Mathematisches Forschungsinstitut Oberwolfach

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Enveloping Algebras and Algebraic Lie Representations

April 28th – May 4th, 2002

This was the tenth Oberwolfach conference on "Enveloping Algebras" of Lie algebras after the previous ones in the years 1973, 1975, 1978, 1982, 1985, 1987, 1990, 1995 and 1997.

As for the last two meetings, W. Borho (Wuppertal), M. Duflo (ENS Paris), A. Joseph (Paris 6 und Rehovot) and R. Rentschler (Paris 6) have been in charge of the conference.

These meetings on "Enveloping Algebras" started by studying the structure of enveloping algebras of finite dimensional Lie-algebras, their representations and their primitive ideals. This time to the title was added "Algebraic Lie Representations"; so more generally "representation theory of Lie algebraic systems" has been regarded. The wording "Algebraic Lie Representations" is also the title of a European Network (1997-2002) in which more than half of the participants have been involved for several years.

Since the last Oberwolfach conference in 1997 a lot of new questions and new methods have appeared, especially those related to quiver varieties, to canonical bases and to infinite dimensional situations.

The subject also presents various links to neighbouring areas as : noncommutative algebra, differential operators on flag varieties, algebraic groups actions and invariant theory, topological invariants, K-theory, algebraic singularities.

Especially the new methods of the last years: quantum groups, crystal (canonical) bases, adapted algebras, cluster algebras, Hall algebras, vertex operator algebras, tilting modules, quiver varieties, double affine Hecke algebras introduced new challenges.

This time the following subjects have been in the centre of interest :

(1) Quantized enveloping algebras and relations with knot theory and braid group actions
(2) Kac-Moody algebras, affine algebras, quantum affine affine algebras and toroidal algebras
(3) Yangians and Young symmetrizers
(4) Lie superalgebras and their representations

(5) Tilting modules (in char p and for quantized enveloping algebras at roots of unity)
(6) Special aspects of representation theory
(7) Canonical (or crystal) bases, dual canonical bases and its multiplicative properties, standard monomial theory
(8) Cluster algebras
(9) Orbital varieties and Young tableaux
(10) Quivers, path algebras and quiver varieties
(11) Hecke algebras

First of all, some of the talks have been in the classical framework of finite-dimensional semisimple Lie-algebras. There was given an upper bound for multiplicities of composition factors of primitive quotients of enveloping algebras using Kazhdan-Lusztig polynomials. In certain cases, the top Borel-Moore homology for generalized Steinberg varieties has been identified with spaces of intertwining operators between Weyl group representations. An exciting recent development is the use and the study of orbital varieties which are special Lagrangian subvarieties of co-adjoint orbits; results has been presented on orbital varieties in nilpotent orbits on rank two in $\mathfrak{gl}_n$ and on the description of their closures with the help of Young tableaux. Orbital varieties are also involved in the study of Lusztig's nilpotent variety. For the Duflo map between the symmetric algebra and the enveloping algebra, there has been given in case of a quadratic Lie algebra an interesting extension where the symmetric algebra is multiplied by the exterior algebra and the enveloping algebra is multiplied by the Clifford algebra of the given quadratic Lie algebra. There was given a new formulation (using Ginzburg's principal nilpotent pairs) of the recently proved $n!$-conjecture which makes sense for arbitrary semisimple algebraic groups.

A quarter of the talks was dealing with the infinite dimensional cases : especially affine Lie algebras, quantum affine algebras, Yangians and toroidal Lie algebras (toroidal Lie algebras are natural multi-variable generalizations of affine Kac-Moody algebras). Results on particular representations have been given. For the orthogonal and for the symplectic (finite dimensional) groups there have been presented explicitly constructed analogues of the Young symmetrizer for the general linear group GL(N); these analogues are then identified as special cases of intertwining operators between principal series representations of twisted Yangians.

