Mathematisches Forschungsinstitut Oberwolfach

Report No. 14/2002

Probability and Statistics of Random Algebraic Structures

March 10th – March 16th, 2002

The meeting was organized by Jean-Dominique Deuschel (Berlin), Persi Diaconis (Stanford), and Friedrich Götze (Bielefeld). The 28 lectures that were presented during the morning and afternoon sessions covered probabilistic, combinatoric and statistical aspects of random algebraic structures, and 44 participants took part in the meeting.

The meeting highlighted the recent connections between the statistics and the probability of random matrices, longest increasing subsequences in random permutations and random words, stochastic processes like the polynuclear growth model, free probability theory, as well as Toeplitz matrices, and representation theory. The lectures at this meeting illuminated the different statistical, analytic, discrete algebraic, and even algebro-geometric aspects of these connections. A considerable number of talks concentrated various new aspects of the polynuclear growth model and its relations to different versions of the Knuth-Robinson-Schensted correspondence, longest increasing subsequences in random permutations, and eigenvalue distributions of random matrices.

The abstracts of the talks are given below (in alphabetical order). The inspiring atmosphere and the excellent working conditions at Oberwolfach made this meeting between representatives of different fields very fruitful.

Abstracts

Properties of the general Wishart distribution on homogeneous cone

Steen Anderson (Bloomington, IN, USA)

The classical family of Wishart distributions on a cone of positive definite matrices and its fundamental features and properties are extended to a family of generalized Wishart distributions on a homogeneous cone using the theory of exponential families. The generalized Wishart distributions include all known families of Wishart distributions as special cases. The relations to graphical models and Bayesian statistics are indicated. Several examples are presented. The theory contains many new testing problems and eigenvalue problems.

Convergence Rates of Empirical Spectral Distributions of large Dimensional Random Matrices

Zhi Dong Bai (Singapore)

The talk covers several works of mine and joint works with my colleagues. I will introduce an inequality, similar to Berry-Esseen inequality, but in terms of Stieltjes transforms instead of Fourier transform, which provided a methodology to establish convergence rates of empirical spectral distributions of large dimensional random matrices. As illustration, I give some examples of Wigner matrix and sample covariance matrix to demonstrate how this inequality be used to establish the convergence rates. Some recent developments are also presented.

Some universality for sample covariance matrices

Gérard Ben Arous (Lausanne)
(joint work with Sandrine Peche (EPFL))

We give a result of universality in the bulk, asymptotic distributions of the spacings, universality at the hard edge for square sample covariance matrices.

Representations of symmetric groups, random matrices and free Probability

Philippe Biane (Paris)

We show that the representation theory or large symmetric groups $\mathrm{S}(n)$ can be understood using free probability theory. If one takes an irreducible representation of $\mathrm{S}(n)$, whose Young diagram has largest row and column of the order $\sqrt{n}$, then we find the shape of the typical Young diagrams which appears in the restriction to a subgroup $\mathrm{S}(m)$ with $m \sim tn$ for some $t \in [0, 1]$. It can be described in terms of free probability theory and is related to the problem of finding the typical spectrum of a random $N \times N$ matrix $\Pi X \Pi$ where X is some fixed matrix and Π is a random orthogonal projection of rank $\sim tN$. We also obtain results on the problem of induction of representations. This is based on new asymptotic results for characters of large symmetric groups.

Correlations between Zeros of Random Polynomials of several complex variables

Pavel Bleher (Indianapolis)

We study the limit as $N \to \infty$ of the correlations between simultaneous zeros of random sections of the powers L^N of a positive holomorphic line bundle L over a compact complex manifold M, when distances are rescaled so that the average density of zeros is independent of N. We show that the limit correlation is independent of the line bundle and depends only on the dimension of M and the codimension of the zero sets. We also provide some explicit formulas for pair correlations. In particular, we provide an alternate derivation of Hannay's limit pair correlation function for SU(2) polynomials, and we show that this correlation function holds for all compact Riemann surfaces. This is a joint project with Bernie Shiffman, Steve Zelditch and Denis Ridzal.

