corresponding versal unfoldings and discriminants.

This talk addressed the basic question of whether the discriminants of versal unfoldings for such equivalences retain the basic property of freeness possessed by versal deformations of ICIS. In general, they do not. We identify when they do in terms of conditions involving the $\mathcal{G}$-normal space being Cohen-Macaulay and the genericity of the appropriate analogue of “Morse singularities”. Besides covering a number of situations where the results hold, we also show their failure as one moves even beyond the simple cases for functions on manifolds with singular boundaries, and for the simplest classes of non-isolated complete intersection singularities (through a joint computation with Anne Frühbis-Krüger using a Singular package which she developed).

For cases where the conditions fail, we describe a method of “Cohen-Macaulay Reduction” $\mathcal{G}^*$ of the group $\mathcal{G}$, which frequently applies and yields instead a “Free* Divisor” structure for the discriminant.

We indicate the usefulness of this structure for determining the vanishing topology, and the role of the $\mathcal{G}^*$-normal space such as in generalizations to non-isolated singularities of the Lê–Greuel formula.

Motivic Serre invariant and degenerations of algebraic varieties

François Loeser (Paris)

In this talk we describe recent work in collaboration with J. Sebag. The problem we address is the following: let k be a field; given a variety X over $K = k((t))$, and a model $\mathfrak{X}$ of X over $k[[t]]$, what information on the special fibre $\mathfrak{X}_0$ depends only on X and not on the chosen model?

The results are obtained by analogy with Serre's invariant. The latter attaches to a smooth compact locally p-adic analytic variety X an invariant $s(X) \in \mathbb{Z}/(p-1)\mathbb{Z}$. Two varieties X and X' of dimension $d > 0$ are isomorphic iff they have the same Serre invariant.

Let R be a complete discrete valuation ring with residue field k and field of fractions K. Assume that k is perfect. Let $\mathfrak{X}$ be a formal R-scheme of finite type and consider its Greenberg scheme $\mathrm{Gr}(\mathfrak{X})$, which is an analogue of the space of arcs. Set $R_n = R/t^{n+1}$ (t a uniformizing parameter), $\mathfrak{X}_n = \mathfrak{X} \otimes_R R_n$. Then $\mathrm{Gr}(\mathfrak{X}) = \varprojlim \mathrm{Gr}_n(\mathfrak{X}_n)$, with $\mathrm{Gr}_n(\mathfrak{X}_n)$ a k-scheme such that (for A any k-algebra)

$$\mathrm{Gr}_n(\mathfrak{X}_n)(A) = \mathfrak{X}\big(L(A) \otimes_R R_n\big)\,,$$

$L(A) = A$ in the equicharacteristic case, $L(A) = W(A)$ in the mixed characteristic case. J. Sebag developed a theory of motivic integration on $\mathrm{Gr}(\mathfrak{X})$ with values in $K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]$. Here, $K_0(\mathrm{Var}_k)$ denotes the "Grothendieck ring of algebraic varieties over k", and $\mathbb{L} = [\mathbb{A}^1_k]$. Now if X is a smooth rigid K-space and ω is a gauge form on X then one may define an integral $\int_X |\omega|$ in $K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]$ by using a formal model $\mathfrak{X}$ with generic fibre $\mathfrak{X}_K = X$. To prove that the value of $\int_X |\omega|$ does not depend on the model $\mathfrak{X}$, one uses

- Néron's weak desingularization,
- a fundamental change of variables formula.

Now, if one has two gauge forms ω and ω' on X, one proves that

$$\int_X |\omega| - \int_X |\omega'| \in (\mathbb{L}-1)K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]\,.$$

Since, locally, gauge forms do always exist, this allows to define the motivic Serre invariant

$$\lambda(X) \in K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]/(\mathbb{L}-1)K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]$$

when $K = \mathbb{Q}_p$ one can show that $s\big(X(\mathbb{Q}_p)\big)$ is equal to $N_p\big(\lambda(X)\big)$ where N_p stands for "number of points in $\mathbb{F}_p$". We also give formulas for $\lambda(X)$ in terms of a weak Néron model. In the special case of Calabi-Yau manifolds over K, one can define an invariant in $K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]$. In particular, the following analogue of Batyrev's theorem is obtained: if $\mathfrak{X}$ and $\mathfrak{X}'$ are two R-smooth models of a Calabi-Yau manifold X over K then their special fibres define the same class in $K_0(\mathrm{Var}_k)[\mathbb{L}^{-1}]$.