Three talks were given on problems on Lie-superalgebras and the structure of some primitive quotients of their enveloping algebras. Especially the loss of complete reducibility for finite dimensional representations requires the use of new methods. The factoring out (in enveloping algebras) of the kernel of a central character in an atypical situation may introduce a nontrivial Jacobson radical and a new nontrivial centre with the corresponding characters. A second factoring out produces generically then still an algebra of twisted differential operators on a parabolic flag variety.

Nearly half of the talks made use of quantum deformations, of quantum groups and of quantum symmetric pairs. If the deformation parameter is a root of unity, then the representation theory of quantum groups is especially rich and relations to other situations are especially striking. This appears for example for multiplicities in tilting modules: In the decompositions of on the one hand side a tilting module for a semisimple algebraic group in characteristic p and on the other hand the corresponding tilting module for the quantized enveloping algebra at a p-th root of unity, the multiplicities of each indecomposable summand with defining highest weight in the so-called "second weight cell" are the same. One talk established a relation of irreducible representations of quantum enveloping algebras at roots of unity and certain topological invariants, namely "invariants of links with flat connections in the complement".– Also quantum Weyl group actions have been studied in relation with new flat connections on the set of regular elements of the Cartan subalgebra of a semisimple Lie algebra.

Still very exciting are questions related to the crystal (canonical) basis of Kashiwara and Lusztig. Five talks used crystal (canonical) bases or dual canonical bases. There are especially tricky questions related to multiplicative properties of the dual canonical bases. In this context, a conjecture of Berenstein and Zelevinski has been disproved for all types of simple finite dimensional Lie-algebras with five exceptions (these exceptions are A_1, A_2, A_3, A_4 and B_2).– In the homogeneous coordinate ring of a highest weight orbit of a complex semisimple algebraic group one may compare standard monomial theory and dual canonical basis; inside so-called adapted algebras there is a coincidence between the standard monomial basis and elements of the dual canonical basis (as has been presented). The study of the multiplicative properties of the dual canonical basis give rise to a complete new class of algebras, the so-called cluster algebras (these algebras bave been presented and studied in a talk by A. Zelevinsky).

Three talks were dealing with affine or double affine Hecke algebras. Especially all "semisimple" irreducible representations (and in particular those which are of finite dimension) of double affine Hecke algebras have been described. Furthermore K-theory of some affine version of the Steinberg varieties has been used for the construction of a class of simple modules.

An important part of the meeting was devoted to the use of quivers, quiver varieties and path algebras. They give new insight in several important questions. Indeed quivers are related to Kac-Moody Lie algebras as Dynkin quivers are related to finite dimensional Lie algebras. One talk showed, that using the theory of moduli of vector bundles applied to moduli of quiver representations and using the Hall algebra approach to quantum groups, it is possible to construct a canonical orthogonal system in the quantum enveloping algebra of a Kac-Moody Lie algebra and to deduce Betti number formulae. In one talk, a formula for the number of irreducible components of Lusztig's nilpotent variety is given for a large class of quivers. Finally a conjecture of Kac (on the polynomial counting the number of absolutely indecomposable representations of a quiver over a finite field) has been proven for indivisible dimension vectors.

Al the end, let us mention that at many points, geometric questions, subjects and methods are appearing, such as algebraic group actions, co-adjoint orbits, nilpotent orbits, Steinberg varieties, singularities, the Springer fibre and the Springer representation.

The conference gave a good overview on recent challenging problems and results and an impression of the liveliness of the subject of Enveloping Algebras and Algebraic Lie Representations.

Abstracts

Clifford algebras, Duflo maps and the cubic Dirac operator of Kostant
Anton ALEKSEEV

The Duflo map is a map from the symmetric algebra $S(\mathfrak{g})$ of a Lie algebra $\mathfrak{g}$ to the universal enveloping algebra $U(\mathfrak{g}$ which restricts to an algebra isomorphism of

$$(S(\mathfrak{g}))^{ad\mathfrak{g}} \longrightarrow (U(\mathfrak{g}))^{ad\mathfrak{g}} = Z(U(\mathfrak{g})).$$

For $\mathfrak{g}$ a quadratic Lie algebra we suggest an extension of the Duflo map

$$Q_{\mathfrak{g}} : S(\mathfrak{g}) \otimes \Lambda(\mathfrak{g}) \longrightarrow U(\mathfrak{g}) \otimes Cl(\mathfrak{g})$$

which is functorial with respect to quadratic subalgebras. This can be used to extend recent results of Huang-Pandzic and Kostant on the conjecture of Vogan on the Dirac cohomology.

