Mathematisches Forschungsinstitut Oberwolfach

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Optimization and Applications

January 13th – January 19th, 2002

The meeting was organized by Florian Jarre (Düsseldorf), Claude Lemaréchal (Saint Ismier), and Jochem Zowe (Erlangen).

The workshop was focused on most recent topics in continuous optimization, including

- semidefinite programming
- mathematical programs with equilibrium constraints
- interior-point and smoothing methods
- stochastic programming

Although these topics represent separate disciplines, they are linked with each other in many ways. Solution techniques can be adapted, problems of one type appear as subproblems in another discipline or serve as approximations for problems in other disciplines. Due to the wide range of topics this conference provided a good opportunity for the attending experts to profit from many informal discussions.

Special emphasis was given to applications of continuous optimization. Several talks showed how continuous problems and techniques can be used to provide excellent approximations for discrete problems. Other talks showed applications in control and examples where modern optimization methods can substantially reduce costs of industrial processes and/or speed up the corresponding analysis.

The workshop was attended by 46 participants. Among these were many young participants and many participants from overseas. Most participants of either of these two categoies were given the possibility to give a presentation resulting in a somewhat higher number of a total of 35 talks of 30 to 40 minutes each. Nevertheless there was still plenty of time for informal exchanges. The pleasant atmosphere in Oberwolfach contributed to the overall succsess of the meeting. The feedback from many participants during and after the conference was very positive.

Abstracts

Properties of a New Semidefinite Relaxation for the Max-Cut Problem

MIGUEL F. ANJOS
(joint work with Henry Wolkowicz)

Semidefinite programming (SDP) has become an area of intense research in recent years. One reason for this is its application to obtain tight convex relaxations for quadratic boolean optimization. We consider the Max-Cut problem for a complete graph with arbitrary edge weights.

Anjos and Wolkowicz recently introduced two new SDP relaxations for the Max-Cut problem that are obtained using a "second lifting". After briefly sketching the derivation of the tighter of these relaxations, we describe some connections between the rank of the matrix variable of the relaxation and the geometric properties of the matrix obtained by projecting it back to the lower dimensional space. We also present numerical evidence for the strength of this relaxation for several types of graphs and edge weights.

The Steinberg Wiring Problem

KURT ANSTREICHER
(joint work with Nathan Brixius)

In 1961 L. Steinberg posed a problem of arranging 34 computer components on a 4 by 9 grid so as to minimize the length of the wiring required to interconnect them. Steinberg's problem can be written as a quadratic assignment problem (QAP), and has resisted solution attempts for 40 years. We describe a branch and bound (B&B) algorithm based on the Gilmore-Lawler bound that obtains the first optimal solution of the 1-norm version of the problem. The algorithm makes extensive use of strong branching to minimize the growth of the B&B tree. The solution time of approximately 8 days on a single CPU is far less than what would be expected based on previous results for other grid-based QAPs.

The Matrix Cube Problem and Related Uncertain Semi-Definite Problems

AHARON BEN-TAL
(joint work with Arkadi Nemirovski)

We present efficiently verifiable sufficient conditions for the validity Of specific NP-hard semi-infinite systems of Linear Matrix Inequalities (LMI's) arising from LMI's with uncertain data and demonstrate that these conditions are "tight" up to an absolute constant factor. In particular, we prove that given an $n \times n$ "matrix cube" $\mathcal{U}_\rho = \{A \mid \mid A_{ij} - A^*_{ij} \mid \leq \rho C_{ij}\}$, one can build a computable lower bound accurate within the factor $\frac{\pi}{2}$, on the supremum of those ρ for which all instances of $\mathcal{U}_\rho$ share a common quadratic Lyapunov function. We then obtain a similar result for the problem of Quadratic Lyapunov Stability Synthesis. The matrix cube result is also extended to the complex case, and is generalized to more general "structured" uncertainty sets. The latter is used to recover tight bounds on the Structured Singular Value, which plays an important role in Robust Control.

An Error Bound Result for Nonlinear Equations with Nonisolated Solutions and Applications

Andreas Fischer

Let us consider a sufficiently smooth nonlinear equation (with nonisolated solutions) and assume that (close to the solution set) the distance of a point to the solution set is bounded by a constant multiplied with the norm of the residual of the nonlinear equation.