Real cohomology groups of the space of non-singular plane projective quintics

Alexei Gorinov (Paris)

We present a modification of Vassiliev's method of calculating cohomology groups of spaces of non-singular algebraic hypersurfaces of given degree. Due to Alexander duality these groups are isomorphic to the Borel–Moore homology groups of the space of singular hypersurfaces. More precisely, in the complex case one has

$$H^*\big(\Pi_{d,n} \setminus \Sigma_{d,n}\big) \cong \overline{H}_{2D-1-*}\big(\Sigma_{d,n}\big)\,,$$

where $\Pi_{d,n}$ is the space of homogeneous polynomials of degree d in $n+1$ variables, $\Sigma_{d,n}$ is the set of singular polynomials and $D = \dim_{\mathbb{C}} \Pi_{d,n}$. To calculate the groups $\overline{H}_*(\Sigma_{d,n})$ a resolution $\pi : \sigma_{d,n} \to \Sigma_{d,n}$ is constructed, such that π is proper and surjective and $\pi^{-1}(x)$ is contractible. The main result is that the Poincaré polynomial of the space $\Pi_{5,2} \setminus \Sigma_{5,2}$ is equal to $(1+t)(1+t^3)(1+t^5)$.

On the homology of the spaces of long knots

Victor Tourtchine (Paris)

The spaces of long knots are the spaces of regular, injective maps $\mathbb{R}^1 \to \mathbb{R}^d$, $d \geq 3$, coinciding with a fixed linear map outside some compact subset.

One of the main methods to study the homology groups of such spaces is Vassiliev's approach that consists of introducing a simplicial resolution of the discriminant set (complement space) of maps $\mathbb{R}^1 \to \mathbb{R}^d$, $d \geq 3$, having self-intersections or singularities. The resolved discriminant admits a natural filtration. The associated spectral sequence is conjectured to stabilize in the first term.

In the talk the first term of this spectral sequence was described in terms of the Hochschild homology of the $(d-1)$-twisted Poisson algebra operad. This operad is the homology operad of the May operad of little cubes. In particular, it gives a new and purely algebraic point of view on the bialgebra of chord diagrams, that is, the dual of the graded quotient of the space of finite type knot invariants.

Singularities and multiplicities

Terence Gaffney (Boston)

The multiplicity of a module has played a useful role in equisingularity problems. A key property, proved by David Rees, is that if

$$N \subset M \subset \mathcal{O}_X^p$$

are $\mathcal{O}_X$-modules of finite colength, with X an equidimensional complex analytic space, then N is a reduction of M iff $e(N) = e(M)$.

In many situations of interest M is not of finite colength, so $e(M)$ is not defined. In this talk, using the multiplicity of a pair of modules, we introduce new multiplicities which provide a generalization of the theorem of Rees. These new multiplicities are then applied to give necessary and sufficient conditions for a hyperplane to be a limiting tangent hyperplane to an equidimensional complex analytic space.

Singular and singularities

Gert-Martin Greuel, Anne Frühbis-Krüger and Mathias Schulze
(Kaiserslautern)

Algorithmic and computational aspects have become a major, still growing issue in mathematical research and teaching. In this talk, the abilities of Singular were presented. The latter is a computer algebra system for polynomial computations with a special emphasis on commutative algebra, algebraic geometry and singularity theory. In particular, two new algorithms were presented that have numerous applications in singularity theory:

- the partial standard basis algorithm for families allows the calculation of a presentation

$$\mathcal{O}_T^q \xrightarrow{M} \mathcal{O}_T^p \longrightarrow T^1_{X/T}$$

modulo a power of the maximal ideal in $\mathcal{O}_T$ (X a family of hypersurfaces, resp. CM-singularities in codim 2, over a smooth base T). For special cases, it is even possible to determine M exactly. As an application, the adjacency diagram of the simple space curve ICIS, to which Giusti and Goryunov contributed numerous adjacencies, can now be completed.
- based on a suitably modified standard basis algorithm, the monodromy matrix of an IHS, the spectral pairs, and Saito's matrices A_0 and A_1 can be computed (along the lines of M. Saito's paper). During the talk it could be verified for the first time that the singularity given by $x^6 + y^7 + z^7 + x^2y^2z^2 = 0$ has a monodromy matrix with a Jordan block of size 3. This is one of the smallest known examples with this property (Malgrange's technique needed even exponents, so this example was inaccessible for it).

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Valuations and toric geometry

Bernard Teissier (Paris)

We explored the meaning of the slogan "Every singularity is non-degenerate". A singularity germ (say formal) $(X, 0) \subset (\mathbb{A}^N(k), 0)$ is non-degenerate if there exist local coordinates on $\mathbb{A}^N(k)$ and a fan Σ of $\mathbb{R}^N_+$ such that the strict transform of X by the proper birational toric map $\pi(\Sigma) : Z(\Sigma) \to \mathbb{A}^N(k)$ associated to Σ is regular and transversal to the exceptional divisor. The slogan means: every singularity germ can be embedded in some $(\mathbb{A}^N(k), 0)$ in a non-degenerate way.

More modestly, given a valuation ν of $\mathcal{O}_{X,0}$ (assumed to be equicharacteristic, excellent, with algebraically closed residue field) one may ask if such an embedding exists (depending on ν) with a fan Σ (depending on ν) such that the strict transform of X is regular and transversal to the exceptional divisor at the point picked by ν.

We presented statements which could play a role in a proof of this result.

Existence of curves with prescribed singularities

Thomas Keilen (Kaiserslautern) and Ilya Tyomkin (Tel Aviv)

Given a linear system $|D|$ on a smooth complex projective surface Σ, and topological, resp. analytic, singularity types $S_1, \ldots, S_r$, one may ask whether there exists an irreducible, reduced curve C in $|D|$ having precisely r singular points of the given types. More precisely, one may ask for sufficient numerical conditions for the existence of such curves in terms of invariants of the singularity types and the linear system. Knowing that the genus formula imposes the necessary condition $\sum_i \mu(S_i) \leq D^2 + D.K_\Sigma + 2$, it makes sense to consider a sufficient condition of type $\sum_i \mu(S_i) \leq \alpha D^2 + \beta D.K + \gamma$, for some absolute constants α, β, γ and some fixed divisor K, being a suitable condition. To reach the aim of finding such a condition one can proceed in four steps: 1) One brings the existence of a good curve $C \in |D|$ with ordinary multiple points down to a vanishing theorem. 2) One derives such a vanishing theorem with the aid of Kawamata–Vieweg and Nakai–Moishezon from numerical conditions just depending on the multiplicities and on the linear system. 3) One finds good local conditions representing the singularity types $S_1, \ldots, S_r$. 4) One glues these local equations with a Viro glueing type method into a curve with ordinary multiple points. In this way one replaces the multiplicities in the conditions of 2) by bounds for the degree of the good equations in 3), which are given in terms of the Milnor numbers $\mu(S_1), \ldots, \mu(S_r)$, and, finally, the desired sufficient condition for the existence can be derived.

Adjacencies between planar curve singularity types

Joaquim Roé (Barcelona)

Let us consider reduced curves $C : f = 0$, $f \in \mathbb{C}[[x, y]]$, classified by topological equivalence or equisingularity. The question we addressed is that of adjacencies between classes, that is, given two equisingularity classes S, S', is it true that $S' \subset \overline{S}$? The same question can be put in the following way: given S, S', is it true that for all $f \in S'$ there is a family f_t with $f_0 = f$ and $f_t \in S$ for $t \neq 0$, $|t| < \varepsilon$? In this form, a particular case of special interest is that of *linear adjacency*, that is, the case that the family is of the form $f_t = f + tg$.

