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Nichtkommutative Geometrie

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The meeting was organized by Alain Connes (Paris), Joachim Cuntz (Münster) and Marc Rieffel (Berkeley). In the following we include the abstracts of the talks in alphabetical order.

Abstracts

Periodic cyclic homology of Iwahori-Hecke algebras and noncommutative algebraic geometry

Paul F. Baum

Iwahori-Hecke algebras play a central rôle in the representation theory of (reductive) p-adic groups. In order to apply noncommutative geometry to this representation theory it is essential to calculate the periodic cyclic homology of these algebras. This calculation has been done (using results of G. Lusztig) by P. Baum and V. Nistor. This talk explains the calculation and indicates how it can be viewed as a result in noncommutative algebraic geometry. Based on this, one is led to conjectures within the representation theory of p-adic groups.

Gravity in Noncommutative Spaces

Ali H. Chamseddine

We review the construction of Riemannian noncommutative geometry from the spectral triple data. This construction is applied to the noncommutative space defined as the product of a Riemannian manifold times a two point space, and it is shown that this yields the same action as that given by the Randal-Sundrum construction of a five dimensional manifold with two branes corresponding to the fixed points of Z_2 discrete symmetry. The construction of gravitational actions invariant under the star product are described briefly.

Non-commutative spherical manifolds and elliptic functions

A. Connes

We discuss the notion of noncommutative manifold starting from the framework of spectral triples. A spectral triple (A, H, D) is given by a von Neumann algebra A represented in a Hilbert space H, and an unbounded self-adjoint operator D in H.
The "smoothness" of an element f of A is governed by the regularity of the function $\exp(it|D|)f\exp(-it|D|)$ of the real variable t, which plays the role of the geodesic flow. The density of the subalgebra of smooth (resp. Lipschitz, resp. analytic) elements reflects the regularity of the triple.
The basic point is that the triple encodes both the K-homology fundamental class and the metric.
The axioms of NC-manifolds were first written in the most conservative manner in order to encode ordinary Riemannian spin manifolds, the NC-torus and the standard model.
I described my recent joint work with G. Landi and M. Dubois-Violette, in which the basic equation of degree n fulfilled by n-dimensional spectral triples led to discover that any compact Riemannian spin manifold whose isometry group has rank $r \geq 2$ admits iso-spectral deformations to noncommutative geometries, fulfilling all the "conservative" axioms.
The new examples include the NC-4-spheres S^4_θ. The noncommutative algebras A of functions on spherical manifolds are solutions to the vanishing, $\mathrm{ch}_j(e) = 0\,, j < 2$, of the Chern character in the cyclic homology of A of an idempotent $e \in M_4(A)\,, e^2 = e\,, e = e^*$. (of a unitary in the odd case). We describe the noncommutative geometry of S^4_θ as given by a spectral triple (A, H, D) and check all axioms of noncommutative manifolds. For any

Riemannian metric $g_{\mu\nu}$ on S^4 whose volume form $\sqrt{g}\, d^4x$ is the same as the one for the round metric, the corresponding Dirac operator gives a solution to the quartic equation $< (e-1/2)[D,e]^4 >= \gamma_5$ where $< >$ is the projection on the commutant of 4 by 4 matrices. We show how to construct the Dirac operator D on the noncommutative 4-spheres S^4_θ so that the previous equation continues to hold without any change. With Michel Dubois-Violette we gave a complete classification of noncommutative three-dimensional spherical manifolds. The corresponding algebras are labelled by three real parameters. In the generic case the corresponding $\mathbb{R}^4$ are intimately related to the Sklyanin algebras. The analysis of these NC-spaces and of their moduli space is intimately related to the theory of elliptic functions in the sense of Jacobi and is work in progress with Michel.

Symplectic realizations

M. Crainic

(joint work with R.L. Fernandes)

Question: When does a Poisson manifold M admit a complete symplectic realization (i.e. S =symplectic, $\mu : S \to M$ complete, Poisson, submersion) ?

This is a basic question in differential geometry. Symplectic realizations give the possibility of unravelling intricacies specific to Poisson manifolds by moving up to symplectic geometry. They go back to Lie's "function groups" (1890). We give the following answer.