Quantum chaos: universal versus system-specific fluctuations

Oriol Bohigas (Orsay)

Are there signatures in the quantum spectra of the underlying regular or chaotic nature of the corresponding classical motion? Are there universality classes? Within this framework, the status of some current problems are reviewed: comparison of Wigner-Dyson random matrix theories with compound nuclear data, numerical experiments,... The results indicate that there exist universality classes of spectral fluctuations characterized by the regular or chaotic nature of the underlying classical motion as well as by their global symmetries. From a theoretical point of view, the semiclassical approaches a la Berry are discussed. They lead to a clear identification of energy (or time) regimes for which universalities hold, beyond which properties become system-specific and governed by properties of short periodic orbits. These ideas are illustrated by considering two quantities, spacing autocovariances and the total energy of a system of fermions. Starting from the Gutzwiller trace formula, spacing autocovariances are written in terms of classical dynamical zeta functions. The role of the spectrum of the Perron Frobenius evolution operator of the classical system is exhibited. The fluctuations of the total energy are expressed in terms of an integral of the form factor which is dominated by the short periodic orbits. The theory is also tested with the Riemann zeta function, whose zeros are known to admit a semiclassical interpretation (Berry, Keating). The dynamical zeta functions corresponding to this case are simple and can be clearly identified. The distribution of the fluctuation of the total energy of the 'Riemannium' (adding imaginary parts of successive zeros) is worked out. Its moments are expressed in terms of rapidly convergent sums extended over prime numbers. The distribution is asymmetric, in contrast to a random matrix evaluation, which gives a gaussian distribution.

O.Bohigas, P.Leboeuf, M.J.Sanchez, 'Spectral spacing correlations for chaotic and disordered systems', Found.of Physics 31 (2001)489-517, nlin.CD/0012049

P.Leboeuf, A.Monastra, O.Bohigas, 'The Riemannium', Reg.Chaot.Dyn. 6 (2002) 205-210, nlin.CD/0101014

O.Bohigas, P.Leboeuf, 'Nuclear masses: evidence of order-chaos coexistence', Phys.Rev.Lett. 88 (2002) 092502

A fixed point approach to self-avoiding random walks

Erwin Bolthausen (Zürich)
(joint work with Christine Ritzmann (Zürich))

The lace expansion for weakly self-avoiding random walks, introduced by Brydges and Spencer in 1985, leads to an expansion of the two-point functions $C_n(x)$, $x \in \mathbb{Z}^d$, of the form

$$C_n = C_{n-1} * D + \sum_{k=1}^{n} c_k B_k * C_{n-k}$$

where $c_n := \sum_x C_n(x)$, and D is the jump distribution of the random walk. We prove a (nearly local) central limit for sequences $(C_n(\cdot)/N_n)_{n\geq 0}$ which are solutions of the above equation for a given input. For self-avoiding walks, the B_k can be estimated in terms of Feynman diagrams which itself can be estimated by the two-point functions. By an easy induction, one can prove that the B_k for SAW satisfy the conditions required for our CLT provided $d \geq 5$ and the coupling parameter is small. The resulting estimates are sharper than those obtained by other methods.

Gap probabilities and discrete Painlevé equations

Alexei Borodin (Philadelphia)

We show that the distribution function for the longest increasing subsequence of poissonized random permutations can be expressed through a specific solution of the discrete Painlevé II equation. In a similar but more general model the distribution of the first row of the random Young diagram leads to the discrete Painlevé V equation. We also show that the distribution function of the first particle in an orthogonal polynomial ensemble whose weight function has rational (discrete) log-derivative, can be expressed through a solution of a recurrence procedure involving factorization of matrix polynomials.

Large Deviations for Random Graphs

Amir Dembo (Stanford)

Based on Joint Work with F. Comets.
Consider the random graph (directed on undirected) on $n \gg 1$ vertices with each edge chosen independently with probability $0 < p < 1$. Let $W(n)$ count the number of wedges (i connected to j connected to k) in the graph. We provide large deviations principle at speed n^2 for $W(n)$ and for certain versions of $L(n)$ – counting open path of length 4. Key to this work is the representation of $W(n)$, $L(n)$ and other local patterns of random graphs via the connectivity matrix X (with $X(i,j) = 1$ indicating an edge from i to j). Indeed, the results apply to same functionals applied on any matrix X with bounded i.i.d. entries (symmetric, when analogous to the undirected graph). I shall provide a general lower bound that should be sharp in many cases, but give an example when it is clearly not, and explain why $R(n)$ – counting simple cycles of length 4 is related to the partition function for SK model. Finally, the original question of Götze and Bolthausen, what are the large deviations of $T(n)$, the number of triangles in the graphs, is as unsolved as it was before.

