Mathematisches Forschungsinstitut Oberwolfach

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Combinatorics

January 6th – January 12th, 2002

The workshop was organized by László Lovász (Redmond) and Hans Jürgen Prömel (Berlin). During the meeting, 37 talks were delivered. They covered a wide range of aspects within combinatorics, thus providing a forum where both surveys and in-depth expositions of the most important new results and methods from many combinatorial areas were given.

In total, 45 scientists participated in this meeting, coming from no less than 11 different countries. The organizers and participants thank the "Mathematisches Forschungsinstitut Oberwolfach" for providing a comfortable and inspiring setting for this conference.

In the following we include the abstracts in alphabetical order.

Abstracts

Interlace Polynomials

Martin Aigner

(joint work with Hein van der Holst)

Arrátia, Bollobás and Sorkin introduced an interesting polynomial $q(G,x)$ of a simple graph G, called the *interlace polynomial*: i) $q(\overline{K}_n, x) = x^n$, ii) $q(G,x) = q(G \setminus u, x) + q(G^{uv} \setminus v, x)$ for $uv \in E$, where G^{uv} denotes the following graph: Let A, B and C be the sets of vertices $\neq u, v$ adjacent to u but not to v, to v but not to u, to both u and v. Now interchange edges $\longleftrightarrow$ non–edges between any two different sets from $\{A, B, C\}$, leaving the rest unchanged. They showed that $q(G,x)$ is independent of the order of removal.

The following results are presented. Let $V = [n] = \{1, \ldots, n\}$.

1. $q(G,x) = \sum\limits_{S \subseteq [n]} (x-1)^{\mathrm{co}(A_S)}$ where A_S is the principal $S \times S$–submatrix of the adjacency matrix A, and co is the corank.
2. Let G be bipartite on color–classes of sizes r and s. Let A be the (shortened) $r \times s$–adjacency matrix, and M the binary matroid generated by $(I_r|A)$. Then $q(G,x) = T_M(x,x)$ where T_M is the Tutte polynomial of M.
3. $q(G,-1) = (-1)^n(-2)^{\mathrm{co}(A+I)}$ where A is the adjacency–matrix.
4. Let $S = \{AnX + BX : X \subseteq V\}$ be the isotropic system (after Bouchet) corresponding to G. Then $q(G,x)$ is the Martin polynomial $m(S,x)$.
5. Another interlace polynomial, defined by a 3–term recursion, is also discussed.

Voting paradoxes and digraphs realizations

Noga Alon

A family of permutations F forms a realization of a directed graph $T = (V,E)$ if for every directed edge uv of T, u precedes v in more than half of the permutations. The quality $q(F,T)$ of the realization is the minimum, over all directed edges uv of T, of the ratio $(|F(u,v)| - |F(v,u)|)/|F|$, where $|F(x,y)|$ is the number of permutations in F in which x precedes y.The study of this quantity is motivated by questions about voting schemes in which each individual has a linear ordering of all candidates, and the individual preferences are combined to decide between any pair of possible candidates by applying the majority vote.

After a brief discussion of related topics in Social Choice Theory, including Condorcet Paradox and Arrow Paradox, I will sketch a proof of a recent result that asserts that every simple digraph T on n vertices, with no anti-parallel edges, admits a realization F with quality at least $c/\sqrt{n}$ for some absolute positive constant c. This is tight up to the constant factor c.

Combinatorics and topology of graph properties

Anders Björner

Fix node set $[n] = \{1, 2, \ldots, n\}$ and identify graphs on these nodes with the sets of edges $E \subseteq \mathbf{2}^{\binom{[n]}{2}}$. A monotone graph property is a family of graphs $([n], E)$ closed under deletion of edges and isomorphisms. Such a property P is therefore in particular an abstract simplicial complex, a subcomplex of the complete simplex on $\binom{n}{2}$ vertices.

In this talk I gave a survey of the main results known concerning the topology of P, for properties such as "not i-connected" and "matching". Some motivations for asking such questions was given, with examples coming from knot invariants, discrete geometry and algebra. Similar results for directed graphs and hypergraphs were briefly mentioned.

The number of k-Sat Funktions

Graham Brightwell
(joint work with Béla Bollobás and Imre Leader)

A k-Sat function is a Boolean function on n variables that can be represented by a k-Sat formula. Most k-Sat formulae are equivalent to the identically zero function; our interest is in determining the asymptotics of the number $\textsc{Sat}(k;n)$ of different k-Sat functions. Here $n \to \infty$, and k may be a function of n.

There is a simple lower bound of $2^{\binom{n}{k}}$, as this is the number of k-Sat formulae in which only positive literals occur – it is easy to see that these all give rise to different function.

For $k = 2$, we show that $\textsc{Sat}(k;n) = 2^{\binom{n}{2} + o(n^2)}$. We conjecture that a similar result holds whenever $k \leq (1-\varepsilon)^{n/2}$, and we can prove that $\textsc{Sat}(k;n) \leq 2^{\sqrt{\pi(k+1)}\binom{n}{k}}$ in this range.

The case $k = 3$ is of particular interest : the most we have proved is $\textsc{Sat}(3;n) \leq 2^{\frac{16}{5}\binom{n}{3}}$, whereas we conjecture $\textsc{Sat}(3;n) \leq 2^{\binom{n}{3} + o(n^3)}$.