Answer: That happens iff the monodromy groups $N_x \subset T^*_x M$ of M are discrete, locally uniform with respect to $x \in M$.

The "monodromy groups" N_x are well understood conceptually (they reflect the relation between the topology of the symplectic leaves and the geometry of the isotropy Lie algebras of M), and are also quite computable (precise algorithms e.g., if x =regular, N_x =groups of variations of symplectic areas.

Method of proof: use Lie algebroids, "contravariant differential geometry" and M. Crainic, R.L. Fernandez. "Integrability of Lie brackets", Annals of Math., to appear.

Some examples of quantum principal bundles

L. Dabrowski

Few examples of noncommutative principal fibre bundles (Hopf-Galois extensions, coalgebra-Galois extensions) will be presented, including quantum group coverings of the Lorentz group (arising from the polynomial function algebra of $SL_q(2)$ at roots of unity $q^n = 1$), monopole bundles on S^2_q and instanton bundles on S^4_q, which are of interest for mathematical physics.

The Baum-Connes conjecture for connected and p-adic groups

S. Echterhoff
(joint work with J. Chabert, R. Nest)

Using recent results of V. Lafforgue on the Baum-Connes conjecture for reductive groups we show that all connected groups and all linear algebraic $\mathbb{Q}_p$-groups satisfy the Baum-Connes conjecture, i.e. the assembly map

$$\mu : K_*^{top}(G) \to K_*(C_r(G))$$

is an isomorphism for such groups.

Baum-Connes conjecture for Sp(n,1)

P. Julg

We prove the Baum-Connes conjecture with coefficients in any C^*-algebra

$$K_*^{top}(G; A) \to K_*(A \rtimes_{red} G)$$

for $G = Sp(n, 1)$, which is a rank one Lie group having property T. This is equivalent to the fact the Kasparov's element $\gamma \in KK_G(\mathbb{C}, \mathbb{C})$ acts as identity on $K_*(A \rtimes_{red} G)$ though not equal to one. The proof uses

1. a realization of γ by hypoelliptic operators on the boundary sphere
2. a homotopy through uniformly bounded representations constructed by M. Cowling.

Rooted trees and effective acions

Thomas Krajewski

We describe a solution to the exact renormalization group equation based on the Hopf algebra introduced by Connes and Kreimer. We first recall some basic properties of the trees and interpret the characters of the Hopf algebra they generate as coefficients of formal power series of non-linear operators satisfying a natural identity

$$\sum_t a_t X^t \circ \sum_t b_t X^t = \sum_t (b * a)_t X^t.$$

Then we apply this construction to the solution of the renormalization group equations

$$\frac{dx}{dt} = Ax + X(x, t)$$

that describes the scale dependence of the effective action. Renormalization is then interpreted as a change of boundary conditions and yields a factorization theorem. This is therefore extended to Feynman diagrams which are considered as indices for formal power series of operators acting on the space of effective actions. Finally, we mention the rôle of this construction in the study of non-perturbative truncations of the exact renormalization group.

Ideal Feynman graphs and Dyson-Schwinger equations

Dirk Kreimer

Dyson-Schwinger equations are the quantum equations of motions of a given QFT. Their solution is a formal series of primitives, graphs which have no sub-divergencies in the ultraviolet. This amounts to a complete factorization into these primitives. For the linearized Dyson-Schwinger equations, this is a factorization with respect to the shuffle product, with the generalization to the general partial order case being straightforward.

In gauge theories, one need identities between Feynman graphs to make the factorization unique, these identities being provided by the Wand-Takahashi/Slaunov-Taylor identities. Radiative corrections establish an identity between corrections to the Dirac operator which clarifies its gauge-dependence in connection with the factorization into primitives.

The dual of quantum $\widetilde{SU}(1,1)$

Johan Kustermans

(joint work with Erik Koelink)

In 1991, S.L. Woronowicz showed that quantum $SU(1,1)$ does not exist as a locally compact quantum group. L.I. Korogodski explained in 1994 that one should focus on quantizing a certain extension $\widetilde{SU}(1,1)$ of $SU(1,1)$ instead. Woronowicz picked up on this idea and constructed quantum $\widetilde{SU}(1,1)$ to a large extent, but was unable to prove the Co-associativity of the coproduct completely.