Weight cells in affine Weyl groups
Henning Haahr ANDERSEN

Let G be a semisimple algebraic group over a field of characteristic $p > 0$. The characters of the simple modules, the Weyl modules and the indecomposable tilting modules for G give us three different bases for $\mathbb{Z}[X]^W$ (X the weight lattice, W the Weyl group). We denote these bases $[L(\lambda)]_{\lambda \in X^+}$, $[V(\lambda)]_{\lambda \in X^+}$ and $[T(\lambda)]_{\lambda \in X^+}$, respectively. If $f \in \mathbb{Z}[X]^W$ we have therefore integers $[f : L(\lambda)]$, $[f : V(\lambda)]$, and $[f : T(\lambda)]$ such that

$$(1) \quad f = \sum_{\lambda \in X^+} [f : L(\lambda)][L(\lambda)] = \sum_{\lambda \in X^+} [f : V(\lambda)][V(\lambda)] = \sum_{\lambda \in X^+} [f : T(\lambda)][T(\lambda)].$$

When U_q is the quantized enveloping algebra corresponding to G with q a p'th root of unity in $\mathbb{C}$ then we have analogously defined modules $L_q(\lambda)$, $V_q(\lambda)$, $T_q(\lambda)$ giving rise to expressions similar to (1).

We shall discuss the following results together with some applications and further remarks on cells.

Theorem (T.E. Rasmussen '01). *For all λ in the second weight cell we have $[f : T(\lambda)] = [f : T_q(\lambda)]$ for all $f \in \mathbb{Z}[X]^W$.* (Here a weight cell is a set of dominant weights attached to a Kazhdan-Lusztig cell in the affine Weyl group for G; the second weight cell corresponds to the second highest Kazhdan-Lusztig cell.)

Representations of toroidal Lie algebras
Yuly Billig

In this talk we discuss the representation theory of toroidal Lie algebras, which are natural multi-variable generalizations of affine Kac-Moody algebras. A toroidal Lie algebra is the universal central extension of the Lie algebra of maps of a torus into a finite-dimensional simple Lie algebra $\mathfrak{g}$, taken together with a Lie algebra of vector fields on a torus. We construct the modules using the technique of vertex operator algebras. For the subalgebras of a toroidal Lie algebra, corresponding to the divergent free vector fields, we obtain a result that has a striking resemblance to the formula for the critical dimension in bosonic string theory.

Let V_{hyp}^{+} be a subalgebra of a hyperbolic lattice VOA , and let $V_{Vir}(c_1)$ be a Virasoro VOA of rank c_1. Then $V_{aff}(c) \otimes V_{hyp}^{+} \otimes V_{Vir}(c_1)$ has a structure of a module for an $(N+1)$-toroidal Lie algebra when

$$\frac{c\, dim\, \mathfrak{g}}{c+\check{h}} + 2(N+1) + c_1 = 26$$

where $\check{h}$ is the dual Coxeter number of $\mathfrak{g}$.

Weyl modules and fusion product

Vijayanthi CHARI

In this talk, we explain the connections between : irreducible finite dimensional representations of quantum affine algebras, the finite-dimensional highest weight modules of the affine algebras and the fusion product of representations of the current algebra.

Semisimple and finite-dimensional representations of double affine Hecke algebra

Ivan CHEREDNIK

We describe all "semisimple" irreducible representations of the double affine Hecke algebras of type $GL(N)$. They appear to be in one-to-one correspondence with periodic skew diagrams with the fundamental domain of cardinality N when q is a fractional power of t for the structural constants q, t of double affine Hecke algebras. The finite-dimensional irreducible representations correspond to either the infinite 1-column or the infinite 1-row. The q is the N-th power of t or its inverse in this case.