The problem changes character for $k \geq n/2$; here it is more natural to think in terms of an alternative formulation: how many subsets of the n-cube 2^n can be written as unions of $(n-k)$-cubes?

We show that, if $r = n - k$ is a constant, then

$$\textsc{Sat}(n-r;n) = 2^{2^n - 2^{\frac{r-1}{2}\log^2 n + o(\log^2 n)}}.$$

For $r = n-k$ between about $\log\log\log n$ and $\log\log n$, $\textsc{Sat}(n-r;n)/2^{2^n}$ grows very rapidly, approximately as $2^{-2^{2^{2^r}}}$. For $r = n-k = \alpha n$ with $\alpha < \frac{1}{2}$, we show $2^{\beta 2^n} \leq \textsc{Sat}(k;n) \leq 2^{\beta' 2^n}$, for some $0 < \beta = \beta(\alpha) < \beta' = \beta'(\alpha) < 1$.

Combined connectivity orientation and augmentation problems

András Frank

We call an undirected graph $G = (V, E)$ (k,ℓ)-*partition-connected* $(0 \leq \ell \leq k)$ if $e(\mathcal{P}) \geq k(t-1) + \ell$ for every partition $\mathcal{P} = \{V_1, \ldots, V_t\}$ of V where $e(\mathcal{P})$ denotes the number of edges connecting distinct sets V_i.

When $\ell = 0$, this is equivalent, by a classical theorem of W.T. Tutte, to the existence of k edge-disjoint spanning trees. When $\ell = k$, this is k-edge connectivity. A digraph $D = (V, E)$ is called (k,ℓ)-edge connected, if there is a root–node s so that $\rho(X) \geq k$, $\delta(X) \geq \ell$ for every subset $X \subseteq V - s$, where $\rho(X)$ denotes the number of edges entering X

and $\delta(X) := \rho(V - X)$. By Menger's theorem this is equivalent to requiring that there are k edge-disjoint paths from s to V and ℓ edge-disjoint paths from v to s for every $v \in V$.

In 1980 I proved (in a more general form) that a graph G has a (k, ℓ)-edge-connected orientation iff G is (k, ℓ)-partition connected.

Theorem (Frank and T. Kiràly) Given a graph $G = (V, E)$ and degree specification $m : V \to \mathbb{Z}^+$, there is a graph $H = (V, E)$ so that $d_H(v) = m(v)\ \forall v \in V$ and $G + H$ is (k, ℓ)-partition-connected iff $m(V) := \sum m(v)$ is even,

$$\begin{array}{lcll} m(V)/2 & \geq & k(t-1) + \ell - e_G(\mathcal{P}), & \text{for every partition } \mathcal{P} = \{V_1, \ldots, V_t\} \text{ of } V, \text{ and} \\ m(V - V_i) & \geq & k(t-1) + \ell - e_G(\mathcal{P}), & \text{for } 1 \leq i \leq t. \end{array}$$

We also developed a min-max formula for the minimum number of new edges whose addition to G leaves a (k, ℓ)-partition-connected graph. Another result is an extension of Tutte's theorem for hypergraphs. We call a hypergraph *partition-connected* if there are at least $t - 1$ hypergraphs intersecting at least two members of every partition of V into t parts. (For graphs, this is equivalent to connectivity.)

Theorem (Frank, T. Kiràly, and M. Kriesell) A hypergraph can be partitioned into k spanning partition-connected subhypergraphs if and only if there are at least $k(t-1)$ hyperedges intersecting at least two members of every partition of V into t parts.

Corollary If the largest hyperedge of a hypergraph H has q elements and H is (qk)-edge-connected (i.e., there exist qk hyperedges intersecting X and $V - X$ for every $X < V$), then H can be decomposed into k connected (spanning) subhypergraphs.

A weighted version of Shearer's Entropy Lemma

Ehud Friedgut

Shearer's Lemma: Let $H(V, E)$ be a hypergraph. Let $F_1, \ldots F_r \subseteq V$ such that every vertex belongs to at least t of the sets F_i. Let $E_i = \{e \cap F_i : e \in E\}$, then

$$|E|^t \leq \prod_{i=1}^{r} |E_i|.$$

The weighted version: Let H, F_i, E_i be as above. For each i let $w_i : E_i \to \mathbb{R}^+$. Also, assign weights $\alpha_j(F_j)$ such that for every $v \in V$

$$\sum_{j: v \in F_j} \alpha_j \geq 1\ .$$

Then

$$\sum_{e \in E} \prod_{i=1}^{r} w_i(e_i)\ \leq\ \prod_{i=1}^{r} \left[\sum_{f \in E} w_i(f)^{\frac{1}{\alpha_i}} \right]^{\alpha_i}$$

Here's a nice consequence. Let X, Y, Z be measure spaces and consider

$f : X \times Y \to \mathbb{R}$
$g : Y \times Z \to \mathbb{R}$
$h : Z \times X \to \mathbb{R}$.

Then

$$\sqrt{\int f^2(x,y)dxdy \int g^2(y,z)dydz \int h^2(z,x)dzdx} \geq \int f(x,y)g(y,z)h(z,x)dxdydz.$$

Perfect matchings in random graphs with minimum degree 1 or 2

Alan Frieze
(joint work with Boris Pittel)

We consider the existence of perfect matchings in random graphs with n vertices, m edges and minimum degree at least 1 or 2. For minimum degree 1 we extend a result of Bollobas and Frieze to bipartite graphs. We show that roughly $n \log n/2$ edges are needed to obtain a perfect matching in a random bipartite graph with $n+n$ vertices, about 1/2 the number needed in the unconditioned case. We further show that with high probability a random graph with cn edges, $c > 2$ and minimum degree at least 2 has a perfect matching. For non-bipartite graphs we establish the limiting probability that a random graph with cn edges and minimum degree at least 2 has a perfect matching.