In 2001, Erik Koelink and myself constructed quantum $\widetilde{SU}(1,1)$ as a full blown locally compact quantum group, thereby relying on the theory of q-hypergeometric functions and some of the ideas of Korogodski and Woronowicz

In the first half of this talk we explain how quantum $\widetilde{SU}(1,1)$ is constructed, thereby indicating the relevance of q-special functions. In the second half of the talk we discuss how to get a better picture of de dual of quantum $SU(1,1)$ by using the Casimir operator and its spectral decomposition, that, again, is obtained by relying on techniques from de theory of q-hypergeometric functions.

Deformation Quantization and the Baum-Connes conjecture

N.P.Landsman

Let a groupoid H be a bundle of groupoids over the interval I; this means that there is a surjection $p : H \to I$ such that $p \circ r = p \circ s = p$ (where r and s are the range and source projections of H). It follows that $H = \coprod_{\hbar \in I} H_\hbar$, where each $H_\hbar = p^{-1}(\hbar)$ is a subgroupoid of H. If H is a Lie groupoid one requires that p is a smooth submersion; if H is etale, p should be continuous and open. In either case, one can form a field of C^*-algebras $(A_\hbar)_{\hbar \in I}$, defined by a C^*-algebra of sections A, by taking $A = C^*(H)$ and $A_\hbar = C^*(H_\hbar)$. The key lemma, due to B. Ramazan, is that this field is continuous at all points $\hbar$ where $H_\hbar$ is amenable. Examples include: 1) Weyl-Moyal quantization 2) C^*-algebras of Lie groups 3) C^*-algebras of Lie groupoids 4) Noncommutative tori 5) Noncommutative 4-spheres (Connes and Landi) 6) The Baum-Connes conjecture for Lie groupoids

In Examples 1, 2, 3, and 6 the fields are trivial away from $\hbar = 0$, and therefore continuous, despite the possible lack of amenability, whereas continuity at $\hbar = 0$ follows from the above lemma. In examples 4 and 5 the field is not locally trivial anywhere, but all fibres are amenable, so that continuity follows from the lemma as well.

On Connes's spectral interpretation of zeros of L-functions

Ralf Meyer

Let K be a global field, and let $\mathbb{A}$ be its adele ring. Studying the action of the invertible group $K^\times$ on $\mathbb{A}$, Alain Connes has obtained a spectral interpretation for the nontrivial zeros of L-functions associated to characters of the idele class group of K. However, in his spectral interpretation zeros off the critical line appear only as resonances. Sharpening the functional analysis behind his approach, we obtain a variant of his spectral interpretation in which all zeros appear directly.

On the classification of regular Lie groupoids

Ieke Moerdijk

Any regular Lie groupoid G over a manifold M fits into an extension $K \to G \to E$ of a foliation groupoid E by a bundle of connected Lie groups K. If J is the foliation on M given by (the connected components of) the orbits of G and if T is a complete transversal for J, this extension restricts to an extension $K_T \to G_T \to E_T$ of an étale groupoid by (non-abelian) cohomology classes in a new Čech cohomology for étale groupoids. On the other hand, given K and E and an extension $K_T \to G_T \to E_T$ over T, we present an obstruction class in $H^2(M, K)$ to the problem of whether this extension is the restriction of an extension $K \to G \to E$ over M. If these obstructions vanish, all extensions $K \to G \to E$ over M which restrict to a given extensions over the transversal form a principal bundle under the "group" (gr-category) of K-K bitorsors over M.

Godbillon-Vey class in adelic context

H. Moscovici
(joint work with A. Connes)

This is preliminary report on joint work in progress with A. Connes. We show that the modular forms of all levels acted upon by the Hecke operations can be fitted with a non-commutative geometric structure analogous to that of a codimension one foliation. The fact that this structure has an interesting internal dynamics is illustrated by the non-vanishing of the appropriate analogue of the Godbillon-Vey class. The latter class is defined by means of the Hopf algebra of codimension one transverse geometry, which acts by a Hopf action on the crossed product of the modular forms by the group of finite adeles. The invariant trace is provided by the Manin residue.