We attacked the problem by solving first the linear adjacencies, showing that there are none exceptional. This criterion is combinatorial and expressed in terms of Enriques diagrams. Then a necessary and a sufficient condition are obtained by studying how the resolution may vary in families and by studying adjacencies in the variety of all clusters.

Semistable $K3$-surfaces with icosahedral symmetry

Jan Stevens (Göteborg)

In a type III degeneration of $K3$-surfaces the dual graph of the central fibre is a triangulation of S^2. By results of Friedman every combinatorial possibility occurs as central fibre. This raises the question of explicitly realising one's favourite triangulation.

The tetrahedron gives a nice illustration of the occuring phenomena. One way to realise the tetrahedral triangulation starts from a pencil $\lambda T + \mu F$, where $T = 0$ is the product of 4 planes in general position, and $F = 0$ is a general smooth quartic. The total space has A_1-singularities where the smooth quartic intersects the edges of the tetrahedron, which have to be resolved by small resolutions. The most symmetric of possible choices gives a central fibre which consists of four cubic surfaces. One can also start from the normal crossings variety obtained by glueing four cubic surfaces along triangles. Embedded deformations can be computed explicitly. Both constructions lead to different 19-dimensional families. For the icosahedral triangulation one can construct a dodecahedron consisting of 12 Del Pezzo surfaces of degree 5, admitting icosahedral symmetry. This object of degree 60 in $\mathbb{P}^{31}$ is described by 406 equations. An infinitesimal smoothing deformation can be computed, but extending it to higher order is beyond all hope.

By breaking the symmetry, the dodecahedron can also be realised with surfaces of degree 12. The special fibre consists of 6 planes with triangles as double curve and three quadric surfaces with rectangles as double curve. The three remaining divisors arise by resolving the singularities of the total space of a one-parameter smoothing. Explicit equations were presented in the talk, they can also be found in our preprint math.AG/0106133.

Third differential and Thom polynomials for isolated hypersurface singularities

MAXIM KAZARIAN (MOSCOW)

Thom polynomials are characteristic classes dual to different strata of the discriminant set in the parameter space of a family of some singular objects (maps, fields, actions, sections, varieties, etc.). The computation of the Thom polynomials for isolated hypersurface singularities by the method of resolution of singularities leads to the classification of marked singularities of functions. The marking of a germ of a critical point of a function is a point on the cubic given by the third differential of the function. The classification of marked singularities is parallel to the usual classification of critical points.

In the talk the classification of marked singularities of functions and its application to the computation of Thom polynomials was discussed.

Topology of hypersurfaces: applications to polar Cremona transformations and hyperplane arrangements

ALEXANDRU DIMCA (BORDEAUX)

Let $f \in \mathbb{C}[x_0, \ldots, x_n]$ be a non-constant homogeneous polynomial. Let $d(f)$ be the degree of the associated gradient map $x \longmapsto (\frac{\partial f}{x_0}(x) : \ldots : \frac{\partial f}{x_n}(x))$, regarded as a rational map $\mathbb{P}^n \dashrightarrow \mathbb{P}^n$.

We show that $d(f)$ has a simple topological interpretation in terms of the topology of the hypersurface $V \subset \mathbb{P}^n$ defined by $f = 0$. This completes some of the results of I. DOLGACHEV (Michigan J. Math. **48** (2000)) on polynomials with $d(f) = 1$.

When applied to hyperplane arrangements, this result implies that the complement $M = \mathbb{P}^n \setminus \bigcup_i H_i$ has a very special CW-complex structure. This in turn gives information on the higher homotopy groups $\pi_j(M)$ for $j > 1$.

These results are available on the web (math.AG) and represent a joint work with S. PAPADIMA in Bucharest.

Elliptic genera of singular varieties

ANATOLY LIBGOBER (CHICAGO)
(joint work with LEV BORISOV)

We give two extensions of the two-variable elliptic genus (introduced by KRICHEVER, HÖHN, TOTARO, WITTEN, etc.) to singular varieties.