Regular spacings of complex eigenvalues of the non-Hermitian Anderson model

Ilya Ya. Goldsheid (London)
(joint work with B. Khoruzhenko)

The non-Hermitian Anderson Model is described by the following operator:

$$(H_n^g y)_k = -e^g y_{k+1} + q_k y_k - e^{-g} y_{k-1}, \quad 1 \le k \le n$$

with periodic boundary conditions $y_0 = y_n,\ y_1 = y_{n+1}$. Here $g \ge 0$ is a real parameter. This operator is self-adjoint if and only if $g = 0$. We introduce a class of selfaveraging potentials. Namely, a potential $q \equiv \{q_j\}_{j=1}^{\infty}$ is selfaveraging if the integrated density of states of associated self-adjoint Anderson model exists:

$$N(\lambda) = \lim_{n\to\infty} N_n(\lambda)$$

where

$$N_n(\lambda) = n^{-1}\#\{\lambda_i \ :\ \lambda_i \in \text{spectrum of } H_n^0 \text{ and } \lambda_i < \lambda\}.$$

For operators H_n^g with selfaveraging potentials we prove that:

1. The non-real eigenvalues of H_n^g belong to analytic curves $y = f_n(x)$ given by the equation

$$\int_{-\infty}^{\infty} \ln|x + iy - \lambda| dN_n(\lambda) = n^{-1}\ln(e^{ng} + e^{-ng}). \tag{1}$$

2. These curves converge, as $n \to \infty$, to a limiting curve $y = f(x)$ given by the equation

$$u(x, y) \equiv \int_{-\infty}^{\infty} \ln|x + iy - \lambda| dN(\lambda) = g. \tag{2}$$

Remark. The solutions to (2) exist iff $g > \inf_x \int_{-\infty}^{\infty} \ln|x + i0 - \lambda| dN(\lambda)$. Obviously, if $y = f(x)$ is a solution then $y = -f(x)$ is a solution too. From now on we consider only non-negative solutions to (2).

3. The non-real eigenvalues are regularly spaced. Namely, suppose that the interval $[a, b]$ is such that $y = f(x)$ solves equation (2) for every $x \in [a, b]$ and moreover there is a $\delta > 0$ such that $f(x) \ge \delta$ on $[a, b]$. Then, for n sufficiently large,

(a) equation (1) can also be solved for every $x \in [a, b]$,

(b) if $z = x + iy$ and $\tilde z = \tilde x + i\tilde y$ are two neighbouring eigenvalues of H_n^g with $x, \tilde x \in [a, b]$ then

$$|z - \tilde z| = 2\pi(u_x^2(x, y) + u_y^2(x, y))^{-\frac{1}{2}} n^{-1} + o(n^{-1}).$$

Remark. The eigenvalues of H_n^g whose real parts belong to $[a, b]$ can be ordered in a natural way. This explains what is meant by the expression "neighbouring eigenvalues".

A random signed measure, associated with Hammersley's process

Piet Groeneboom (Delft and Amsterdam)

To analyze the behaviour of Hammersley's interacting particle process, starting from the empty configuration, a 2-parameter signed measure process V_t is introduced, which converges almost surely, in the vague topology, to a positive deterministic measure V, as $t \to \infty$. It is shown that the process V_t satisfies the stochastic integral equation

$$V_t(x, y)^2 = 2\int_B V(u, v-)\, dV_t(u, v) + t^{-1}\{V_t^+(x, y) + V_t^-(x, y)\},$$

where $B = [0,x] \times [0,y]$, and V_t^+ and V_t^- are the positive and negative part of V_t, respectively. This relation corresponds to the equation

$$V(x,y)^2 = 2\int_B V(u,v)\,dV(u,v) + 2xy$$

for the (absolutely continuous) deterministic limit process, which implies that the universal constant in Ulam's problem is equal to 2. The expectation of the integral $\int_B V_t(u,v-)\,dV_t(u,v)$ is shown to be decomposable into two parts, one involving the measure EV_t and its mixed second derivative, and another part, involving the covariance of the "corner spacings" with the random signed measure V_t.

Harmonic Analysis and Random Matrices

THOMAS GUHR (LUND)

Random Matrices are powerful tools in the theory of complex systems and quantum chaos. The resulting matrix models lead to a harmonic analysis in ordinary and also in superspaces. The corresponding kernels are the Gelfand spherical functions. The simplest case is the unitary one, the so-called Itzykson-Zuber integral. However, we are interested in the orthogonal and unitary-symplectic cases where the standard approaches do not work. We find a new and very general recursive structure for the Harish-Chandra and the Gelfand spherical functions.

Our explicit construction implies, as will be argued, that the spherical functions are collectively integrable systems in the sense of Guillemin and Sternberg. Moreover, our construction proves even more general than the theory of classical Lie groups and supergroups.

On the practical side, we find explicit results which make possible exact solutions of various Random Matrix Models.