Under this error bound condition it can be shown that the norm of the residual of the nonlinear equation which arises from the necessary optimality conditions applied to the least squares formulation of the original nonlinear equation is again an error bound for the distance of a point to the solution set of the original nonlinear equation.

This result is then used to show local Q-quadratic convergence of a Levenberg-Marquardt type algorithm where the regularization parameter can be chosen proportional to the norm of the residual of the nonlinear equation.

As a particular application we consider a nonlinear complementarity problem (with possibly nonisolated solutions) reformulated as a semismooth equation. By means of an active inequality identification this equation is modified so that it becomes sufficiently smooth and the Levenberg-Marquardt type algorithm can be applied. In combination with a globally convergent algorithm for the semismooth equation, an algorithm for solving complementarity problems with global and local superlinear convergence is obtained.

Complexity of Convex Optimization based on Geometry-based Measures and a Reference Point

Robert M. Freund

We consider the convex optimization problem: minimize $c^T x$ subject to constraints $Ax = b$ and $x \in P$, where P is a closed convex set. We bound the complexity of computing an ϵ-optimal solution of this problem using an interior point algorithm. Our complexity bound involves natural geometric measures of the feasible region and the ϵ-optimal level set, measured from a given reference point that is specified as part of the input to the algorithm.

Nonconvex Semidefinite Programs Arising in the Design

Roland W. Freund
(joint work with Florian Jarre)

Matrix-Padé approximation is a powerful tool for generating reduced-order models of large-scale time-invariant linear dynamical systems. However, when applied to passive descriptor systems, matrix-Padé reduced-order models do not preserve passivity in general. One approach to restoring passivity of the reduced-order models is to use structured low-rank modifications of the matrices describing the non-passive matrix-Padé model. The use of such low-rank modifications ensures that the new model is almost as accurate as the Padé model. In order to determine appropriate low-rank modifications that yield passive models, we first describe extensions of the classical positive real lemma for regular linear dynamical systems to general descriptor systems. These extensions characterize passivity of a descriptor system via the solvability of certain nonconvex semidefinite programming problems. By determining the low-rank modifications via solutions of such semidefinite programs, one can guarantee passivity of the reduced-order model. We also sketch two interior-point methods for the solution of these semidefinite programs, and we present some numerical results.

Analytic Centers Cutting Plane Methods and Mixed Integer Programming

Jean-Louis Goffin
(joint work with Samir Elhedhli)

The most effective way to solve realistic MIPs (mixed integer programs) is branch and price, which is based on Lagrangean relaxation. Lagrangean relaxation provides better bounds than the traditional branch and bound method, which relax the integer requirement.

At every node of the B&B tree, a nondifferentiable convex function (NDO) needs to be optimized. The classical NDO techniques, such as the Dantzig-Wolfe decomposition algorithm or subgradient optimization, have weaknesses, such as unreliable convergence or the lack of a rigorous termination criterion. The analytic center cutting plane method (ACCPM) attempts to improve over this.

We will sketch a full branch and price method that uses extensions of ACCPM, including hot starts at the child nodes, using a dual Newton method.

Numerical results will be presented in problems arising in supply chain optimization.

Maximal Monotone Operators: Theory, Applications, Special Cases

Osman Güler

The concept of maximal monotone operators (mmo) offers a unification of convex functions, concave-convex saddle functions, and the variational inequalities connected with them. The first applications of the area was to partial differential equations through the evolution equations associated with the semigroup on a mmo. The first applications to convex programming were suggested by Martinet and Rockafellar in the early seventies and gave rise to proximal point methods, augmented Lagrange methods, and decomposition techniques. The subject is essentially built upon on a seminal result by Minty, and a few other related techniques peculiar to the area. However, these techniques allow one to obtain very sharp results. In our talk, we first give an overview of the subject and show how convex and saddle functions are part of the subject. We then explain a result of Krauss showing how mmo are intimately connected with (skew-symmetric) concave-convex functions, thus closely related to convexity. Next, Rockafellar's theorem on cyclic mmo is viewed as a result in potential theory. Finally, we give a characterization of mmo with affine graphs, and indicate directions for future research on related topics, including applications in interior-point methods.