The structure and K-theory of the Boutet de Monvel algebra

Ryszard Nest
(joint work with S. Melo, E.Schrohe)

For a compact manifold M with boundary ∂M, the algebra of pseudo-local operators is described by the Boutet de Monvel calculus. The norm closure A of the algebra of bounded pseudo-local operators fits into the exact sequence

$$0 \to K \to A \to S \to 0$$

where K is the algebra of compact operators and the symbol algebra S is an extension of the algebra $C(S^*M)$ of continuous functions on the cosphere bundle of M by the reduced Toeplitz algebra T. The Index map

$$K_0(S) \to K_1(K)$$

is always surjective. The group $K_1(S)$ has the form

$$K_1(S) = K^0(T^*M) \oplus K^1(M),$$

where the canonical projection onto the first factor is the topological index map of Boutet de Monvel. The same decomposition holds for K_0 modulo torsion.

Conjecture by J. Bellisard

H. Oyono-Oyono
(joint work with M.T. Benameur)

Let Ω be a Cantor set equipped with an action of $\mathbb{Z}^n$ and with a $\mathbb{Z}^n$-invariant measure μ. The action of $\mathbb{Z}^n$ induces an action of the C^*-algebra $C(\Omega)$ of complex continuous functions on Ω and we can form the crossed product $C(\Omega) \rtimes \mathbb{Z}^n$. The measure μ induces a trace τ^μ on $C(\Omega) \rtimes \mathbb{Z}^n$ and we obtain in this way a morphisms $K_0(C(\Omega) \rtimes \mathbb{Z}^n) \to (R)$. If we denote by $\mathbb{Z}[\mu]$ the subgroup of $\mathbb{R}$ generated by the measure of the compact open subsets of Ω, J. Bellisard stated the following conjecture.

The image of $K_0(C(\Omega) \rtimes \mathbb{Z}^n)$ under τ^μ is equal to $\mathbb{Z}[\mu]$.

We give in this lecture a proof of this conjecture by using the measure index theorem for foliated spaces of Alain Connes.

Bivariant Chern character and BRS cohomology

D. Perrot

The aim of this talk is two-fold. First, we provide a bivariant Chern character in terms of heat kernel methods. It is based on the following general construction, inspired by the work of Quillen: given a differential graded (Co-associative) Co-algebra C and a differential graded (associative) algebra R, we choose a "connection form" θ as a linear map of odd degree $C \to R$. Its Chern character $ch(\theta)$ is a cocycle in the complex Hom $(X(C), X(R))$, roughly obtained by exponentiating the curvature of θ. Let now (E, ρ, D) be an unbounded Kasparov bimodule over two locally convex algebras $\mathcal{A}$ and $\mathcal{B}$. We choose C as the differential graded bar Co-algebra of $\mathcal{A}$, R as the trivially graded tensor algebra of $\mathcal{B}$ (with zero differential), and θ as the "superconnection" $\rho + D$. Then modulo suitable completions, the complex Hom $(X(C), X(R))$ computes the bivariant cyclic cohomology of $\mathcal{A}$ and $\mathcal{B}$, and $ch(\theta)$ actually gives a map from the entire cyclic homology of $\mathcal{A}$ to the periodic cyclic homology of $\mathcal{B}$

$$ch(\theta) : HE_*(\mathcal{A}) \to HP_*(\mathcal{B})$$

automatically incorporating the heat kernel $\exp(-tD^2)$ as a regulator of traces. This defines the bivariant Chern character of (E, ρ, D).

In the second part of the talk, we adapt the previous construction to Hopf algebras in order to describe the relationship between characteristic maps à la Connes-Moscovici and the BRS formalism in Quantum Field Theory. More precisely, let $(\mathcal{A}, H, D)$ be a spectral triple over the algebra $\mathcal{A}$. We suppose that all objects are endowed with the coaction of a Hopf algebra $\mathcal{H}$ whose antipode is involutive. These data give rise to a Kasparov bimodule for $\mathcal{A}$ and $\mathcal{H}$, and the bivariant Chern character yields a map $HE_*(\mathcal{A}) \to HP_*^{inv}(\mathcal{H})$, whose target is the invariant cyclic homology of $\mathcal{H}$ in the sense of Connes-Moscovici and Crainic. This characteristic map and related constructions provide a description of anomalies and BRS cohomology in noncommutative geometry.