Discrete polynuclear growth and determinantal transition functions

KURT JOHANSSON (STOCKHOLM)

The discrete polynuclear growth model (PNG) is a local random growth model with a one-dimensional interface. A limit of the model is the continuous PNG model studied by Prähofer and Spohn. Define $G(M,N) = \max_\pi \sum_{(i,j)\in\pi} w(i,j)$, $w(i,j)$ independent geometric and the maximum taken over all up/right paths through the integer points from (1,1) to (M,N). Then the discrete process $K \to G(N+K, N-K)$, $-N < K < N$, is embedded in the PNG process. Based on the RSK correspondence the growth model can be extended to a cascade of growth models with non-intersecting paths, height curves. These non-intersecting paths are given by determinantal transition functions which give rise to determinantal correlation functions using the method of Eynard and Mehta for chains of random matrices. These type of determinantal correlation functions have been computed by Forrester-Nagao-Honner, Prähofer and Spohn and Okounkov-Reshetikhin. Using these correlation functions we prove that a rescaled version of the process $k \to G(N+K, N-K)$ converges to the Airy process introduced by Prähofer and Spohn. We prove this in the strong sense of a functional limit theorem. Using this we obtain some results for the law of the transversal fluctuations, i.e. the fluctuations of the point(s) K whose $G(N+K, N-K)$ is maximal, relating them to the limiting Airy process.

Some statistical questions: especially largest eigenvalues in canonical correlations and principal components analysis

Iain Johnstone (Stanford)

A description is given of the role of sample eigenvalues in some classical problems of multivariate statistics, including a review of two common methods, principal components and canonical correlations analysis. It is suggested that results from random matrix theory can provide practically useful approximations when the ratio of the number of variables to sample size is not necessarily small. Earlier work by the author showed that centered and scaled largest eigenvalue of a $W_p(n, I)$ matrix converged to the Tracy-Widom F_1 distribution as $n, p \to \infty$ and $n/p \to \gamma > 0$. This talk describes a similar result, in collaboration with Peter Forrester, for the largest squared canonical correlation between independent $n \times p$ and $n \times q$ matrices of i.i.d. standard Gaussian variables in the case $p \leq \min(q, n-q)$ all large and $p/n \to c_1 > 0, q/n \to c_2 > 0$.

Moderate Deviations for Longest Increasing Subsequences in Random Permutations

Franz Merkl (Bielefeld)

(joint work with Matthias Löwe and (for the lower tail) with Silke Rolles)

Let L_n denote the length of a longest increasing subsequence in a random permutation $\pi \in S_n$, drawn uniformly from the symmetric group S_n. Then the following moderate deviation results hold, as was shown by Löwe, M., and Rolles:

$$\frac{\log P[L_n > (2 + tn^{-\eta})\sqrt{n}]}{t^{3/2} n^{(1-3\eta)/2}} \overset{n\to\infty}{\longrightarrow} -\frac{4}{3},$$

$$\frac{\log P[L_n \leq (2 - tn^{-\eta})\sqrt{n}]}{t^3 n^{1-3\eta}} \overset{n\to\infty}{\longrightarrow} -\frac{1}{12}.$$

Refinements of these formulas with small quantitative error terms are shown, too.
In the talk, some of the main steps for the proof of these facts are presented. The analysis of 2×2-matrix Riemann Hilbert problems is the central technique in the proof.
The moderate deviation results are consistent with an extrapolation of large deviation results by Seppäläinen, Deuschel, Zeitouni, Logan, and Shepp on the one hand, and with the tail behaviour of the nonstandard central limit theorem for L_n shown by Baik, Deift, and Johansson on the other hand.

A path-transformation for random walks and the Robinson-Schensted correspondence

Neil O'Connell (Paris)

In [O'Connell and Yor (2002)] a path-transformation $G^{(k)}$ was introduced with the property that, for X belonging to a certain class of random walks on $\mathbb{Z}_+^k$, the transformed walk $G^{(k)}(X)$ has the same law as that of the original walk conditioned never to exit the Weyl chamber $\{x : x_1 \leq \cdots \leq x_k\}$. The proof of this representation theorem is based on symmetry and reversibility properties of queues in series (or, equivalently, the asymmetric exclusion process). I will recall the main ideas of the proof.

It turns out that $G^{(k)}$ is closely related to the Robinson-Schensted algorithm, and this connection leads to a new proof of the above representation theorem. The new proof is valid for a larger class of random walks and yields additional information about the joint law of X and $G^{(k)}(X)$. The corresponding results for the Brownian model are recovered by Donsker's theorem. These are connected with Hermitian Brownian motion and the Gaussian Unitary Ensemble of random matrix theory. The connection we make between the path-transformation $G^{(k)}$ and the RS algorithm also provides a new formula and interpretation for the latter. This can be used to study properties of the RS algorithm and, moreover, extends easily to a continuous setting.