The centre of a simple P-type Lie superalgebra

Maria GORELIK

The subject of my talk is the centre $Z(\mathfrak{g})$ of the universal enveloping algebra of a simple Lie superalgebra $\mathfrak{g} = P(n)$. M. Scheunert proved that all homogeneous elements of $Z(\mathfrak{g})$ have degree $-n$: $Z(\mathfrak{g}) = \mathbb{C} \oplus Z(\mathfrak{g})_{-n}$, $Z(\mathfrak{g})_{-n} := Z(\mathfrak{g}) \cap U(\mathfrak{g})_{-n}$. In particular, $Z(\mathfrak{g})$ is an algebra with trivial multiplication : $ab = 0$ for all $a, b \in Z(\mathfrak{g})_{-n}$. In my talk I described a linear isomorphism between $Z(\mathfrak{sl}_n)$ and $Z(\mathfrak{g})_{-n}$.

The isomorphism is given by $z \mapsto (adX)(Yz)$ where X and Y are nonzero elements of the top external degree of $\mathfrak{g}_1$ and $\mathfrak{g}_{-1}$ respectively. The action of elements of $Z(\mathfrak{g})_{-n}$ on Verma modules can be expressed in terms of a linear injective map $P : Z(\mathfrak{g})_{-n} \to S(\mathfrak{h})$ whose image is equal to $tS(\mathfrak{h})^W$ where $W = S_n$ is the Weyl group of $\mathfrak{sl}(n)$ and $t = \prod_{\alpha\in\Delta^+}(\check{\alpha}+(\rho,\alpha)-1)$.

Littelmann's path crystal ans some integrable $\hat{\mathfrak{sl}}_n$-modules

Jacob GREENSTEIN

Let $\hat{\mathfrak{g}}$ be an untwisted affine Lie algebra. After Chari and Pressley, a simple integrable $\hat{\mathfrak{g}}$-module with finite dimensional weight spaces is either a highest/lowest weight integrable module or a simple factor of the loop space of a simple finite-dimensional $\hat{\mathfrak{g}}' = [\hat{\mathfrak{g}}, \hat{\mathfrak{g}}]$-module. The former class of modules admits a cystal realization in the framework of Littelmann's path model, namely the subcrystal of the Littelmann path crystal generated by the linear path connecting the origin with a dominant weight λ has the same character as the highest weight integrable module of highest weight λ. The purpose of our talk is to establish a similar result for a class of $\hat{\mathfrak{sl}}_n$-modules of the second type. Then we use the isomorphism theorem of Littelmann in order to describe the decomposition of the tensor product of a highest weight crystal and our crystal of level zero.

The irreducible components of Lusztig's nilpotent variety and crystal bases

Lutz HILLE

The principal aim of this talk is to find formulas for the number of irreducible components of Lusztig's nilpotent variety $\mathcal{R}(\Pi(Q), d)_0$ for a quiver Q and a dimension vector d. If we fix Q and vary d then the irreducible components of it form the so-called global crystal in the sense of Kashiwara/Saito. These varieties are also closely related to Kleinian singularities and their resolutions (for Q affine and d the imaginary root) by the methods of Kronheimer/Nakajima and Cassens/Slodovy. Moreover, it is also related to the orbital varieties for $\mathfrak{gl}_n$.

For a partition $\lambda \vdash n$ we denote by $C(\lambda)$ the corresponding nilpotent class in $\mathfrak{gl}_n$. For two partitions λ and μ we define a number $NA(\lambda, \mu)$ as the number of irreducible components of $\{(A, B) | AB \in C(\lambda), BA \in C(\mu)\}$. This number can be computed as a number of certain nilpotent representations of the quiver$\tilde{A}_1$.