On Cycles in Matroids

Martin Grötschel

A circuit in a matroid is a minimal dependent subset of the ground set; a cycle is the disjoint union of circuits. A matroid is binary if and only if every cycle is the symmetric difference of some circiuts. Cycles in binary matroids are a very useful common generalization of Eulerian subgraphs and of cuts in a graph. In fact, the polyhedral characterizations of the cut and the Eulerian subgraph polytope carry nicely over to the cycle polytope, i.e., the convex hull of all incidence vectors of cycles, if the matroid is binary. Nothing, though, is known about cycle polytopes of non-binary matroids so far.

I will present in this talk complete and nonredundant characterizations of the circuit and the cycle polytopes of the uniform matroids $U(k,n)$. This result is derived from a slightly more general characterization of polytopes associated with cardinality homogeneous set systems.

On phase transition in the hard-core model on $\mathbf{Z}^d$

Jeff Kahn

It is shown that the hard-core model on $\mathbf{Z}^d$ exhibits a phase transition at activities above some function $\lambda(d)$ which tends to zero as $d \to \infty$. That is:

Consider the usual nearest neighbor graph on $\mathbf{Z}^d$, and write $\mathcal{E}$ and $\mathcal{O}$ for the sets of even and odd vertices (defined in the obvious way). Set

$$B_N = B_N^d = \{z \in \mathbf{Z}^d : \|z\|_\infty \le N\}, \ \partial B_N = \{z \in \mathbf{Z}^d : \|z\|_\infty = N\},$$

and write $\mathcal{I}(B_N)$ for the collection of independent sets (sets of vertices spanning no edges) in B_N. For $\lambda > 0$ let $\mathbf{I}$ be chosen from $\mathcal{I}(B_N)$ with $\Pr(\mathbf{I} = I) \propto \lambda^{|I|}$.

Theorem [D.Galvin and J. Kahn] For suitable constant C, if $\lambda > Cd^{-1/4}\log^{3/4} d$, then

$$\lim_{N\to\infty} \Pr(\underline{0} \in \mathbf{I} | \mathbf{I} \supseteq \partial B_N \cap \mathcal{E}) \neq \lim_{N\to\infty} \Pr(\underline{0} \in \mathbf{I} | \mathbf{I} \supseteq \partial B_N \cap \mathcal{O}).$$

Thus, roughly speaking, the influence of the boundary on behavior at the origin persists as the boundary recedes.

Reconstructing a Simple Polytope from its Graph

Volker Kaibel

(joint work with Michael Joswig and Friederike Körner)

Blind and Mani proved in 1987 that every isomorphism of the (abstract) graphs of two simple (convex) polytopes can be extended to an isomorphism of their face lattices. In particular, the entire combinatorial structure of a simple polytope is determined by its graph. Kalai (1988) found a short and elegant proof of that result, from which an algorithm can be devised that computes the vertex-facet incidences (from which the face lattice can be computed in polynomial time) of a simple polytope from its graph. However, the complexity status of the latter problem remained unclear. We describe a strongly dual pair (A), (B) of combinatorial optimization problems on the graph G of a simple polytope P with the property that the optimal solution of (A) is the set of 2-faces of P (from which one can compute the vertex-facet incidences in polynomial time) and the optimal solutions of (B) are the "AOF-orientations" of G (which are those acyclic orientations whose linear extensions correspond to the shelling orders of the boundary complex of the dual polytope of P). Thus, we obtain good characterizations of both the vertex-facet incidences of a simple polytope P and for its AOF-orientations in terms of the graph of P. This might eventually lead to a polynomial time (primal-dual) algorithm to compute the vertex-facet incidences of a simple polytope from its graph.

A coding problem for pairs of sets

Gyula O.H. Katona

Let X be an n–element finite set, $0 < k < n/2$ an integer. Suppose that (A_1, B_1) and (A_2, B_2) are pairs of disjoint k-element subsets of X (that is, $|A_1| = |B_1| = |A_2| = |B_2| = k, A_1 \cap B_1 = \emptyset, A_2 \cap B_2 = \emptyset$). Define the distance of these pairs by $d((A_1, B_1), (A_2, B_2)) = \min\{|A_1 - A_2| + |B_1 - B_2|, |A_1 - B_2| + |B_1 - A_2|\}$. One can see that this is really a distance on the space of such pairs. $C(n, k, d)$ denotes the maximum number of pairs (A, B) with pairwise difference at least d. The motivation comes from database theory. The lower and upper estimates use Hamiltonian type theorems, traditional code constructions and Rödl's method for packing.

Sperner type theorems and Sperner capacity: new results via intertwining of sequences

János Körner

(joint work with A. Monti)

Sequence intertwining is a construction method introduced by Gargano, Körner and Vaccaro (Graphs and Combinatorics, 1993) to build a construction achieving separation for any two graphs based on constructions achieving the same for the two graphs individually, and this without loss in the exponential asymptotics. This technique is not functioning for corresponding problems involving hypergraphs.