Shape Optimization in Contact Problems with Coulomb Friction

Jaroslav Haslinger and Jiří Outrata
(joint work with Michal Kočvara)

We investigate a discretized problem of shape optimization of elastic bodies in unilateral contact. The aim is to extend the existing results to the case of contact problems following the Coulomb friction law. Mathematical model of the Coulomb friction problem leads to a quasi-variational inequality. It is shown that for small coefficients of friction, the discretized problem with Coulomb friction has a unique solution and that this solution is Lipschitzian as a function of a control variable, describing the shape of the elastic body.

The shape optimization problem belongs to a class of so-called mathematical programs with equilibrium constraints (MPECs). The uniqueness of the equilibria for fixed controls enables us to apply the so-called implicit programming approach. Its main idea consists in minimization of a nonsmooth composite function generated by the objective and the (single-valued) control–state mapping. In this problem, the control–state mapping is much more complicated than in most MPECs solved so far in the literature, and the generalization of the relevant results is by no means straightforward. Numerical examples illustrate the efficiency and reliability of the suggested approach.

A Semidefinite Cutting Plane Approach to Quadratic 0–1 Programming

Christoph Helmberg

The recent spectral bundle method allows to compute, within reasonable time, approximate dual solutions of large scale semidefinite quadratic 0-1 programming relaxations. We show that it also generates a sequence of primal approximations that converge to a primal optimal solution. Separating with respect to these approximations gives rise to a cutting plane algorithm that converges to the optimal solution under reasonable assumptions on the separation oracle and the feasible set. We have implemented a practical variant of the cutting plane algorithm for improving semidefinite relaxations of constrained quadratic 0-1 programming problems by odd-cycle inequalities. We also consider separating odd-cycle inequalities with respect to a larger support than given by the cost matrix and present a heuristic for selecting this support. Our preliminary computational results for max-cut instances on toroidal grid graphs and balanced bisection instances indicate that warm start is highly efficient and that enlarging the support may sometimes improve the quality of relaxations considerably.

Smoothing-Type Methods for Semidefinite Programs

Christian Kanzow

(joint work with Christian Nagel)

Motivated by some results for linear programs and complementarity problems, we present some new characterizations of the central path conditions for semidefinite programs. These new characterizations are obtained by suitable modifications of the Minimum- and the Fischer-Burmeister-Function. Exploiting these characterizations, some smoothing-type methods for the solution of semidefinite programs are derived. The search directions generated by these methods are automatically symmetric, and the overall methods are globally and locally quadratically convergent under certain conditions. An interesting difference between the modified Minimum- and the modified Fischer-Burmeister-Function is also pointed out. Finally, some numerical results are presented which indicate that the proposed methods are very promising and comparable to several interior-point methods.

PENNON—A Generalized Augmented Lagrangian Method for Semidefinite Programming

Michal Kočvara

(joint work with Michael Stingl)

We describe a generalization of the PBM method by Ben-Tal and Zibulevsky to convex semidefinite programming problems. The algorithm used is a generalized version of the

Augmented Lagrangian method. We present details of this algorithm as implemented in a new code PENNON. The code can also solve second-order conic programming (SOCP) problems, as well as problems with a mixture of SDP, SOCP and NLP constraints. Results of extensive numerical tests and comparison with other SDP codes are presented.

An Interior Point Trust Region Method for Nonlinear Semidefinite Programs

Friedemann Leibfritz

An interior point trust region algorithm for the solution of a class of nonlinear semi–definite programming (SDP) problems is described and analyzed. Such nonlinear and non–convex programs arise, e. g. , in the design of optimal static or reduced order output feedback control laws and have the structure of abstract optimal control problems in a finite dimensional Hilbert space. The algorithm treat the abstract states and controls as independent variables. In particular, an algorithm for minimizing a nonlinear matrix objective functional subject to a nonlinear SDP–condition, a positive definiteness condition and a nonlinear matrix equation is considered. The algorithm is designed to take advantage of the structure of the problem. It is an extension of an interior point trust region method to nonlinear and non–convex SDP's with a special structure which applies sequential quadratic programming techniques to a sequence of barrier problems, and uses trust regions to ensure robustness of the iteration. Some convergence results are given, and, finally, several numerical examples demonstrate the applicability of the considered algorithm.