Further we have to define for any set of nilpotent classes $\lambda^1, \ldots, \lambda^t \vdash d$ a number $N\ P(\lambda^1, \ldots, \lambda^t)$ counting certain irreducible components in a subvariety of the representation space of the free associative algebra. Then our formula

$$\sum_{\underline{\lambda}=(\lambda^{p,\alpha})} \prod_{p \in Q_0} NP(\lambda^{p,\alpha}) \prod_{\alpha \in Q_1} NA(\lambda^{s(\alpha),\alpha}, \lambda^{t(\alpha),\alpha^\star})$$

counts the number of irreducible components of some larger variety, the weakly nilpotent variety $\mathcal{R}_w$. Thus, the formula gives an upper bound for the number of irreducible components we wish to determine.

Finally we determine the remaining correcting term to get the number of irreducible components of Lusztig's nilpotent variety $\mathcal{R}_0$ for a large class of quivers Q.

Some new formulations and generalizations of the $n!$ conjecture to arbitrary groups

Shrawan KUMAR

(joint work with Jesper Funch THOMSEN)

In an attempt to prove the Macdonald positivity conjecture, Garsia-Haiman gave a conjecture known as the $n!$ conjecture in 1993. Last year Haiman proved this conjecture by using Hilbert scheme of n-points in $\mathbb{C}^2$. We present a formulation of the conjecture in geometric terms using the principal nilpotent pairs introduced by Ginzburg. This formulation readily generalizes to an arbitrary semisimple groups.

Dual canonical bases, quantum shuffles and q-characters

Bernard LECLERC

Rosso and Green (independently) have constructed an embedding of $U_q(\mathfrak{n})$ in a quantum shuffle algebra. In this talk I discuss the the image of the dual canonical basis in this embedding. In the type A case this gives immediately a notion of q-character for irreducible modules of the Hecke algebras of affine type GL_m. In general it also allows to implement an efficient algorithm for calculating the dual canonical basis. Using this algorithm one can obtain, for all types except A_1, A_2, A_3, A_4, B_2, some elements $b^\star$ of the dual canonical basis whose square does not belong to the dual canonical basis (up to a power of q). This disproves a conjecture of Berenstein and Zelevinsky (of 1993).

Quantum zonal spherical Functions

Gael LETZTER

We study the space ob bi-invariants and zonal spherical functions associated to quantum symmetric pairs in the maximally split case. Under the obvious restriction map, the space of bi-invariants is proved to be isomorphic to the Weyl group invariants of the character group ring associated to the restricted roots. As a consequence, there is either a unique set, or an (almost) unique two-parameter set of Weyl group invariant quantum zonical spherical functions associated to an irreducible symmetric pair. When the restricted root system is reduced and there is either no parameter, or the parameters are set to zero, then the zonical spherical functions can be realized as Macdonald polynomials.

Standard Monomial Theory (SMT) and adapted algebras

Peter LITTELMANN

Let $G/Q \longrightarrow \mathbb{P}(V(\lambda))$ be the highest weight orbit of a complex semisimple algebraic group G. The dual canonical basis $B(\lambda)^\star$ forms a basis of $V(\lambda)^\star$ with special nice properties. The aim of SMT is to give a presentation of the homogeneous coordinate ring $R = \mathbb{C}[G/Q]$ of the embedded variety $G/Q \subset \mathbb{P}(V(\lambda))$, so the elements of $B(\lambda)^\star$ look like a perfect candidate as a set of generators for R. But the multiplicative properties of these elements are not very well understood. On the other hand, the standard monomial theory , initiated by Seshadri and Lakshmibai, provide in its general form also a basis for $V(\lambda)^\star$, so it is natural to ask how these two are related, and to investigate in which case they coïncide.

At least, the adapted algebras introduced by Caldero are maximal subalgebras of $\mathbb{C}_q[U^-]$ spanned as vector space by a subset S of the dual canonical basis such that all elements are multiplicative. It turns out, that for these adapted algebras SMT and dual canonical basis coïncide, and from the combinatorics used for indexing the SMT-elements, one gets for these elements a simple expression in terms of the q-minors which have also been used by Berenstein, Fomin and Zelevinski in their work on total positivity.