A significant exception to this is the proof of a theorem (with Angelo Monti, JCT to appear) presented here. If $t_\Delta(p)$ is the asymptotic exponent of the maximum cardinality of a strong Δ system in the sense of Erdős-Szemerédi (JCT, 1978), and t_6 is the asymptotic exponent of the maximum cardinality of a set of bipartitions of an n-set such that the roughest common refinement of any 3 of them has at least 6 atoms, then we show that

$$t_6 = \min_{p\in[0,1]} (t_\Delta(p), t_\Delta(1-p))$$

This implies that $t_6 = 1$ iff $t_\Delta = \max_p t_\Delta(p) = 1$.

A further new application of the sequence intertwining technique is to show that the minmax theorem of Gargano, Körner and Vaccaro (JCT-A, 1994) for the Sperner capacity of a family of digraphs holds for graph separation in the sense in which two sequences are G-separated if $\underline{x} \in X^n, \underline{y} \in X^n$, and for the graph G with vertex set X^2 there exists an index $i \le n-1$ such that $\{x_i x_{i+1}, y_i y_{i+1}\} \in E(G)$.

Coloring uniform hypergraphs with few colors

Alexandr Kostochka

Let $m(r,k)$ (respectively $m^*(r,k)$) denote the minimum number of edges in an r-uniform (respectively, simple r-uniform) hypergraph which is not k-colorable. The study of behaviour of $m(r,k)$ and $m^*(r,k)$ was originated by P. Erdős about 40 years ago and yielded some striking results. The aim of the talk was to survey the state of art of the topic and present the following result:
Theorem. For every k there are $c = c(k)$ and $r_0 = r_0(k)$ s.t. for every $r \ge r_0$

$$m(r, 2^k) \ge \left(2^k\right)^r \cdot c \cdot \left(\frac{r}{\ln r}\right)^{\frac{k}{k+1}}.$$

Induced subdivisions in $K_{s,s}$-free graphs

Daniela Kühn

(joint work with Deryk Osthus)

A classical theorem of Mader states that for every graph H there exists $d = d(H)$ such that every graph G of average degree at least d contains a subdivison of H. (A subdivision of a graph H is a graph obtained by replacing the edges of H with internally disjoint paths.) Obviously, the result becomes false if we ask for an *induced* subdivision of H. However, this stronger assertion does hold if G is 'locally sparse' in the sense that it does not contain a complete bipartite graph $K_{s,s}$:

Theorem. *For every graph H and every s there exists $d = d(H,s)$ such that every graph G of average degree at least d contains either a $K_{s,s}$ as a subgraph or an induced subdivision of H.*

Semidefinite relaxations via sums of squares of polynomials

Monique Laurent

Let $K = \{x \in \mathbb{R}^n \,|\, g_\ell(x) \ge 0 \quad \ell = 1 \dots m\}$ be a semi-algebraic set where g_ℓ are polynomials in $x = (x_1, \dots, x_n)$ of degree 1 and $P = \mathrm{conv}(K \cap \{0,1\}^n)$. Lasserre (2000) proposed the following method for approximating P via semidefinite relaxation.

Given $y \in \mathbb{R}^{P_t(V)}$ a vector indexed by subsets of $V = \{1, \dots, n\}$ of size at most t, its moment matrix of order t is defined as

$$M_t(y) := (y(I \cup J))_{I,J\subseteq V,\ |I|,|J|\le t}.$$

Given two vectors g, y indexed by subsets of V, $g * y$ is the vector with entries $g * y(I) := \sum_H g_H y(H \cup I)$.

Let $Q_t(K)$ denote the projection on $\mathbb{R}^V$ of set of vectors $y \in \mathbb{R}^{P_t(V)}$ for which $M_t(y) \succeq 0$, $M_{t-1}(g_\ell * y) \succeq y0$ for all ℓ. Then, $P \subseteq Q_{n+1}(K) \subseteq \cdots \subseteq Q_1(K) \subseteq K$ and there is finite convergence $P = Q_{n+1}(K)$. The original proof of convergence of Lasserre is based on a result of real algebraic geometry [under some technical assumption on K, every polynomial positive on K can be decomposed as $p_0 + \sum_\ell p_\ell g_\ell$ where p_0, p_ℓ are sums of squares of polynomials] due to Putinar (1993). Laurent (2001) gives a simple direct combinatorial proof. Moreover, she shows that the Lasserre hierarchy refines the hierarchy obtained via the Lovász-Schrijver iterative method (1991). Precisely,

$$Q_t(U) \subseteq N_+^{t-1}(U) \text{ for } t \geq 1 \ .$$

Applications to the maximum stable set problem and the maximum cut problems are presented. In both cases the first order Lasserre relaxation ($t = 1$) coincides with the basic semidefinite relaxation of the problem (the theta body $TH(G)$ in the case of stable sets). A geometric result about moment matrices in the ± 1-variable setting is proved.

Theorem. Let

$$M_t(y) := (y(I \cup J))_{I,J \subseteq V,\ |I|,|J| \leq t,\ |I|,|J| \equiv t \mod 2}.$$

If $Y \succeq 0$ and rank $M_1(y) \leq t$ then Y is a convex combination of 2^{t-1} cut matrices.