From Zero to Hero? Interior Point Methods for MPECs

Sven Leyffer

This talk considers Mathematical Programs with Equilibirum Constraints (MPECs). MPECs generically violate the Mangasarian Fromowitz Constraint Qualification and are commonly regarded as ill-conditioned problems. It is shown that strong-stationarity for MPECs implies the existence of bounded multipliers corresponding to a linearly independent subset of the constraints.

This fact motivates the solution of MPECs by standard NLP solvers. Extensive numerical tests indicate that SQP methods outperform Interior Point Methods (IPM) both in terms of speed and reliability. Some possible explanations for the poor behaviour of IPMs are presented and modifications to the primal-dual system are proposed which are seen to improve the performance and robustness of IPMs.

Optimization Problems with Pseudospectra

Adrian Lewis

Pseudo-eigenvalues of a square matrix A are eigenvalues of matrices in a neighbourhood of A. Large real parts of pseudo- eigenvalues (rather than eigenvalues themselves) often reveal the behaviour of dynamical systems governed by A. I will describe a simple, robust algorithm for maximizing the real part over the pseudospectrum. This subroutine allows us to enhance the stability of a matrix by optimizing its pseudospectrum.

Solving Large Scale Linear Matrix Inequalities with a Reduced Bundle Method

Scott Miller

There are many classes of large-scale problems that can be expressed as linear matrix inequalities or semidefinite programs, each with its own unique structure. These problems are so large – both in the number of decision variables n and the size of the constraint matrix m – that it is inadvisable to construct the m-by-m or n-by-n matrices demanded by interior point methods. Instead, a bundle method may be used, which can easily exploit the structure of the problem, but the bundle in these cases must be highly reduced. That is, the number of subgradients in the bundle must be much smaller than the dimension of the problem, and an aggregate subgradient must be included in the bundle to ensure convergence. In this case, the bundle method still exhibits reasonably fast convergence of the objective values, but the norm of the aggregate subgradient converges very slowly, sometimes sublinearly, so the termination criterion (small aggregate) becomes impractical. To understand the performance issues of a reduced bundle method, the method with just two subgradients in the bundle (one measured subgradient and the aggregate) is analyzed. Near the solution null steps dominate, and when the aggregate becomes small compared to the measured subgradients, the null steps behave like the subgradient method with step size governed by an estimate of the minimum objective value derived from the aggregate linearization error. Therefore, linear or sublinear convergence is observed, depending on whether this estimate is above or below the true minimum. The analysis also indicates that the proximity weight parameter has only a very indirect effect on this behavior, and that the parameter determining the sufficient decrease test for a serious step induces a trade-off between the convergence rate of null steps and the frequency of serious steps. However, practical methods of avoiding sublinear convergence of null steps are still unknown.

Sufficient Optimality in a Parabolic Control Problem

Hans D. Mittelmann

(joint work with Fredi Tröltzsch)

We define a class of parabolic problems with control and state constraints and identify a problem within this class which possesses a locally unique critical point satisfying the second order sufficient optimality conditions. In this solution inequality constraints on the control are strongly active. The second derivative of the Lagrangian is not globally coercive. This is both shown analytically as well as verified numerically for a finite difference discretization.

Primal-Dual Subgradient Methods in Nonsmooth Minimization

Yurii Nesterov

In this talk we present some new subgradient methods for unconstrained minimization of nonsmooth convex functions. These methods have an optimal rate of convergence. On the other hand they provide a direct control of duality gap, which estimates the accuracy of the current test point. We show how to extend these schemes on the problem of constrained minimization, minimax problems and stochastic optimization problems.