On orbital varieties of rank two in $\mathfrak{gl}_n$

Anna MELNIKOV

Orbital varieties in nilpotent orbits of rank two are unions of a finite number of B-orbits and each of them contains a dense B-orbit. Using these facts we can answer basic questions concerning orbital variety closures, like a combinatorial description of the orbital variety closures in terms of Young tableaux, description of orbital varieties intersections, construction of their ideals of definition.

Clifford algebras arising from classical simple Lie superalgebras

Ian M. MUSSON

We work throughout over an algebraically closed ground field K of characteristic zero. Suppose that $\mathfrak{k}$ is a finite dimensional Lie superalgebra and $\mathfrak{k}_0$ is central in $\mathfrak{k}$. Set $R = S(\mathfrak{k}_0)$ and $U = U(\mathfrak{k})$. For a $\mathbb{Z}_2$-graded algebra A, $Spec(A)$ will denote the spaces of $\mathbb{Z}_2$-graded prime ideals of A.

There is a homeomorphism $\pi : Spec(R) \longrightarrow Spec(U)$. If $Q \in Spec(R)$, then the Goldie quotient ring C_Q of $U/\pi(Q)$ is a Clifford algebra of a nonsingular bilinear form over $Fract(R/Q)$.

Now assume that $\mathfrak{g}$ is a classical simple superalgebra and set $\mathbf{U} = U(\mathfrak{g})$. We filter $\mathbf{U}$ by giving elements of $\mathfrak{g}_1$ degree 1, and elements of $\mathfrak{g}_0$ degree 2. Then the associated graded algebra of this filtration has the form $U(\mathfrak{k})$ where $\mathfrak{k} = \mathfrak{g}$ as a graded vector space, but the multiplication is changed so that $\mathfrak{k}_0$ becomes central in $\mathfrak{k}$. Thus the preceding remarks apply to $U(\mathfrak{k})$ and we have $R = S(\mathfrak{k}_0) = S(\mathfrak{g}_0)$.

We study the Clifford algebras C_Q when Q is a prime ideal of R defining a nilpotent orbit and relate them to supergeometry and primitive ideals in $\mathbf{U}$.

Representations of twisted Yangians associated with skew Young diagrams

Maxim NAZAROV

There is a classical realization of all irreducible finite dimensional representations of the orthogonal group $O(N)$ and the symplectic group $Sp(N)$ due to Hermann Weyl. These representations are obtained as certain subspaces in the space of tensors of rank N and degree $n = 1, 2, \ldots$. Namely each of these subspaces is the intersection of the image of any Young symmetrizer corresponding to a partition of n, with the space of so-called "traceless tensors". The image of the Young symmetrizer itself, in the space of all tensors, is an irreducible representation of the general linear group $GL(N)$.

The first aim of the talk is to give a new explicit multiplicative formula for the projector on each of the intersection subspaces. This projector can be regarded as the analogue for $O(N)$ and $Sp(N)$ of the Young symmetrizer for the group $GL(N)$. Moreover, my formula generalizes from the partitions of n (= Young diagrams with n boxes) to skew Young diagrams (pairs of partitions). The generalized formula gives a projector on the multiplicity space in the restriction of irreducible representations from $O(M+N)$ to $O(M)$ and from $Sp(M+N)$ to $Sp(M)$, for any possible M.

The generalized projector is a special case of an intertwining operator between principal series representations of the twisted Yangian (quantum affine symmetric space) introduced by Grigory Olshanski. In my realization, every multiplicity space comes with a natural action of the twisted Yangian corresponding to the group $O(N)$ or $Sp(N)$. This generalizes earlier results of Ivan Cherednik for the Yangian corresponding to the group $GL(N)$.

The Harder-Narasimhan system in quantum groups and cohomology of quiver moduli

Markus REINEKE

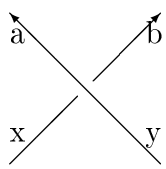
Methods of Harder and Narasimhan from the theory of moduli of vector bundles are applied to moduli of quiver representations. Using the Hall algebra approach to quantum groups, an analogue of the Harder-Narasimhan recursion is constructed inside the quantum enveloping algebra of a Kac-Moody algebra. This leads to a canonical orthogonal system, the HN system, in this algebra. Using a resolution of the recursion, an explicit formula for the HN system is given. As an application, explicit formulas for Betti numbers of the cohomology over quiver moduli are derived, generalizing several results on the cohomology of quotients in "linear algebra type" situations.