Consistency for Partition Regular Equations

Imre Leader

(joint work with Neil Hindman and Dona Strauss)

A matrix A with rational entries is called *partition regular* if, whenever $\mathbb{N}$ is finitely coloured, there is an (integer) vector x with A_x monochromatic. In the *finite* case, the partition regular matrics were characterized by Rado, and as a consequence we have *consistency*: if A and B are partition regular then so is $\binom{A\,0}{0\,B}$.

In the infinite case, consistency is known to fail. But what about consistency for the most trivial systems, namely those just following from Ramsey's theorem – for example, we know that whenever $\mathbb{N}$ is finitely coloured there are distinct $x_1, x_2 \dots$ with $\{x_i + x_j : i \neq j\}$ monochromatic *and* distinct $y_1, y_2 \dots$ with $\{y_i + 2y_j : i < j\}$ monochromatic. Are these consistent (i.e., can we solve both in the same colour class)? Here we give a positive answer. Surprisingly, the proof uses a lot of work in the Stone-Čech compactification $\beta\mathbb{N}$.

Euclidian distortion in graphs of high girth

Nati Linial

Let (X, d) be a finite metric space. We denote by $c_2(X)$ the least metric distortion (Lipschitz constant) with which (X, d) can be embedded in ℓ_2. It is known (Bourgain) that $c_2(X) \leq O(\log n)$ for every n-point metric X and this bound is tight (joint work with London and Rabinovich) and is attained for the metric of constant degree expander. It is also known that $c_2(X)$ is poly-time computable. It was conjectured that if (X, d) is the metric of a graph of girth g where all vertex degrees are ≥ 3, then $c_2(X) \geq \Omega(g)$. In this talk I survey recent work with A. Magen and A. Naor where we show $c_2(X) \geq \Omega(\sqrt{g})$ for regular graphs. The exact bound is still unknown. It is also not known if any lower bound on ℓ_1-embeddings exists which tends to ∞ with g.

Ramsey properties of families of graphs

Tomasz Łuczak

(joint work with R.L. Graham, V. Rödl, A. Ruciński, and S. Urbański.)

For two families of graphs $G_1, \ldots, G_r$ and $H_1, \ldots, H_s$ we write $(G_1, \ldots, G_r) \to (H_1, \ldots, H_s)$ if for any graph F such that $F \to (G_1, \ldots, G_r)$ we have also $F \to (H_1, \ldots, H_s)$. We survey the known results on the relation $(G_1, \ldots, G_r) \to (H_1, \ldots, H_s)$ and its vertex-partition analog, and state some of many open questions concerning them.

Unique-sink orientations of cubes

Jiří Matoušek

The n-cube is considered as a graph (with vertex set $\{0,1\}^n$). An unique-sink orientation (USO) is an orientation of the edges of the n-cube such that every face of the cube has exactly one sink (directed cycles are allowed). Such orientations arise from several sources, such as linear programs (considering a generic linear function on a polytope isomorphic to the cube), certain linear complementarity problems, and certain convex programs. Algorithms have been studied for finding the global sink in a USO; the USO is specified by an oracle that, given a vertex, returns the orientation of the edges incident to that vertex. Upper and lower bounds for the complexity of such algorithms have recently been given by Welzl, Szabó, and Schurr; improving them significantly is the main challenge. The speaker has proved that the number of USO is $2^{\Theta(2^n \log n)}$. The number of acyclic USO is easily seen to be between $2^{\Omega(2^n)}$ and $2^{O(2^n \log n)}$; it would be nice to find better bounds.

Some new results on hypergraph Turán number

Dhruv Mubayi

We obtain new bounds for the Turán numbers of several classes of hypergraphs, both degenerate and nondegenerate. The main result is summarized below.

Let t, n be integers with $n \geq 3t$. Let F be any family of at least $t^4\binom{n}{2}$ triples from an n-element set X. Then there exist $2t$ triples $A_1, B_1, \ldots A_t, B_t$ and distinct elements $a, b \in X$ such that $A_i \cap A_j = \{a\}$ and $B_i \cap B_j = \{b\}$, for all $i \neq j$, and

$$A_i \cap B_j = \begin{cases} A_i - \{a\} = B_j - \{b\} & \text{for } i = j \\ \emptyset & \text{for } i \neq j \end{cases}$$

When $t = 2$, the upper bound $t^4\binom{n}{2}$ is improved to $3\binom{n}{2} + 6n$. This improves upon the previous best known bound of $3.5\binom{n}{2}$ due to Füredi.

Bounds for graphs

Jaroslav Nešetřil

The lecture covered the solution of two problems which were posed at the last Oberwolfach meeting on combinatorics (2000):

1. **Antisymmetric flows**
   Thm. Every orientation of a 3-connected graph G has a $\mathbb{Z}_3^6 \cdot \mathbb{Z}_6^6$-flow f which satisfies
   $$f(e) \neq -f(e')$$
   for any edges $e, e' \in E(G)$.
   This solves a problem of Nešetřil and Raspaud and extends a result of Seymour and DeVos and Johnson (who first established a universal bound). It is due to DeVos, Nešetřil and Raspaud.
2. **K_k-free bounds**
   The coloring order $\mathcal{C}$ (induced by graphs and homomorphisms) has the following property (established jointly with P. Orsona Mendes):
   Thm. For any minor closed class $\mathcal{K}, \mathcal{K} \neq$ all graphs and any $k \geq 3$ there exists a graph $H(\mathcal{K}, k) = H$ with the following property:
   1. $K_k \not\to H$
   2. For any $G \in \mathcal{K}, K_k \not\to G$ implies $G \to H$.