Electricity Pricing and Complementarity

Jong-Shi Pang

We discuss nonlinear complementarity problem formulations of oligopolistic equilibrium models in an electricity power market and present existence and uniqueness of solutions to the models. The models contain four major components: the electricity generators, an independent system operator, a resource allocator, and an arbitrager. Another important ingredient is a set of price function conjectures that describe the firms's beliefs of their rivals' reactions to prices. The models further distinguish themselves in the firms' anticipation of the arbitrager

An Interior Point Method in the Large Neighbourhood of the Central Path

Florian A. Potra

We propose a predictor-corrector method for monotone linear complementarity problems based on a δ_∞^- neighborhood of the central path that has the same structure as the MTY algorithm: each predictor is followed by exactly one corrector. We prove that this algorithm has $O(nL)$ iteration complexity under general conditions, and quadratic convergence of the primal-dual gap under the assumption that the LCP is nondegenerate.

By using a higher order predictor we reduce the iteration complexity and prove superlinear convergence even in the degenerate case. More precisely, by using a predictor of order m we obtain algorithm with $O(n^{1/2} + (n+2)^{1/(m+1)}L)$ iteration complexity. If $m > 1$, then the duality gap is superlinearly convergent under general conditions. The Q-order of convergence of the primal-dual gap is $m+1$ in the nondegenerate case, and $(m+1)/2$ in the degenerate case. Here m can be a constant, or can depend on the problem dimension. Since $\lim_{n\to\infty} n^{(1/n^\omega)} = 1$ for any constant $\omega > 0$, (in fact, $n^{(1/n^\omega)} \leq e^{1/(\omega e)}$, for all n), it follows that by taking $m = \lceil (n+2)^\omega - 1 \rceil$, for some $\omega \in (0,1)$, the iteration complexity of our predictor-corrector algorithm reduces to $O(\sqrt{n}L)$, the same as the best complexity results for small neighborhoods (actually the best iteration-complexity known so far for any interior point method for LP).

Robust Versions of Convex Quadratic and Conic-Quadratic Problems

Cornelius Roos

(joint work with Aharon Ben-Tal and Arkadi Nemirovski)

We consider a conic-quadratic (and in particular a quadratically constrained) optimization problem with uncertain data, known only to reside in some uncertainty set $\mathcal{U}$. The robust counterpart of such a problem leads usually to an NP-hard semidefinite problem; this is the case for example when $\mathcal{U}$ is given as intersection of ellipsoids, or as an n-dimensional box. For these cases we build a single, explicit semidefinite program, which approximates the NP-hard robust counterpart, and we derive an estimate on the quality of the approximation, which is essentially independent of the dimensions of the underlying conic-quadratic problem.

Modelling Techniques for Optimal Control Problems in Food Processing

Ekkehard Sachs

In this talk we review various models which have been used in modelling the sterilization process of food production. The deficiency of the classical model by Ball and Olsen is demonstrated and several remedies are presented. For the sterilization of fluid food products the Navier-Stokes equation is used as a model. In this context we use a reduced order model obtained by proper orthogonal decomposition (POD) to perform the optimization in an efficient way. A trust region approach for nonlinear model functions is used to manage the accuracy of the POD model.

Componentwise Fast Convergence in the Solution of Full-rank Systems of Nonlinear Equations

Annick Sartenaer
(joint work with Nick Gould and Philippe Toint)

The asymptotic convergence of parameterized variants of Newton's method for the solution of nonlinear systems of equations is considered. The original system is perturbed by a term involving the variables and a scalar parameter which is driven to zero as the iteration proceeds. The exact local solutions to the perturbed systems then form a differentiable path leading to a solution of the original system, the scalar parameter determining the progress along the path. A path-following algorithm, which involves an inner iteration in which the perturbed systems are approximately solved, is outlined. It is shown that asymptotically, a single linear system is solved per update of the scalar parameter. It turns out that a componentwise Q-superlinear rate may be attained, both in the direct error and in the residuals, under standard assumptions, and that this rate may be made arbitrarily close to quadratic. Numerical experiments illustrate the results and we discuss the relationships that this method shares with interior methods in constrained optimization.