Markov equivalence and essential graphs for graphical Markov models

Michael D. Perlman (Seattle)

Graphical Markov models (GMM) use graphs, either undirected, directed, or mixed, to represent global dependences among statistical variables by means of local specifications, thereby achieving substantial computation efficiencies. Examples of GMMs include (finite) Markov random fields, Bayesian networks, and influence diagrams. We shall review the basic Markov properties of GMMs determined by undirected graphs (UG), acyclic directed graphs (ADG = DAG), and adicyclic graphs (= "chain graphs" (CG), which allow both undirected and directed edges with no partially directed cycles) and briefly discuss their statistical analysis. Three topics of current interest will be noted: characterizing Markov-equivalent graphs, unique representation of a Markov equivalence class by means of its "essential graph", and the existence of competing Markov interpretations of the same chain graph.

Asymptotic Limit of the PNG Droplet and the Airy Process

Michael Prähofer (München)
(joint work with Herbert Spohn)

We determine the asymptotic fluctuations of a simple growth model, the PNG droplet, in the appropriate scaling limit. The limiting process is the Airy process, which, roughly speaking, is the last line of Dyson's Brownian motion model (with beta=2, the GUE case) in a linear potential. We describe some basic properties of the Airy process and give an idea of the convergence proof, which involves the notion of determinantal line processes.

Reinforced random walks

Silke Rolles (Eindhoven)
(joint work with Michael Keane)

Reinforced random walk was introduced by Coppersmith and Diaconis in 1987 as follows: Let G be a locally finite graph. Initially all edges are given weight 1. In each step, the random walker jumps to a nearest-neighbour vertex with a probability proportional to the weight of the connecting edge. Each time an edge is traversed, its weight is increased by 1.

It is known that the process is recurrent in one dimension, but the recurrence problem is open in dimension two. Even for $\mathbb{Z} \times \{1, 2\}$ there seems to be no recurrence proof.

In the talk, I present a recurrence result for directed-edge-reinforced random walk on $\mathbb{Z} \times G'$ with G' a finite directed graph.

Law of large numbers for a second class particle in an asymmetric exclusion process

Timo Seppäläinen (Madison)

A second-class particle in an exclusion-type particle system is a particle with lower priority than the regular particles. Through a variational coupling representation of the exclusion process, a second class particle can also be represented by a variational formula that involves a last-passage percolation model. From this representation one can derive laws of large numbers and large deviation bounds for the location of a second class particle. The laws of large numbers state that the second class particle follows the characteristics and shocks of the macroscopic conservation law of the particle density. As a corollary of the large deviation bounds one gets variance bounds for the occupation time of a site in equilibrium exclusion.

CLT for Linear Spectral Statistics of Large Dimensional Sample Covariance Matrices

Jack W. Silverstein (Raleigh, NC, USA)
(joint work with Zhidong Bai)

Let $B_n = (1/N)T_n^{1/2}X_nX_n^*T_n^{1/2}$ where $X_n = (X_{ij})$ is $n \times N$ with i.i.d. complex standardised entries having finite fourth moment, and $T_n^{1/2}$ is a Hermitian square root of the nonnegative definite Hermitian matrix T_n. The limiting behaviour, as $n \to \infty$ with n/N approaching a positive constant, of functionals of the eigenvalues of B_n, where each is given equal weight, is discussed. Due to the limiting behaviour of the empirical spectral distribution of B_n, it is known that these linear spectral statistics converges a.s. to a non-random quantity. This paper shows their rate of convergence to be $1/n$ by proving, after proper scaling, they form a tight sequence. Moreover, if $\exp X_{11}^2 = 0$ and $\exp |X_{11}|^4 = 2$, or if X_{11} and T_n are real and $\exp X_{11}^4 = 3$, they are shown to have Gaussian limits.

Free Probability and Invariant Subspaces

Roland Speicher (Kingston, Canada)

After a short introduction to the connection between random matrices and free probability and to the notions of invariant subspaces and decomposable operators, I show how free probability and the concept of Brown measure allow to derive new decompositions for interesting classes of non-normal operators. (This line of research was initiated by Haagerup.) I will concentrate here on a result of Sniady and myself about decompositions of so-called R-diagonal operators.

The rate of convergence in semicircular laws

Alexander Tikhomirov (Syktyvkar, Russia)
(joint work with F. Götze)

We study the rate of convergence of the normalized spectral distribution functions of the Wigner or Hermitean random matrices.