Topological minors in graphs of large girth

Deryk Osthus

(joint work with Daniela Kühn)

Theorem *Every graph of minimum degree at least r and girth at least 186 contains a subdivision of K_{r+1} and for large r a girth of at least 15 suffices.*

This improves a result of Mader, who gave a bound on the necessary girth which is linear in r. It also implies that the conjecture of Hajós that every graph of chromatic number at least r contains a subdivision of K_r (which is false in general) is true for graphs of girth at least 186 (or 15 if r is large). For ordinary minors we obtained the following result:

Theorem *Every $K_{s,s}$-free graph of average degree at least r contains a K_t-minor for all $t \leq r^{1+\frac{1}{2(s-1)}+o(1)}$.*

This implies the conjecture of Hadwiger for $K_{s,s}$-free graphs whose chromatic number is sufficiently large compared to s.

Remarks on the construction of Hadamard matrices

Alexander Pott

(joint work with U.T. Arasu and Y.Q. Chen)

A Hadamard matrix H of order n is a ± 1-matrix of size n such that $H \cdot H^t = n \cdot I_n$. It is an open question whether they exist for all n divisible by 4. There are many constructions known. However, they are mostly limited in the sense that it can be shown that they are not powerful enough to construct Hadamard matrices for all $n \equiv 0 \mod 4$. In the talk I suggest a method to construct H-matrices which seems to be rather powerful. The matrices are cocyclically developped; the idea to use such matrices goes back to Ito and DeLauney/Horadam. We know of no single value $n \equiv 0 \mod 4$ for which such matrices

cannot exist. But, on the other hand, there are still many integers n for which we do not know whether a construction works.
I also discuss a similar approach to conference matrices. These are matrices in $\{0, +1, -1\}^n$ such that $C \cdot C^t = (n-1)I_n$. It is known that no circulant conference matrices can exist. Negacirculant matrices are known if $n = p^f + 1$ (p prime). It has sometimes been conjectured that no other sizes of group developped conference matrices can exist. It is therefore surprising that we can find cocyclic conference matrices of size $2(p^f + 1)$ if $p^f \equiv 3 \mod 4$).

$\mathcal{S}$-paths

Alexander Schrijver

We present a short proof of Mader's theorem on the maximum number of disjoint $\mathcal{S}$-paths in a graph. The method is based on an exchange property of $\mathcal{S}$-paths, which also leads to a matroid. Moreover, a new derivation is given of the disjoint $\mathcal{S}$-path problem from the matching problem.

Independent sets, lattice gas and the Lovász Local Lemma

Alexander Scott
(joint work with Alan Sokal)

Let G be a graph with vertex set X. We say that G is a *dependency graph* for a collection of events $(A_x)x \in X$ (in some probability space) if for each $x \in X$ and $Y \subset X \backslash (\Gamma(x) \cup \{x\})$ the event A_x is independent of the σ-algebra $\sigma(A_y : y \in Y)$. Given such a collection of events, with probabilities $(p_x)_{x\in X}$, what is the largest real r such that we can guarantee $\mathbb{P}(\bigwedge_{x\in X} \overline{A}_x) > 0$? In particular, for what $(p_x)_{x\in X}$ can we guarantee $r > 0$? An important tool for dealing with this setup is the Lovász Local Lemma, which gives a qualitative lower bound on $\mathbb{P}(\bigwedge_{x\in X} \overline{A}_x)$.

The partition function of the hard-care lattice gas, with variables $(z_x)_{x\in X}$, is the polynomial

$$Z_G = \sum_{I \text{ independent}} \prod_{x\in I} z_x.$$

Mathematical physicists have devoted significant effort to finding complex polydiscs in which the partition function Z_G is nonvanishing.

This talk discusses the close relationship between these problems: in particular, $(p_x)_{x\in X}$ guarantees $\mathbb{P}(\bigwedge_x \overline{A}_x) > 0$ if and only if the partition function Z_G is nonvanishing in the polydisc $(|z_x| \leq p_x : x \in X)$. Furthermore, the Locász Local Lemma turns out to be equivalent to a result found twenty years later by the mathematical physicist Dobrushin. Finally, a softened version of Dobrushin's argument translates back into the probabilistic context to give a softened version of the Lovász Local Lemma.

On the number of $\mathcal{L}$-free graphs

Miklós Simonovits
(joint work with Jósef Balogh and Béla Bollobás)

Given a family $\mathcal{L}$ of graphs, let

$$p = \min_{L \in \mathcal{L}} \chi(\mathcal{L}) - 1$$

and $\mathcal{P}(n, \mathcal{L})$ be the set of $\mathcal{L}$-free graphs on $\{1, \ldots, n\}$ (labelled). Extending, and sharpening the Erdős-Frankl-Rödl theorem we prove that

$$|\mathcal{P}(n, \mathcal{L})| \leq 2^{\frac{1}{2}(1-\frac{1}{p})n^2 + O(n^{2-\gamma})}$$

where we also characterize the best γ in this estimate, in terms of the "decomposition class" of $\mathcal{L}$, by connecting this problem to the extremal problem $ex(n, \mathcal{L})$: the error terms in these two cases basically are the same.