Dirac index classes and the noncommutative spectral flow

Paolo Piazza

(joint work with E. Leichtnam)

Let Γ be a finitely generated discrete group and let $\Gamma \to \widetilde{N} \to N$ be an odd dimensional Γ-Galois covering. Let D_N be a Dirac-type operator and let $\mathcal{D}_N$ be the associated Mishchenko-Fomenko Λ-operator, with $\Lambda = C^*_r(\Gamma)$. Suppose now that we have a 1-parameter family of such operators, $\{\mathcal{D}_N(t)\}_{t\in[0,1]}$. I have first addressed the question of how to define a notion of noncommutative higher spectral flow for such a 1-parameter family. In order to proceed we need a way of dividing the spectrum of each Λ-operator in a positive and negative part. This is achieved through the notion of (noncommutative) spectral section associated to any Λ-linear Dirac operator $\mathcal{D}$; this is a Λ-linear self-adjoint projection P having the following additional property: there exist real continuous functions $\chi_1(x), \chi_2(x)$ on the real line that are equal to 1 for large positive x and equal to 0 for large negative x, are such that $\chi_2(x) = 1$ $\forall x \in \operatorname{supp}\chi_1$ and $\operatorname{Im}(\chi_1(\mathcal{D})) \subset \operatorname{Im}(P) \subset \operatorname{Im}(\chi_2(\mathcal{D}))$. The notion of spectral section was introduced for the first time by Melrose-Piazza in the commutative context (families of Dirac operators parametrized by a compact space X). It is a fundamental result, due to Melrose-Piazza in the commutative case and Wu and Leichtnam-Piazza in the noncommutative case, that a spectral section exists if and only if $\operatorname{Ind}(\mathcal{D}) = 0$ in $K_1(C^*_r(\Gamma))$. If one spectral section exists then there are an infinite number of spectral sections and given two spectral sections P and Q for $\mathcal{D}$ there is a well-defined difference class $[P - Q] \in K_0(C^*_r(\Gamma))$. The notion of higher noncommutative spectral flow for the 1-parameter family $\{\mathcal{D}_N(t)\}$ is given under the assumption that $\operatorname{Ind}(\mathcal{D}_N(t)) = 0$ for one (and thus any) t. The definition employs two reference spectral sections P_1 and P_0 for $\mathcal{D}_N(1)$ and $\mathcal{D}_N(0)$ respectively and a total spectral section $\{Q(t)\}_{t\in[0,1]}$ for the whole family $\{\mathcal{D}_N(t)\}_{t\in[0,1]}$:

$$\operatorname{hsf}(\{\mathcal{D}_N(t)\}_{t\in[0,1]}; P_0, P_1) := [P_1 - Q(1)] - [P_0 - Q(0)] \in K_0(C^*_r(\Gamma))\,.$$

The definition, which is due to Dai and Zhang in the commutative context, does not depend on the choice of $\{Q(t)\}$ and can be proved to coincide with the classical one in the numerical case.

In the talk I then described how spectral sections for the boundary operator associated to a Dirac-type operator $\mathcal{D}_M$ on an even-dimensional Galois covering $\Gamma \to \widetilde{M} \to M$

with boundary, can be used to define Atiyah-Patodi-Singer index classes $\mathrm{Ind}(\mathcal{D}_M, P) \in K_0(C^*_r(\Gamma))$ and how the noncommutative spectral flow fits into a variational formula for such index classes once a family $\{\mathcal{D}_M(t)\}$ of such operators is given. I also described a gluing formula for the index class, in $K_0(C^*_r(\Gamma))$, associated to a pair $(N, r : N \to B\Gamma)$ with N closed and union along a hypersurface F of two manifolds with boundary N_+, N_-.