Under the condition that the 4^{th} moment of matrix entries are finite, we prove that the rate of convergence to the semi-circular law in Kolmogorov distance is $O(n^{-1/2})$, where n is the dimension of the matrices. The bound for the rate of convergence depends on the 4^{th} moment, such that we can obtain the rate of convergence of order $O\left(\frac{1}{p_n n}\right)$ for the sparse matrices, where $p_n = \mathrm{E}\varepsilon_{lj}^{(n)}$. The sparse matrix here means a matrix with entries $\xi_{lj} = \varepsilon_{lj}^{n)} X_{lj}/\sqrt{p_n(1-p_n)}$. In the Gaussian case, we have a rate of convergence of order $O(n^{-2/3})$ for the Hermitean matrices.

Random matrices with invariant distributions and its applications to metric geometry

Anatoly M. Vershik (St. Petersburg)

One of the sources of random matrices is the so called matrix distribution of measurable functions of two variables, e.g. an invariant (w. r. to the permutations of the rows and columns) ergodic measure on the space of matrices. Each matrix distribution is in a sense a degenerated measure which means that it has an additional property, (so called "simplicity") which e.g. the Wigner ensemble (GOE or GUE) does not satisfy.
A special case of such functions is a metric on the topological space with Borel measure. The corresponding matrix distribution is a measure on the cone of distance matrices and defines a random metric on the natural numbers. Gromov's reconstruction theorem asserts that there is only one metric space with a Borel probability measure and with a given matrix distribution. This result is a partial case of the theorem of the speaker which claims that a complete invariant of a pure measurable function of two variables is the matrix distribution. The main part of the proof is an application of the ergodic theorem; this gives a new simple proof of the reconstruction theorem.

In the twenties P. Urysohn introduced a remarkable metric space which is now called Urysohn space and which is universal (in the category of separable metric spaces) and homogeneous (with respect to compact subsets). He proved existence and uniqueness up to isometry of this space. It is possible to define the universum of separable complete metric spaces, the so called cone of distance matrices, and to prove that indeed the Urysohn space is generic in this cone.

Using previous facts we can analyse the notion of randomness of metric spaces.

Theorem (A.Vershik, 2002): The random metric space is the universal Urysohn space.

It means that if we choose a metric on the natural numbers at random with respect to generic measures on the cone of distance matrices then with probability one we obtain the Urysohn space as completion of the set of natural numbers under this metric. The theorem could be considered as a far generalization of the Erdös-Rényi theorem about random graphs. The calculation of the spectrum of random distance matrices for concrete metric spaces or for concrete measures seems to be an important problem in the theory of random matrices.

Asymptotic Distribution Theory for Shell-Sort

JON A. WELLNER (SEATTLE)

We analyze the Shell Sort algorithm under the usual random permutation model. Using empirical distribution functions, we recover Louchard's result that the running time of the 1-stage of $(2,1)$-Shell Sort has a limiting distribution given by the area under the absolute Brownian bridge. The analysis extends to $(h,1)$-Shell Sort where we find a limiting distribution given by the sum of areas under correlated absolute Brownian bridges. A variation of $(h,1)$-Shell Sort which is slightly more efficient is presented and its asymptotic behaviour analyzed.

We also analyze $(3,2,1)$-Shell Sort using Poissonization methods followed by a local limit theorem for additive functionals of a Markov chain due to Kolmogorov (1949) to "de-Poissonize".

Fluctuations of a disordered growth model

HAROLD WIDOM (SANTA CRUZ, USA)

In joint work with J. Gravner and C. A. Tracy we determined the limiting shape and fluctuations for a discrete growth model in two dimensions. This was equivalent to determining the limiting distribution for the length of the longest increasing sequence of ones in a large random zero-one matrix where the probability of a one in any position is a fixed number p. We describe this work and the generalization in which each column of the matrix (each site, in the original problem) has its own probability, these probabilities themselves being i.i.d random variables. Via combinatorics and a theorem of Gessel it becomes the problem of determining the asymptotics of a Toeplitz determinant, random in our case. We establish the asymptotics, in two different regimes, using an identity of Borodin and Okounkov and steepest descent techniques.

Computing permanents via random matrices

OFER ZEITOUNI (HAIFA)
(joint work with B. Rider and S. Friedland)

Let $A_n = (a_{ij})_{i,j=1}^n$ be an $n \times n$ positive matrix with entries in $[a,b]$, $0 < a \leq b$. Let $X_n = (\sqrt{a_{ij}} x_{ij})_{i,j=1}^n$ be a random matrix where $\{x_{ij}\}$ are i.i.d. $N(0,1)$ random variables. We show that for large n, $\det(X_n^T X_n)$ concentrates sharply at the permanent of A_n, in the sense that $n^{-1} \log(\det(X_n^T X_n)/\operatorname{perm} A_n) \to_{n\to\infty} 0$ in probability.