Frequency of subgraphs in (G_n)

Vera T. Sós

We say that a graph L (respectively induced L) is p-uniformly distributed in (G_n), if the number of copies (respectively induced copies) of L in every $S \subseteq V(G_n)$ is

$$p^{\ell}|S|^v + o(n^v)$$

respectively

$$p^{\ell}(1-p)^{\binom{v}{2}-e}|S|^v + o(n^v)$$

(where $v = |V(L)|$, $e = |E(L)|$).

We proved with M. Simonovits, that for an arbitrary $L_0 \neq \overline{K}$ if L_0 is p-uniformly distributed in (G_n), then this implies that every $L \neq \overline{K}$ is p-uniformly distributed.

For induced L the analogous statement does not hold for every L_0. E.g., $L_0 = P_3$ (or $L_0 = \overline{P_3}$) is a counterexample. For every p there is a (G_n) s.t. P_3 is p-uniformly distributed in (G_n) but even edges ($L = K_2$) are not p-uniformly distributed in (G_n). (We determine all graph sequences (G_n) with that property.)

We proved that if L_0 is regular and induced L_0 is p-uniformly distributed in (G_n), then this implies that every induced L is p-uniformly distributed in (G_n). We conjecture that $L_0 = P_3$ and $L_0 = \overline{P_3}$ are the only counterexamples.

K_4-free subgraphs of random graphs revisited

Angelika Steger
(joint work with S. Gerke, H.J. Prömel, T. Schickinger, and A. Taraz)

In Combinatorica 17(2), 1997, Kohayakawa, Łuczak and Rödl state a conjecture which would permit the application of Szemerédi's regularity lemma for the estimation of ex($G_{n,p}$, G), where ex($G_{n,p}$, G) denotes the maximum number of edges in a G-free subgraph of a random graph $G_{n,p}$. In this talk we outline a proof of their conjecture for $G = K_4$, thereby providing a comparatively short proof for the original result of Kohayakawa, Łuczak and Rödl .

On Ramsey numbers of sparse graphs

Benny Sudakov
(joint work with Sasha Kostochka)

The Ramsey number of a graph G, denoted by $r(G)$ is the minimum integer N such that in any 2-coloring of the edges of the complete graph K_N on N vertices, there is always a monochromatic copy of G. In 1975 Burr and Erdős posed a problem of estimating Ramsey numbers of d-degenerate graphs. They conjectured that for all d there exist a constant $c(d)$ such that all graphs G of order n in which every subgraph has minimum degree at most d satisfy

$$r(G) \leq c(d)n \ .$$

This conjecture attracted a lot of attention in the last two decades and is still open despite attempts by various researchers to attack it.

In this talk we prove the following slightly weaker result. Let d be a fixed integer and G be a d-degenerate graph of order n. Then the Ramsey number of G is bounded by

$$r(G) \leq n^{1+o(1)}$$

where $o(1) \to 0$ as $n \to \infty$.

On the density of sequences of integers the sum of no two of which is a square

Endre Szemerédi
(joint work with Ayman Khalfalah and Sachin Lodha)

Erdős and Silverman proved the problem of determining the maximal density attainable by a set $S = \{s_i\}$ of positive integers having the following property NS: $s_i + s_j$ is not a perfect square whenever $i \neq j$.

Massias discovered that the set S_1 consisting of all $x \equiv 1 \pmod 4$ together with $x \equiv 14, 16, 30 \pmod{32}$ has property NS and density $\frac{11}{32}$. Lagasios, Odlyzko, Shearer proved that if the set S is the union of arithmetical progressions than Massias's example is the best. We prove it for general sets S.

Lattice gases and the number of partial orders

Anusch Taraz
(joint work with H.J. Prömel and A. Steger)

Denote by $\mathcal{P}_{n,d}$ the set of labelled partial orders on n points with $\lfloor dn^2 \rfloor$ comparable pairs. Define the function $c(d)$ by letting $|\mathcal{P}_{n,d}| = 2^{c(d)n^2+o(n^2)}$. The question of determining $c(d)$ was raised by Deepak Dhar, who suggested a model for a lattice gas where the system is described by a partial order, the energy be proportional to the number of comparable pairs, and the entropy given by $c(d)$.

In a series of papers (around 1980), Dhar, as well as Kleitman and Rothschild, determined $c(d)$ in the range $0 < d \leq \frac{3}{10}$. We complete the picture for the whole range $0 < d \leq \frac{1}{2}$ and show that infinitely many phase transitions in the structure of a "typical" partial order occur.

On the Strong Perfect Graph Conjecture

Robin Thomas

A *hole* in a graph is an induced cycle of length at least four. An *antihole* in a graph is an induced subgraph isomorphic to the complement of a cycle of length at least four. A graph is *Berge* if it has no odd hole and no odd antihole. The Strong Perfect Graph Conjecture of Berge from 1960 asserts that every Berge graph is *perfect* (that is, $\chi(H) = \omega(H)$ for every induced subgraph H).

Neil Robertson, Paul Seymour and I have been working on the SPGC for the past two years. In my talk I have given an outline of our strategy for attacking the SPGC. I have also mentioned the following:

Theorem: Let G be a graph with an odd hole and no K_4 subgraph. Then G is 4-colorable.

This is special case of a conjecture of Gyarfas, and the proof method may give a clue for the general case.