Quantized Universal Enveloping Algebras at roots of 1 and invariants of links with flat connections in the complement

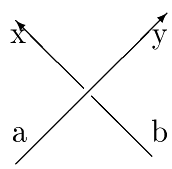
Nicolai RESHETIKHIN

Invariants of tangles with flat connections in the complement are constructed using solutions to the holonomy Yang-Baxter equations. This is done in two steps.

Step 1. Gauge classes of flat connections in a trivial principle G-bundle over the complement to a tangle are parametrized by G-colourings of a diagram of the tangle. If $\mathcal{D}_t$ is a diagram of a tangle, denote by $E(\mathcal{D}_t)$ the set of edges of $\mathcal{D}_t$ (vertices are double points of the projection). A G-colouring of $\mathcal{D}_t$ is a map $c: E(\mathcal{D}_t) \longrightarrow G$ such that elements assigned to adjacent edges satisfy the relations



$$a = x_- y x_-^{-1}$$
$$b = a_+^{-1} x a_+$$

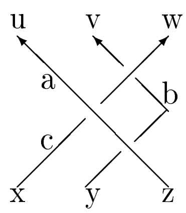


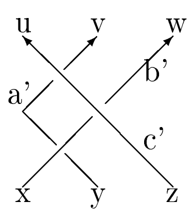
Here G is assumed to be a factorized group, i.e. a triple (G, G_+, G_-), $G\pm \subset G$ are subgroups, s.t. any $x \in G$ (or any $x \in$ open dense subset of G) admits a unique decomposition $x = x_+x_- = \tilde{x}_-\tilde{x}_+$, with $x_\pm, \tilde{x}_\pm \in G$

Step 2. Reidemeister equivalence classes of G-coloured diagrams form a category. Suppose we have a system of maps $R^{VW} : G \times G \longrightarrow End(V \otimes W)$ $(V,\, W)$

$$(\star) \qquad R^{VU}_{12}(c,b)R^{VW}_{13}(x,a)R^{UW}_{23}(y,z) = R^{UW}_{23}(a',c')R^{VW}_{13}(b',z)R^{VU}_{12}(x,y)$$

for each triple V,U,W from the corresponding system of vector spaces. The arguments in these equations are G-colourings of the following diagrams :





Then one can show that the representation theory of quantized universal enveloping algebras $U_\epsilon(\mathfrak{g})$ at roots of 1 provide solutions to the hYBE $(\star)$ ($U_{\epsilon(g)}$ is a "large quantized universal enveloping algebra" at roots of 1). Here $\mathfrak{g} = Lie(G)$, G is a simple complex algebraic group and the irreducible representation theory of these algebras $U_{\epsilon(g)}$ is described by DeConcini, Kac, Procesi.

The geometry of general Steinberg varieties

Gerhard ROEHRLE

(joint work with J.Matthew DOUGLASS)

For a complex, reductive algebraic group G, the Steinberg variety $\mathcal{Z}$ is the set of triples consisting of a nilpotent element x in $Lie(G)$, the Lie algebra of G together with two Borel subgroups of G whose Lie algebras contain x.

We define generalization of these varieties that depends on four parameters and analyze in more detail two special cases that turn out to be related to distinguished double coset representations in the Weyl group W of G.

The equidimensionality of the generalized varieties in these special cases allow to apply Ginzburg's convolution construction for the Borel-Moore homology groups of these varieties. Then using Ginzburg's algebra isomorphism between the Borel-Moore homology of the Steinberg variety $\mathcal{Z}$ and the rational group algebra of W, the top Borel-Moore homology groups of these varieties in the two special cases can be shown to be isomorphic to spaces of intertwing operators of representations of W induced from the trivial respectively sign representation of parabolic subgroups of W.