One is given in terms of resolutions of singularities and has BATYREV's string theoretical Euler characteristic as its limit. If a singular variety admits a crepant resolution then its elliptic genus is the elliptic genus of the crepant resolution.

Another is defined for quotient spaces X/G, where X is a complex manifold and G a finite group of biholomorphic automorphisms. It specializes to the orbifold Euler characteristic considered by HARVEY, DIXON, VAFA, WITTEN, HIRZEBRUCH, BATYREV, etc.. We conjecture that the two types of elliptic genera of X/G coincide.

Our definition of the orbifold elliptic genus yields the following formula of DIJKGRAAF, MOORE, VERLINDE and VERLINDE: If Σ_n is the symmetric group acting on X^n in the standard way then

$$\sum_{n\geq 0} p^n Ell_{\mathrm{orb}}(X^n/\Sigma_n; y, q) = \prod_{i=1}^{\infty}\prod_{\ell,m} \frac{1}{(1-p^i y^\ell q^m)^{c(mi,\ell)}},$$

where $Ell(X) = \sum_{m,\ell} c(m,\ell) y^\ell q^m$.

Moduli spaces of polynomials in two variables

Javier Fernández de Bobadilla de Olazábal (Nijmegen)

To study families of polynomials in two variables up to right equivalence the space to look at is $\mathbb{C}[x,y]/\operatorname{Aut}(\mathbb{C}^2)$. This space is infinite dimensional and non-Hausdorff, hence quite difficult to be studied geometrically. To understand it better we define a geometrically meaningful stratification on it, and consider the quotient of each stratum by the induced equivalence relation.

Given any $f \in \mathbb{C}[x,y]$, we will associate to it a blowing-up process π_f, that is, roughly speaking, the blowing-up process such that the centres of the appearing blowing-ups are at the same time:

(1) infinitely near to the projective completion of $V(f) \subset \mathbb{C}^2 \subset \mathbb{P}^2$, and
(2) centres of a blowing-up involved in the minimal resolution of the indetermination of any $\phi \in \operatorname{Aut}(\mathbb{C}^2)$ viewed as a birational transformation from $\mathbb{P}^2$ to $\mathbb{P}^2$.

Using π_f it is possible to associate a graph β_f to f. We define the stratum associated to β_f to be the set of polynomials whose graph is isomorphic to β_f.

We show that the quotient of each stratum by the induced equivalence relation is the quotient by a finite group of an affine algebraic variety, and that the properties of the group and of the variety can be read off directly from the combinatorics of the graph associated to the stratum.

The Poincaré series of a quasi-homogeneous surface singularity

Wolfgang Ebeling (Hannover)

S.M. Gusein-Zade, F. Delgado, and A. Campillo have shown that for an irreducible plane curve singularity the Poincaré series of the ring of functions on the curve coincides with the zeta function of its monodromy transformation.

We show that there is also a relation between the Poincaré series $p_A(t)$ of the coordinate algebra of a two-dimensional quasi-homogeneous isolated hypersurface singularity and the characteristic polynomial of its monodromy operator. More precisely, we define

$$\psi_A(t) := (1-t)^{2-r}\prod_{i=1}^{r}(1-t^{\alpha_i}), \quad \phi_A(t) := p_A(t)\psi_A(t),$$

where $\{g; b; (\alpha_1,\beta_1),\ldots,(\alpha_r,\beta_r)\}$ are the orbit invariants. We show that the dual (in Saito's sense) of the rational function $\widetilde{\phi}_A(t) := \phi_A(t)/(1-t)^{2g}$ is the characteristic polynomial of the monodromy operator of (X,x). Similar results can be proved for ICIS of certain types.

If (X,x) is a Kleinian singularity, not of type A_{2n}, then $\psi_A(t)$ is the characteristic polynomial of the affine Coxeter element of the corresponding root system, and the above result implies that $\phi_A(t)$ is the characteristic polynomial of the Coxeter element. We derive this result also directly from the McKay-correspondence.