On the Design of Controllers Suppressing Uncertain Noise

Carsten Scherer

The design of controllers which suppress stochastic disturbances has a long history in control. In these classical H_2-design techniques it is required that the noise's statistical properties are known. In practice it is more realistic to rather assume only knowledge of bounds on the statistical properties of the disturbances as they are acquired through experiments. This leads to the task of designing controllers which achieve robust suppression of a whole class of disturbances with specific structure. In this talk we survey recent results on robust controller analysis and synthesis techniques against structured noise that are based on semi-definite programming.

Combinatorial Structures in Nonlinear Programming

Stefan Scholtes

Non-smoothness and non-convexity in optimization problems often arise because a combinatorial structure is imposed on smooth or convex data. The combinatorial aspect can be explicit, e.g. through the use of "max", "min", or "if" statements in a model, or implicit as in the case of bilevel optimization and decomposition where the combinatorial structure arises from the choice of active constraints in the lower level problem.

In analysing such problems, it is desirable to decouple the combinatorial from the nonlinear aspect and deal with them separately. We suggest a problem formulation which explicitely decouples the two aspects. Such nonlinear combinatorial problems, despite their inherent non-convexity, allow for a convex first order local optimality condition which is generic and tight. Under suitable assumptions, the optimality condition can be rephrased as a condition involving Lagrange multipliers in the classical sense with the non-negativity condition replaced by a more complex condition which depends on multiplier vector and the combinatorial structure in the vicinity of the given point, but not on the behaviour of the nonlinear data functions in a neighbourhood of this point. The verifiability of this condition, and therefore the computational effectiveness of the approach, depends on the representation of the combinatorial structure. We illustrate the potential of this approach by applying it to optimization problems with max-min constraints. An active set method illustrates the algorithmic benefits of the problem formulation which opens the door to SQP-type methods for a variety of nonsmooth nonlinear optimization problems.

On Variable Index Sets in Semi-Infinite Programming

Oliver Stein

Optimization problems in finite-dimensional variables which are subject to infinitely many constraints are called semi-infinite. The interest in these problems stems from applications in approximation theory, and to date a large number of other applications from engineering and economics have been treated in a semi-infinite formulation, e.g. robust optimization, minimax problems, design centering and defect minimization for operator equations. In many of these examples the index set of the inequality constraints inevitably depends on the decision variable.

The feasible sets of problems of the latter type have turned out to possess a very unusual topological structure, even in the generic case. We motivate these genericity results geometrically and show which types of stationarity conditions optimal points satisfy under different assumptions on the structure of the defining functions. The derivation of these results heavily depends on the so-called bilevel structure of semi-infinite optimization problems. Under a convexitiy assumption, this bilevel structure also admits the design of an easily implementable solution algorithm for semi-infinite optimization problems. We discuss convergence results for this algorithm and illustrate it with numerical examples.

A Bundle Method to Solve Multivalued Variational Inequalities

Jean-Jacques Strodiot
(joint work with Geneviève Salmon and Van Hien Nguyen)

In this paper we present a bundle method for solving a generalized variational inequality problem. This problem consists in finding a zero of the sum of two multivalued operators defined on a real Hilbert space. The first one is monotone and the second one is the subdifferential of a lower semicontinuous proper convex function. The method is based on the auxiliary problem principle due to Cohen and the strategy is to approximate, in the subproblems, the nonsmooth convex function by a sequence of convex piecewise linear functions as in the bundle method in nonsmooth optimization. This makes these subproblems more tractable. Moreover to ensure the existence of subgradients at each iteration, we also introduce a barrier function in the subproblems. This function prevents the iterates to go outside the interior of the feasible domain. First we explain how to build, step by step, a suitable piecewise linear approximation and we give conditions to ensure the boundedness of the sequence generated by the algorithm. Then we study the properties that a gap function must satisfy to obtain that each weak limit point of this sequence is a solution of the problem. In particular, we give existence theorems of such a gap function when the first multivalued operator is paramonotone, weakly closed and Lipschitz continuous on bounded subsets of its domain and when it is the subdifferential of a convex function. When it is strongly monotone, we obtain that the sequence generated by the algorithm strongly converges to the unique solution of the problem.