Using the variational formula and the gluing formula I finally described how it is possible to give sufficient conditions on F and Γ ensuring that Novikov higher signatures are *cut and paste invariants.*

K-theory and Langlands multiplicity formula

F. Pierrot

We give a generalization of Langlands' formula computing the multiplicity of the integrable discrete series in $L^2(G/\Gamma)$, Γ discrete co-compact torsionless subgroup of G, valid in particular for G a reductive group over a p-adic field, by very recent results for the K-theory of group algebras.

Periodic cyclic homology of the Hecke algebra of $Gl(n)$

R.T. Plymen

(joint work with J. Brodzki)

Let F be an non-archimedean local field, so that F is a finite extension $\mathbb{Q}_p$, or F is the power series field $\mathbb{F}_q[[x]]$. Let $G = Gl(n) = Gl(n, F)$, let $\Pi(G)$ denote the smooth dual of G, let $\Pi(\Omega)$ denote the Bernstein component of $\Pi(G)$ and let $H(\Omega)$ denote the Bernstein ideal in the Hecke algebra $H(G)$. With the aid of Langlands parameters, we equip $\Pi(\Omega)$ with the structure of a complex manifold, and prove that the periodic cyclic homology of $H(\Omega)$ is isomorphic to the de Rham cohomology of the Bernstein component $\Pi(\Omega)$. We show how the structure of the variety $\Pi(\Omega)$ is related to Xi's affirmation of a conjecture of Lusztig for $Gl(n, (C))$. The smooth dual of G admits a deformation retraction onto the tempered dual $\Pi^t(G)$ and we show that this deformation retraction is a geometric counterpart of the Baum-Connes map for $Gl(n)$.

Noncommutative geometry of warped cones

John Roe

(joint work with Nigel Higson)

Let (X, d) be a metric space (proper and noncompact). A map $\alpha : X \to X$ is a translation if $\sup_{x \in X} d(x, \alpha(x)) < \infty$. For instance if Γ is a discrete group equipped with a left invariant metric, then the right action of Γ on itself is an action by translations.

Suppose that a finitely generated group Γ acts on a metric space (X, d). We want to "warp" the metric on X so that the action becomes one by translations. Let δ be the largest metric on X such that $\delta(x, x') \leq d(x, x')$ and $\delta(x, \gamma x) \leq |\gamma|$ (word length) for all $\gamma \in \Gamma$, $x, x' \in X$. Let X_Γ denote X with metric δ. Consider the case $X = \mathcal{O}Z$ (open metric cone). The warped cone $\mathcal{O}_\Gamma Z = X_\Gamma$ associated to an action of Γ on Z is a slowly growing space (diameter growth $O(\log r)$). We show that among these warped cones are some counterexamples to the coarse Baum-Connes conjecture. A key role in the proof is played by a transfer homomorphism $C^*(\mathcal{O}_\Gamma Z)/I \to (C^*(\mathcal{O}Z) \otimes C^*_r\Gamma)/I'$ (where I, I' are the ideals generated by compactly supported elements); this homomorphism exists when Γ has finite asymptotic dimension in the sense of Gromov.

L^2-Betti numbers and braid groups

Thomas Schick

Definition: Given a discrete group Γ and $\Delta \in M(n \times n, \mathbb{Z}\Gamma)$, Δ gives rise by left convolution to a bounded operator $\Delta : (l^2\Gamma)^n \to (l^2\Gamma)^n$ with kernel $ker(\Delta)$. We define the L^2-Betti number $b^{(2)}(\Delta) := dim_\Gamma ker(\Delta) = tr_\Gamma(pr_{ker\Delta})$, using the canonical trace $tr_\Gamma(\sum \lambda_g g) = \lambda_1$. This gives, via the combinatorial Laplacian of the cellular chain complex, rise to the L^2-Betti numbers of (a Γ-covering of) a finite CW-complex.
Atiyah conjecture If Γ is torsion free, $b^{(2)}(\Delta) \in \mathbb{Z}$ for every Δ as above.
The Atiyah conjecture for a group Γ implies the
Zero divisor conjecture If $a, b \in \mathbb{Z}\Gamma$ satisfy $ab = 0$ and $b \neq 0$, then $a = 0$.
The following permanence property of the Atiyah conjecture is true: if in an extension $1 \to H \to G \to A \to 1$ H is torsion free and satisfies the Atiyah conjecture, and A is torsion free and elementary amenable, then G satisfies the Atiyah conjecture (result due to Linell). By a result of the speaker, the Atiyah conjecture is true for subgroups of inverse images of torsion free elementary amenable groups, e. g. for residually torsion free solvable groups.
The latter result applies to the pure braid groups P_n. To deal with the full braid groups B_n we factorize the projection $B_n \to \Sigma_n$ (with kernel P_n) through a torsion free elementary amenable group Q_n (the kernel being $\gamma_N(P_n)$, a lower central series subgroup for N sufficiently large). To prove torsion-freeness of the quotient, pro-p-completions of the groups in question and their Galois cohomology is used.
This factorization also implies that the Baum-Connes conjecture is true for the full braid groups.