Conjecturally, the top Borel-Moore homology groups of the generalized Steinberg varieties should be isomorphic to a space of intertwining operators of induced Springer representations of parabolic subgroups on the Weylgroup of G.

On the reduction method in representation theory of Lie superalgebras

Vera SERGANOVA

Let $\mathfrak{g}$ be a contragredient Lie superalgebra, U the universal enveloping algebra and Z be the center of U. We study the quotient algebra $U_\chi = U/(Ker\chi)$, where χ is a central character $\chi : Z \longrightarrow \mathbb{C}$. It was known, that if χ is typical, then U_χ is isomorphic to the algebra $\mathcal{D}^\mu$ of twisted differential operators on G/B. For atypical χ, we show that U_χ has a non-trivial Jacobson radical. The quotient algebra $\bar{U} = U_\chi/rad$ has a new non-trivial center $\bar{Z}$. We show that for a generic central character $\theta : \bar{Z} \longrightarrow \mathbb{C}$, $\bar{U}_\theta = \bar{U}/(Ker\theta)$ is isomorphic to an algebra of twisted differential operators on G/P for a suitable choice of a parabolic subalgebra $P \subset G$.

Composition factors of primitive quotients of $U(\mathfrak{g})$

Catharina STROPPEL

Let $\mathfrak{g}$ be a complex semisimple Lie algebra and let U be its universal enveloping algebra. We consider the quotient $U/Ann_U(L)$ for L a simple U-module. Let us assume $L \cong L(x.0)$ is a simple module of highest weight $x(\rho) - \rho$, i.e. it is isomorphic to the simple quotient of the Verma module $M(x.0) \in \mathcal{O}$.

We explain how a graded version of the category $\mathcal{O}$ (introduced by Beilinson, Ginzburg and Soergel) can be used to get some information about the composition factors of $U/Ann_U(L(x.0))$.

The main result is an upper bound $B_{x,y}$ such that

$$[U/Ann_U(L(x.0)) \otimes_U M(0) : L(y.0)] \leq B_{s,y} \,.$$

We explain how $B_{x,y}$ can be computed using KL-polynomials.

Flat connections and quantum groups

Valerio TOLEDANO LAREDO

I will review the definition of a new flat connection on the set of regular elements of the Cartan subalgebra of a complex semisimple Lie algebra $\mathfrak{g}$ with simple poles on the root hyperplanes and values in any finite dimensional $\mathfrak{g}$-module V.

I will also relate its monodromy to the quantum Weyl group action of the generalized braid group of type $\mathfrak{g}$.

Finally, I will explain how all this is related to recent work of P. Boalch on isomonodromic deformations of connections on the unit disk with irregular singularities at the origin.

Absolutely indecomposable representations and Kac-Moody Lie algebras

Michel Van den BERGH
(joint work with William CRAWLEY-BOEVEY)

A conjecture of Kac states that the polynomial counting the number of absolutely indecomposable representations of a quiver over a finite field with given dimension vector has positive coefficients and furthermore that its constant term is equal to the multiplicity of the corresponding root in the associated Kac-Moody Lie algebra. In the talk we outline a proof of these conjectures for indivisible dimension vectors.

On induced modules of double affine Hecke algebras

Eric VASSEROT

The double affine Hecke algebra introduced by Cherednik is an analogue of the usual affine Hecke algebra "containing two copies" of the polynomial algebra. We give a geometric approach to construct a class of simple modules in terms of K-theory of some affine version of the Steinberg varieties.

Cluster algebras of finite type

Andrei ZELEVINSKI

Cluster algebras are a new class of integral domains introduced about two years ago by Sergey Fomin and myself, in an attempt to create an algebraic framework for the dual canonical bases. The main structural result of the theory so far is a classification of cluster algebras of finite type, which turns out to be one more instance of the Cartan-Killing classification. I discuss the structure of these algebras associated to an arbitrary finite root system.

Edited by Rudolf Rentschler

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