Mystics of root distributions for polynomial eigenfunctions of linear ODE

Boris Shapiro (Stockholm)

Consider an operator $\partial_Q = \sum_{i=0}^{k} Q_i(x) \cdot \frac{d^i}{dx^i}$ with polynomial coefficients $Q_i(x)$. We assume that $r := \max_i \deg Q_i(x) - i$ is non-negative. We treat the following "polynomial" spectral problem:

$$(1) \qquad \partial_Q y + \lambda \varphi(x) y = 0 \, ,$$

where $\varphi(x)$ is a polynomial of degree r. If $\deg Q_k - k = r$ then one can show that (after an appropriate choice for $\varphi(x)$) there exist $\binom{n+r}{r}$ polynomial solutions of degree n for (1).

These polynomial eigenfunctions have a remarkable root distribution, asymptotically following curvilinear traces connecting zeroes of $Q_k(x)$. Such a curvilinear tree straightens out by the coordinate change

$$\psi_Q : x \longmapsto \int_{x_0}^{x} \frac{dt}{\sqrt[k]{Q_k(x)}}$$

(case $r = 0$). The angles between edges are determined by the underlying trees, etc.. Relations to WKB-analysis and Strebel differentials were discussed.

Modular deformations

Bernd Martin (Cottbus)

The notion of a modular deformation has been introduced for complete complex varieties and for analytic polyhedra by Palamodov, and later on by Laudal in a formal context.

We are investigating different concepts of modular deformations of germs of isolated singularities (infinitesimal, formal) and construct an obstruction theory for enlarging a modular subgerm, which is induced by the Lie bracket of the tangent cohomology

$$T^0(\mathbf{X_0}) \times T^1(\mathbf{X_0}) \longrightarrow T^1(\mathbf{X_0}) \, .$$

The modular stratum of an ICIS is characterized as flattening of the relative Tjurina algebra $T^1(\mathbf{X}, \mathbf{S})$ of the deformation. Based on implementations of the computation of versal deformations, and of a new flattening algorithm in Singular, explicit computations of non-trivial examples are possible. In particular, we find a modular family whose modular stratum has two components, one of those with a jump in the Tjurina number. Moreover, modular strata of semi-quasihomogeneous functions are connected with the coarse moduli spaces constructed by Pfister and others using different considerations, but have more (non-reduced) structure.

The graph of monomial ideals

Klaus Altmann (Düsseldorf)

Fixing a polynomial ring $R = k[x_1, \ldots, x_n]$ over a field, we define a graph G as follows: its vertices are the monomial ideals M in R, and two monomial ideals M_1 and M_2 are connected by an edge, if there is an ideal having M_1 and M_2 as its only monomial Gröbner degenerations.

If connected, M_1 and M_2 may be found as the only Gröbner degenerations of an ideal admitting an $(n-1)$-dimensional grading. On the one hand, this makes it possible to calculate those so-called edge ideals and to determine whether the M_i are connected or not. On the other hand, since those are exactly the monomial ideals admitting a full n-dimensional grading, this fact leads us to think about G as the 1-skeleton in a huge Hilbert scheme.

The graph G gives rise to interesting subgraphs. First, one might consider only those monomial ideals being contained in a fixed multigraded Hilbert scheme. If they are connected, then this may be done also inside the Hilbert scheme. A special case is that of Artinian ideals. If $n = 2$, then we obtain a new graph of partitions.

Another subgraph is that of the square-free monomial ideals. Via the STANLEY-REISNER construction, this leads to a general notion of flips between simplicial complexes.

Composite functions and families of matrices

VICTOR GORYUNOV (LIVERPOOL)

Consider a diagram $\mathbb{C}^m \xrightarrow{F} \mathbb{C}^n \xrightarrow{f} \mathbb{C}$, where f is an isolated function singularity, and $m \leq n$. We establish a relation between the natural Tjurina number of the diagram and the Milnor number of $f \circ F$. In particular, this provides the $\mu = \tau$ theorem if $m = n, n-1$.

One of the very first applications of this result to the natural classification of simple families of symmetric matrices is a $k(\pi, 1)$-theorem for their discriminants. Classical 2-coloured braids and 2-coloured Brieskorn braids of series D appear in the context.

Edited by Christoph Lossen (Kaiserslautern)

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