Quantum spaces and Cuntz-Krieger algebras

Wojciech Szymanski

A graph algebra $C^*(E)$ corresponding to a directed graph E is the universal C^*-algebra generated by partial isometries and projections indexed by the edges and the vertices of the graph, respectively, subject to certain Cuntz-Krieger type relations. This construction generalizes the classical one of Cuntz and Krieger.
In this talk we show that a number of q-deformed compact manifolds related to compact quantum groups may be described through graph algebras. In particular we show that the C^*-algebra of continuous functions on the quantum $SU(2)$ group and on the Podleś spheres are isomorphic to the C^*-algebras of certain finite graphs. Furthermore, the C^*-algebras corresponding to the odd-dimensional quantum spheres S_q^{2n-1} of Vaksman and Soibelman, as well as those corresponding to the q-deformed projective spaces are of that type too. In all these cases isomorphisms are given by explicit formulae on generators. Building on the Cuntz-Krieger algebra structure of $C(S_q^{2n-1})$ we show how to construct q-deformations of generalized lense spaces. These again turn out to be isomorphic to certain graph C^*-algebras.
As indication of possible future expansion of this line of investigations we show that the

two-parameter deformation of S^3, constructed recently by Calow and Matthes, is a Cuntz-Krieger algebra of rank two.

Toeplitz and Weyl quantization of symmetric domains

Harald Upmeier
(joint work with J. Arazy)

In joint work with J. Arazy, University of Haifa, we study various G-covariant quantization methods for hermitian symmetric domains $D = G/K$ for a semi-simple Lie group G and its maximal compact subgroup K. Using the scalar holomorphic discrete series of representations of G, realized concretely via weighted Bergman spaces of holomorphic functions on D, one defines covariant functional calculi in terms of operator densities with respect to the invariant measure on D. The most important examples are the Toeplitz quantization, a field of rank-one operators related to the reproducing kernel vectors, and the Weyl quantization, a field of unitary operators related to the symmetries of D. The most interesting aspect is the so-called Berezin transform expressed in terms of the invariant differential operators (higher Laplacians). For the Toeplitz transform we express the eigenvalues using multivariable Γ-functions of Koecher-Gindikin type, whereas the more difficult Weyl transform involves multivariable hypergeometric functions.

Semi-regularity of locally compact quantum groups

Stefaan Vaes

In this talk, I will report on a recent joint work with S. Baaj and G. Skandalis on bicrossed product locally compact quantum groups. First, I will recall the definition of a locally compact quantum group from a joint paper with J. Kustermans and I will explain the relation with the multiplicative unitaries of Baaj and Skandalis.

Using the bicrossed product construction, an example of a locally compact quantum group which is not semi-regular will be obtained. This means that the reduced crossed product of this locally compact quantum group acting on itself by left translation does not contain any compact operator. This result is very much in contradiction with the intuition coming from locally compact groups. The underlying C*-algebra of this example is the crossed product of ideles acting on adeles by multiplication.