Solving SDP's by High-Order Infeasible Interior Point Methods

Josef Stoer
(joint work with Martin Preiss)

We consider semidefinite linear programs and, more generally, semidefinite monotone linear complementarity problems (SDLCP) in the space $\mathcal{S}^n$ of real symmetric $n \times n$-matrices equipped with the cone $\mathcal{S}^n_+$ of all symmetric positive semidefinite matrices. One may define weighted (using any $M \in \mathcal{S}^n_{++}$ with $\operatorname{tr} M = n$ as weight) infeasible interior point paths by replacing the standard condition $XS = \mu I$ (that defines the usual central path) by $\frac{1}{2}(XS + SX) = \mu M$. Under some mild assumptions (the most stringent is the existence of some strictly complementary solution of (SDLCP)), these paths have a limit as $\mu \downarrow 0$, and they depend analytically on all path parameters (such as μ and M), even at the limit point $\mu = 0$.

This can be exploited to devise infeasible interior point methods of long-step type with a high order of convergence of the path parameter $\mu_k := \operatorname{tr} X_k S_k / n$ as $k \to \infty$: The μ_k converge Q-superlinearly to 0 with order $p + 1 - 0$, $p \geq 1$ being an arbitrary integer. To achieve this, each iteration of these methods requires the solution of p systems of linear equations in the space $\mathcal{S}^n$, but with the same matrix so that only one matrix factorization is needed.

Primal-Dual Steplengths Differentiation in Interior Point Methods

Jos F. Sturm

Since the primal and dual linear programming problems are nicely de-coupled, there seems no need to restrict to identical step lengths in the primal and the dual. This is certainly true for feasible primal-dual interior point methods. In the infeasible interior point method however, the primal and dual variables are linked in the measure of complementarity-violation, $x^T z$. Similarly, the primal and dual problems are linked in their self-dual embedding by the non-positive duality gap constraint. In this talk, we propose a combination of a merit function and a central path neighborhood that allows for the choice of different step lengths in the primal and the dual. In the polynomial convergence proof, we make use of the self-dual embedding idea, where the residual data changes in each iteration.

Do people optimize?

Michael J. Todd
(joint work with Ana Fostel and Herb Scarf)

Most of the talks at this meeting are concerned with where to optimize (fields of application), what to optimize (modelling), and particularly how to optimize (algorithms). This presentation addresses another (more fundamental?) question: do people optimize? More particularly, does homo economicus optimize? The neo-classical theory in economics assumes that a consumer, faced with prices $p \in \Re^l_{++}$ and an income $I > 0$, chooses her demand bundle by maximizing a utility function $u : \Re^l_+ \to \Re$ over her budget set $B(p, I) := x \in \Re^l_+ : p^T x <= I$. If we have a data set $D := (p_i, x_i) : i = 1, \ldots, n$ of prices and associated demand vectors, can we refute the hypothesis that each x_i maximizes a locally non-satiated utility function u over $B(p_i, p_i^T x_i)$? A fundamental theorem of S. Afriat gives necessary and sufficient conditions for D to be consistent with such utility mazimization. We give a simple inductive proof of this theorem, which also provides a simple $O(n^2 l + n^3)$ algorithm. If the data is consistent with utility maximization, the algorithm in fact gives a concave piecewise-linear strictly monotonic utility function that rationalizes the data.

On Approximating Nonconvex Quadratic Optimization by SDP Relaxation

Paul Tseng

We derive new approximation bounds for nonconvex quadratic optimization by SDP relaxation. For m ellipsoid constraints, $1/O(m)$-optimal solution is found using a rank-1 decomposition result of Sturm and Zhang. If the ellipsoids have common center, $1/m$-optimal solution is found. For $m \leq 11$, this gives the best known bound. Extension to problems with convex cone extreme ray constraints is discussed.

On a Few Topics on Second-Order Cone Programming

Takashi Tsuchiya

In this talk, we deal with two issues on application and theory of second-order cone programming. The first topic is an application to a real-world magnetic shielding design problem which arises in the development of new bullet trains in Japan. This problem is formulated as a minimizing sum of Euclidean norm problem with 1700 variables, and is solved in a few seconds with a Pentium III 700 MHz processor by using a primal-dual interior-point algorithm. A larger problem with 60000 variables is solved in 12 min. We developed a simple numerical procedure to improve robustness of the obtained optimal solution by solving purturbed problems repeatedly. When applied to the original problem, this procedure is completed in a few hours even if we solve several thousands of problems. This is a joint work with T. Sasakawa (RTRI, Japan).