Dual Pairs in Random Matrix Theory

MARTIN ZIRNBAUER (KÖLN)

The traditional tool for computing the correlations of random matrix eigenvalues are orthogonal polynomials. In this talk an alternative method, rooted in the theory of dual pairs and symplectic geometry, is presented. By tensoring the oscillator representation of the metaplectic group with the spinor representation of the spin group, a representation of the orthosymplectic Lie supergroup OSp is obtained. Its character is the square root of a superdeterminant, and serves as a generating function for the eigenvalue correlations of unitary matrices. We focus on the circular random matrix ensembles defined over the classical compact Lie groups: $K = \mathrm{O}(N), \mathrm{U}(N)$, or $\mathrm{USp}(2N)$. Their correlation functions are computed by considering a dual pair $G \times K$ acting inside the oscillator-spinor representation of OSp. On integrating over K with Haar measure, and using Roger Howe's exposition of classical invariant theory, the correlation functions become characters of an irreducible highest-weight representation of G. In the large-N limit, the latter can be evaluated by a supersymmetric generalization of the Duistermaat-Heckman theorem.

Edited by Franz Merkl

Participants

Prof. Dr. Steen Arne Andersson
standers@indiana.edu
Dept. of Mathematics
Indiana University
Bloomington, IN 47405 - USA

Prof. Dr. Zhi Dong Bai
stabaizd@leonis.nus.edu.sg
Department of Mathematics
National University of Singapore
Kent Ridge
Singapore 0511 - SINGAPUR

Prof. Dr. Gerard Ben Arous
Gerard.Benarous@epfl.ch
Departement de Mathematiques
Ecole Polytechnique Federale
de Lausanne
CH-1015 Lausanne

Prof. Dr. Philippe Biane
philippe.biane@ens.fr
Departement de Mathematiques et
Applications
Ecole Normale Superieure
45, rue d'Ulm
F-75230 Paris Cedex 05

Prof. Dr. Pavel Bleher
pbleher@math.iupui.edu
Department of Mathematical Sciences
Indiana University
Purdue University Indianapolis
402 N. Blackford St.
Indianapolis, IN 46202-3216 - USA

Prof. Dr. Oriol Bohigas
bohigas@in2p3.fr
LPTMS
Universite de Paris XI
Batiment 100
F-91405 Orsay Cedex

Prof. Dr. Erwin Bolthausen
eb@amath.unizh.ch
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Prof. Dr. Alexei Borodin
borodine@math.upenn.edu
Department of Mathematics
University of Pennsylvania
Philadelphia, PA 19104-6395 - USA

Dr. Anton Bovier
bovier@wias-berlin.de
Weierstraß-Institut für
Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39
D-10117 Berlin

Prof. Dr. Rainer Dahlhaus
dahlhaus@statlab.uni-heidelberg.de
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294
D-69120 Heidelberg

Prof. Dr. Amir Dembo
amir@stat.stanford.edu
Department of Mathematics
Stanford University
Stanford, CA 94305 - USA

Prof. Dr. Holger Dette
holger.dette@ruhr-uni-bochum.de
Fachbereich Mathematik
Ruhr-Universität Bochum
D-44780 Bochum

Prof. Dr. Jean Dominique Deuschel
`deuschel@math.tu-berlin.de`
Fachbereich Mathematik / FB 3
Sekr. MA 7-4
Technische Universität Berlin
Straße des 17. Juni 136
D-10623 Berlin

Prof. Dr. Peter Eichelsbacher
`peich@math.ruhr-uni-bochum.de`
Fakultät für Mathematik
Ruhr-Universität Bochum
D-44780 Bochum

Prof. Dr. Hans-Otto Georgii
`georgii@mathematik.uni-muenchen.de`
Mathematisches Institut
Universität München
Theresienstr. 39
D-80333 München

Prof. Dr. Friedrich Götze
`goetze@mathematik.uni-bielefeld.de`
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld

Prof. Dr. Ilya Goldsheid
`I.Goldsheid@qmw.ac.uk`
School of Mathematical Sciences
Queen Mary College
University of London
Mile End Road
GB-London, E1 4NS

Prof. Dr. Mikhail Gordin
`gordin@pdmi.ras.ru`
St. Petersburg Branch of
Mathematical Institute of
Russian Academy of Science
Fontanka 27
191011 St. Petersburg - RUSSIA

Prof. Dr. Piet Groeneboom
`p.groeneboom@its.tudelft.nl`
Department of Mathematics and
Computer Science
Delft University of Technology
Mekelweg 4
NL-2628 CD Delft