Equivariant cyclic homology

C. Voigt

We propose a definition of equivariant cyclic homology for actions of discrete groups. This theory generalizes previous constructions studied by various authors. We discuss in particular the resulting (bivariant) equivariant periodic cyclic homology $HP^G_*(A, B)$. There are homological versions of the Green-Julg theorem $HP^G_*(\mathbb{C}, A) \cong HP_*(A \rtimes G)$ for finite groups and its dual $HP^G_*(A, \mathbb{C}) \cong HP^*(A \rtimes G)$ for arbitrary discrete groups. Using the Cuntz-Quillen approach to cyclic homology we show that $HP^G_*(A, B)$ is homotopy invariant, Morita invariant and satisfies excision in both variables.

Operator algebraic analysis of solvable lattive models

Antony Wassermann

In this talk, I started by giving a survey of the general framework of conformal field theory and solvable lattice models. The basic ideas to be incorporated in this subject are the corner transfer matrix (CTM) method of R. Baxter and the boundary conditions in conformal field theory ($BCFT$) due to J. Cardy. Ultimately the goal is to understand why CFT is the scaling limit of a solvable lattice model at criticality. I explained the philosophy of boundary data in lattice models (one adds an extra fixed string to the braid group) together with the bulk boundary formulation of $BCFT$. I also outlined Baxter's CTM method and his predictions for the limits of the CTM and its Hamiltonian. I then gave six mathematical ways in which these ideas from physics have been made precise. Firstly Smirnov's proof of the Cardy-Carleson formula for percolation predicted by $BCFT$. Secondly loop group fusion and its use in finding boundary data in CFT through "quantum subgroups". Thirdly the use of Connes fusion to define a spin structure on loop space in the work of Teichner and Stolz. Fourthly the use of twisted equivariant K-theory to explain Verlinde's fusion ring, due to Freed, Hopkins and Teleman. Fifthly a formulation of Baxter's CTM predictions in terms of C^*-dynamical systems and their ground states. The last point was the realisation of these dynamical systems in terms of vertex operators of quantum affine algebras for $q \in [0, 1)$. The classical limit at $q = 0$ corresponds to the (unitary) theory of cristal bases.

Non-renormalizability of θ-expanded noncommutative QED

Raimar Wulkenhaar

There are good arguments to believe that space-time is a noncommutative manifold, i.e. a spectral triple. Within the noncommutaive geometrical framework it is straightforward to produce action functionals for classical field theories, e.g. via the spectral action principle. However, in order to make predictions for experiments, one has to develop the analogue of a quantum field theory on a given spectral triple. This is an unsolved problem.

The first attempt is to take as the spectral triple a deformation of the commutative one and to try whether ordinary quantum field theory works in the noncommutative case as well. From the computational point of view, the best deformation to take is the noncommutative $\mathbb{R}^4$. It turns out that (in general) quantum field theories on noncommutative $\mathbb{R}^4$ are not renormalizable.

In noncommutative gauge field theories it is possible to quantize another set of degrees of freedom. The traditional field amplitudes are then recovered by a Seiberg-Witten differential equation. Remarkably, the superficial divergences of the photon self-energy (in terms of the new degrees of freedom) are renormalizable to all orders. For noncommutative quantum electrodynamics, however, this method does not give renormalizable Green's functions, but the problem is not as severe as expected from the counting of possible versus actually occurring divergences. This is interpreted as a hint for additional symmetries in noncommutative gauge field theories, the origin of which seems to be the spectral action.

In summary, standard quantum field theory based on path integral quantization and the evaluation of Feynman graphs is not the correct calculus to give renormalizable quantum field theories on noncommutative $\mathbb{R}^4$. There are good chances that within Polchinskis exact renormalization group method the renormalization of quantum field theories on noncommmutative $\mathbb{R}^4$ is possible.

Positive scalar curvature at infinity and K-theory of group C^*-algebras

Guoliang Yu

Let M be a noncompact Riemannian spin manifold with uniform positive scalar curvature at infinity. Bunke constructed an index of the Dirac operator in the K-theory of the fundamental group. We partially compute the K-theoretic index of the Dirac operator for arithmetic manifolds. We also discuss the issue whether the K-theoretic index live in the image of the Baum-Connes map. Much of what I discuss at the conference grew out of my discussions with Shmuel Weinberger.

Edited by Andreas Thom and Christian Voigt

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