Extremal Properties of Graph Minors

Andrew Thomason

Mader defined, and showed the existence of, the function $c(t) = \inf\{\, c : e(G) \geq c|G|$ implies $G \succ K_t\}$, where $G \succ H$ means that H is a subcontraction, or minor, of G. Kostochka proved that $c(t)$ has order of magnitude $t\sqrt{\log t}$ for large t, with random graphs providing the best known lower bounds. Recently the speaker found the asymptotic expression $c(t) \sim (\gamma + o(1))t\sqrt{\log t}$, the constant γ being exactly that given by the random examples. It has also been shown that any graph of positive density contains complete minors at least as large as those found in a random graph of the same order and density (provided the connectivity of the graph is not vanishingly small).

Sós asked whether the only examples attaining equality are pseudo-random graphs; that is, if a graph has no minor larger than that of a random graph of the same order and density, then it is pseudo-random. We describe an argument of Myers that gives a positive answer to this question.

Myers and I have also considered, for a general graph H, the extremal function $c(|H|) = \inf\{\, c : e(G) \geq c|G|$ implies $G \succ H\}$, and ask whether there is some simple property of H that determines the order of magnitude of $c(H)$ (in the same way that the chromatic number determines the ordinary extremal function). It is shown that $c(K_{\beta t,(1-\beta)t}) \sim c(K_{\beta t} + \overline{K}_{(1-\beta)t}) \sim \sqrt{4\beta(1-\beta)}c(K_t)$, and the extremal function for all complete multipartite graphs is found. In general it seems that $c(H) \sim c(|H|)$ provided H is "nowhere sparse" (in a precise sense).

Finally we describe a relationship between minors and linking. Using arguments of Mader and Thomassen it is shown that graphs of connectivity only $2k$ are k-linked provided the girth is at least 133. Elsewhere at this meeting, Kühn and Osthus describe their recent results which are much better than this.

Classification of locally 2-connected compact metric spaces

Carsten Thomassen

A connected metric space M is called locally 2-connected if, for every point x in M and every open set U containing x, there is an open set U' contained in U and containing x such that both U' and $U' - x$ are connected. If we impose the additional conditions that M is compact and contains no infinite complete graph, then surprisingly, M can be embedded in a 2-dimensional compact surface S. Moreover, S can be chosen such that M and S contain precisely the same finite graphs. One application is a complete solution of the k-arc-problem (the problem of finding k pairwise disjoint simple arcs with prescribed ends) for M and each fixed natural number k. The first step in the proof of the classification result is the following generalization of Kuratowski's theorem: A 2-connected, compact, locally connected topological space M can be embedded in the 2-sphere if and only if M is metrizable and contains none of the Kuratowski graphs K_5 or $K_{3,3}$.

Unavoidable Cycles in Graphs

Jacques Verstraete

A set S of integers is called *unavoidable* if there exists a constant c such that every graph of average degree at least c contains a cycle of length in S. We prove an old conjecture of Erdős, stating that there exists an unavoidable set of density zero proving the following Theorem:

Theorem. There exists an unavoidable set S such that $|S \cap \{1, 2, \dots, n\}| = o(n^{1-1/100})$ as $n \to \infty$.

As a consequence, we prove another conjecture of Erdős stating that the number of sets $C(G) = \{\ell : \text{ there exists a cycle of length } \ell \text{ in } G\}$ over graphs G of order n is $o(2^n)$.

The Complexity of some Polynomial Invariants

Dominic Welsh

I shall present results on the computational complexity of computing invariants of three classical polynomials namely the chromatic, flow and reliability polynomials of a graph.

Each of these is intimately related to the partition function of the Ising-Potts partition function of statistical physics and all are specialisations of the Tutte polynomial. In 1990, with F. Jaeger and D.L. Vertigan, we showed that unless #P=P (which is most unlikely) there is no polynomial time evaluation algorithm except at 8 special points and along one special curve of the plane. In this talk I shall concentrate on the complexity of computing various coefficients of these polynomials and show that unless NP=RP, these coefficients do not even have good randomised approximation schemes. We also introduce a quasiorder induced by approximation reducibility. Our nonapproximability results are obtained by showing that various predicates based on the coefficients are NP hard. It also turns out that there is a significant difference between the case of graphs where, using Robertson-Seymour theory many of these predicates can be shown to be in P and nongraphic but representable matroids where we show that in many cases no approximation scheme can exist unless NP=RP.

(This work is joint with James Oxley and will be appearing in Combinatorics, Probability and Computing 2002.)

Combinatorial Proofs for some Kneser-Type Coloring Theorems

Günter M. Ziegler

A general Kneser hypergraph $KG_s^r(\mathcal{S})$ may be defined by all the r-tuples of sets from a set system $\mathcal{S} \subseteq 2^{[n]}$ such that no element is contained in more than s of the sets. We give a lower bound for the chromatic numbers of such hypergraphs in terms of an "s-disjoint r-colorability defect." Our bound is sharp for many (but not all!) instances of the type $\mathcal{S} = \binom{[n]}{k}$, and thus implies results/theorems of Lovász (the Kneser conjecture), Alon-Frankl-Lovász, Dolnikov, Kriz and Sarkaria.

Furthermore, we sketch an entirely combinatorial proof of our result, using a $\mathbb{Z}_p$-analogue of the octahedral Tucker lemma, as well as Lefschetz numbers of self-maps of chain complexes.

(Both our theorem, and the combinatorial proof, extend and were inspired by work (and lectures) of J. Matoušek, specifically his simple proof of Kriz' theorem, and his combinatorial proof of the Kneser conjecture.)

Edited by Anusch Taraz

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