Dr. Thomas Guhr
`thomas@matfys.lth.se`
Matematisk Fysik, LTH
Lunds Universitet
Box 118
S-22100 Lund - Schweden

Prof. Dr. Alice Guionnet
`aguionne@umpa.ens-lyon.fr`
`alice.guionnet@umpa.ens-lyon.fr`
Dept. de Mathematiques, U.M.P.A.
Ecole Normale Superieure de Lyon
46, Allee d'Italie
F-69364 Lyon Cedex 07

Prof. Dr. Kurt Johansson
`kurtj@math.kth.se`
Dept. of Mathematics
Royal Institute of Technology
Lindstedtsvägen 25
S-100 44 Stockholm - Schweden

Prof. Dr. Iain M. Johnstone
`imj@stat.stanford.edu`
Department of Statistics
Stanford University
Sequoia Hall
Stanford, CA 94305-4065 - USA

Dr. Matthias Löwe
`loewe@sci.kun.nl`
Subfaculteit Wiskunde
Katholieke Universiteit Nijmegen
Postbus 9010
NL-6500 GL Nijmegen

Franz Merkl
merkl@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld

Dr. Neil O'Connell
Neil.O.connell@ens.fr
Departement de Mathematiques et Applications
Ecole Normale Superieure
45, rue d'Ulm
F-75230 Paris Cedex 05

Prof. Dr. Michael D. Perlman
michael@stat.washington.edu
Department of Statistics
University of Washington
Box 35 43 22
Seattle, WA 98195-4322 - USA

Michael Prähofer
praehofer@ma.tum.de
Zentrum Mathematik
TU München
Arcisstr. 21
D-80333 München

Prof. Dr. Silke Rolles
rolles@eurandom.tue.nl
rolles@mathematik.uni-bielefeld.de
EURANDOM
Technical University Eindhoven,R.C.
P.O. Box 513
NL-5600 MB Eindhoven

Dr. Yves Romain
romain@cict.fr
Mathematiques
Universite Paul Sabatier
118, route de Narbonne
F-31062 Toulouse Cedex 4

Prof. Dr. Hermann Rost
rost@math.uni-heidelberg.de
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294
D-69120 Heidelberg

Prof. Dr. Timo Seppäläinen
seppalai@math.wisc.edu
University of Wisconsin
Van Vleck Hall
Madison, WI 53706 - USA

Prof. Dr. Jack W. Silverstein
jack@math.ncsu.edu
Dept. of Mathematics
North Carolina State University
P.O.Box 8205
Raleigh, NC 27695-8205 - USA

Prof. Dr. Roland Speicher
speicher@mast.queensu.ca
Department of Mathematics and Statistics
Queen's University
Kingston, Ontario K7L 3N6 - CANADA

Prof. Dr. Steen Thorbjoernsen
steenth@imada.sdu.dk
Institut for Matematik og Datalogi
Syddansk Universitet
Campusvej 55
DK-5230 Odense M

Prof. Dr. Alexander Tikhomirov
tikhomir@ssu.komi.com
tichomir@Mathematik.Uni-Bielefeld.DE
Dept. of Mathematics
Syktyvkarskii Universitet
Oktjabr'skii pr. 55
Syktyvkar 167001 - RUSSIA

Prof. Dr. Craig Tracy
tracy@math.ucdavis.edu
Dept. of Mathematics
University of California
Davis, CA 95616-8633 - USA

Prof. Dr. Anatoli M. Vershik
vershik@pdmi.ras.ru;
avershik@mpim-bonn.mpg.de;
avershik@yahoo.com
St. Petersburg Branch of
Mathematical Institute of
Russian Academy of Science
Fontanka 27
191011 St. Petersburg - RUSSIA

Prof. Dr. Jon A. Wellner
jaw@stat.washington.edu
Department of Statistics
University of Washington
Box 35 43 22
Seattle, WA 98195-4322 - USA

Prof. Dr. Harold Widom
widom@math.ucsc.edu
Dept. of Mathematics
University of California
Santa Cruz, CA 95064 - USA

Prof. Dr. Ofer Zeitouni
zeitouni@ee.technion.ac.il
Dept. of Electrical Engineering
TECHNION
Israel Institute of Technology
Haifa 32000 - ISRAEL

Prof. Dr. Martin Zirnbauer
zirn@thp.uni-köln.de
Institut für Theoretische Physik
Universität Köln
Zülpicher Str. 77
D-50937 Köln

Prof. Dr. Willem R. van Zwet
vanzwet@math.leidenuniv.nl
Mathematisch Instituut
Universiteit Leiden
Postbus 9512
NL-2300 RA Leiden