The second topic concerns with extension of polynomial primal-dual algorithms to infinite dimensional problems. Our approach is based on the generalization of finite-dimensiona Euclidean Jordan algebras to JB algebras of finite rank. This generalization enables us to develop polynomial-time primal-dual algorithms for infinite-dimensional second-order programs. We prove polynomiality of a path-following algorithm using the Nesterov-Todd direction. An application to multi-target tracking problem of control theory is considered to demonstrate how to compute the Nesterov-Todd direction in the infinite-dimensional Hilbert space. This is a joint work with L. Faybusovich (Univ. of Notre Dame, USA).

Engineering Applications of Nonlinear Optimization

Robert J. Vanderbei

We discuss a few nonlinear optimization problems of current interest. The first two are concerned with designing the terrestrial planet finder space telescope. This is a next generation space telescope that will be used to search for planets around nearby stars. Such a planet will be only a fraction of an arc second away from its star and it might be only 10^{-10} as luminous. At such closeness the planet will lie within the first few drffraction rings produced by the star. For circular telescope openings, these diffraction rings are about 10^{-1} to 10^{-3} times as bright as the main Airy disk. The only hope for detecting a planet is to suppress these rings. The diffraction rings can be designed to have a deep null (10^{-10} to 10^{-11}) close to the Airy disk by properly designing the shape of the entrance pupil. Following Kasdin, Littman, and Spergel, we look for masks of the form $\{(x, y) : -a \leq x \leq a, -A(x) \leq y \leq A(x)\}$. This problem is a quadratically constrained convex optimization problem. Such masks ought to be easy to make but only provide deep nulls in certain directions. An alternative is to look for masks consisting of concentric rings $\cup_i\{(x, y) : r_{2i} \leq ||(x, y)|| \leq r_{2i+1}\}$. These masks provide better nulls but are harder to make. This design problem is nonconvex and involves differences of Bessel functions of the first kind. The third problem is to find periodic orbits for the n-body problem by minimizing the action functional. Expressing the trajectories as Fourier series makes it possible to find several new, nontrivial, solutions to this problem including the recently popular "figure-8" solution to the equimass 3-body problem.

A Simple Efficient Iterative Technique for Semidefinite Programming

HENRY WOLKOWICZ

We apply a simple preprocessing step to a Semidefinite Program (SDP) that results in one single, well-conditioned, overdetermined bilinear equation. We use an inexact Gauss-Newton method, within an interior-point framework, to solve this nonlinear system of optimality conditions. To solve the linear least squares problem that arises at each iteration, $F'_\mu \Delta s = -F_\mu$, we use a preconditioned conjugate gradient type method. We include explicit formulae for the diagonal and incomplete Cholesky preconditioners. One of the advantages of this approach is a *crossover* to affine scaling without maintaining positive definiteness once the quadratic region of convergence is reached. As an illustration, we apply this to the SDP relaxation of the Max-Cut problem. Numerical results are included. A paper is available with URL: `http://orion.math.uwaterloo.ca:80/~hwolkowi/henry/reports/ABSTRACTS.html#simpleSDP`

Solving Stochastic Optimization Problems on Computational Grids

STEPHEN J. WRIGHT

We discuss an algorithm for solving solving two-stage linear stochastic programming problems with recourse and its implementation on a computational grid. Grids are parallel computing platforms assembled from a distributed collection of workstations, PCs, and pieces of clusters and conventional parallel computers. We discuss results obtained by using our code in conjunction with sampling techniques to solve many instances of sampled approximations with large sample sizes. In particular, we are able to obtain solutions of much higher quality than before to several benchmark problems in the literature. We are also able to explore various properties of these solutions, such as the size and dimensionality of the space of solutions and their sensitivity.

Edited by Michal Kočvara

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