

Report No. 36/2002

## **Nonlinear and Stochastic Systems and Their Numerics**

July 21st – July 27th, 2002

The workshop was organized by M. Dellnitz (Paderborn, Germany), W. Kliemann (Ames, USA), E. Kreuzer (Hamburg, Germany), and S. Namachchivaya (Urbana, USA). The meeting attracted 40 scientists from seven countries.

Dynamic systems cover a wide spectrum of topics ranging from deterministic nonlinear systems to completely random systems. The last three decades have seen much progress on the mathematical methods dealing with both nonlinear dynamics and stochastic systems. Chaotic behaviour of nonlinear dynamic systems and stochastic systems have common features. The dynamics of such systems is hard to predict and to characterize by means of single trajectories. Therefore, one rather considers the evolution of sets instead of trajectories. Characterization of these systems is based on invariant densities, global measures, and other performance characteristics. Many powerful numerical methods have been developed in recent years in order to compute these measures. The progress has been in the foundation of efficient computational algorithms and visualization. Engineering applications in solid and structure mechanics as well as fluid mechanics and fluid structure interaction problems have shown the need of efficient tools in order to analyze and predict the dynamics of such systems.

This meeting had the aim of allowing researchers from applied mathematics and (mechanical) engineering active in the area to exchange ideas and to report on recent advances. The scientific program, consistent with the general approach advocated by Oberwolfach, was composed of 23 lectures organized in seven sessions. In addition, two after-dinner discussion sessions on Almost Invariant Sets and Numerics were organized. The topics in these discussion sessions were introduced by three short lectures each. The discussion sessions provided many chances to share and discuss ideas with the entire conference without using up a lecture slot. A number of opportunities in this regard would have been missed had there been only unstructured discussion time.

The selection of participants was reflected by the broad spectrum of topics covered by the workshop. A discussion organized after the final session of the workshop proved that the meeting has served the purpose intended and both, mathematicians and engineers, gained much from each other.

The organizers and all participants of the workshop want to thank the Oberwolfach Institute for providing a stimulating atmosphere for discussion and the exchange of ideas.

The staff of the institute made the stay pleasant and memorable.

# Abstracts

## Robotics and Autonomous Machines: Dynamic Planning, Optimization, and Control

SUNIL AGRAWAL

During planning and control of autonomous machines and processes, an important issue is to attain desired end conditions in the presence of constraints, while optimizing given cost criteria. This issue is important in diverse disciplines including robotics, process control, aeronautics & astronautics. The talk describes an approach where the underlying structure of dynamic systems is used explicitly to solve the problem in an efficient manner. This approach applies to linear and nonlinear systems which are 'feedback linearizable' or are 'differentially flat'. The talk will present the theory and experiments on laboratory test-beds using this approach. This approach is being applied to behaviors groups of unmanned ground and aerial vehicles and other robot designs including novel designs of vehicles with expanding wheels and cable suspended cranes.

## Stochastic Bifurcation in Hereditary Systems

S.T. ARIARATNAM

Many physical and physiological systems with memory-dependent properties are governed by integro-differential equations of the form

$$\dot{x} = A(\alpha)\dot{x} + R[x] + f(x), \quad x \in \mathbb{R}^d$$

where  $R[x]_i = \sum_{j=1}^d \int_0^t R_{ij}(t-\tau)x_j(\tau)d\tau$ ,  $\alpha \in \mathbb{R}^n$  is a vector parameter, and  $f$  is a smooth nonlinear vector function of its argument with  $f(0) = 0$ . Suppose that as  $\alpha$  is varied, the response  $x(t)$  undergoes a bifurcation from the trivial solution  $x = 0$  to a nontrivial solution at some value of  $\alpha$ . If the parameter  $\alpha$ , instead of being constant, is subject to fluctuations of the form  $\alpha_t = \alpha + \xi_t$ , where  $\xi_t$  is a zero-mean stationary stochastic process, it is of interest to investigate how the bifurcation is affected by these fluctuations. In this talk, some concepts of stochastic bifurcation are illustrated through an example from mechanics, viz. the dynamics of axially loaded visco-elastic uniform columns and thin rectangular plates in cylindrical bending when the axial load undergoes stochastic fluctuation. Using a single mode approximation and the Bubnov-Galerkin method, the amplitude of the motion is governed by a stochastic integro-differential equation of the form

$$\ddot{q} + 2\zeta w_c \dot{q} + w_0^2 [1 - \xi_t]q + [\alpha q^2 + \gamma(\dot{q}^2 + q\ddot{q})]q - \beta \int_0^t R(t-\tau)(1 + \frac{\pi^2}{8}q^2)qd\tau = 0$$

where  $R(t)$  is the viscoelastic relaxation function and  $\zeta$ ,  $\alpha$ ,  $\gamma$ ,  $\beta$  are constants. For small intensity of the excitation and for material with a large relaxation time, expressions for the Lyapunov exponents and the moment Lyapunov exponent are obtained using the averaging method for integro-differential equations. These are employed to determine the critical values for the damping parameter  $\xi$  at which dynamical (or D-) bifurcation and phenomenological (or P-) bifurcation occur. The P-bifurcation condition is also verified by approximately evaluating the probability density of the nontrivial stationary response and examining its qualitative change in shape.

## Dynamics of milling processes

BALA BALACHANDRAN

In this talk, development of a unified mechanics model, which accounts for time delay effects and loss of contact effects, for milling operations varying from partial immersion operations to slotting operations will be described and numerical results obtained by using this model will be discussed. Loss of stability of periodic motions is examined and numerical predictions of stable and unstable cutting are compared with experimental observations. Bifurcations experienced by periodic motions of the considered workpiece-tool system with respect to quasi-static variation of parameters such as the axial depth of cut are examined. Sensitivity of system dynamics to different parameters is also discussed.

## Lyapunov exponents, stability, and invariant measures for the stochastic Duffing-van der Pol equation

PETER H. BAXENDALE

Let  $\lambda$  denote the almost sure Lyapunov exponent obtained by linearizing the stochastic Duffing-van der Pol equation

$$\ddot{x} = \alpha x + \beta \dot{x} - ax^3 - bx^2\dot{x} + \sigma x \dot{W}_t$$

at the origin  $x = \dot{x} = 0$  in phase space. If  $\lambda > 0$  then the process  $\{(x_t, \dot{x}_t) : t \geq 0\}$  is positive recurrent on  $\mathbf{R}^2 \setminus \{(0, 0)\}$  with stationary probability measure  $\mu$ , say. For  $\lambda > 0$  let  $\tilde{\lambda}$  denote the almost sure Lyapunov exponent obtained by linearizing the same equation along a typical stationary trajectory in  $\mathbf{R}^2 \setminus \{(0, 0)\}$ . The sign of  $\tilde{\lambda}$  is important for stability properties of the two point motion associated with the original equation. It also determines whether or not the random statistical equilibrium measure associated with  $\mu$  is almost surely a single point mass.

It is fairly easy to calculate  $\lambda$  numerically (and in the case  $\alpha < 0$  there is an exact formula due to Imkeller and Lederer). However the calculation of  $\tilde{\lambda}$  is much harder; the circle  $\mathbf{S}^1$  used in the calculation of  $\lambda$  must now be replaced by the non-compact 3-dimensional space  $(\mathbf{R}^2 \setminus \{(0, 0)\}) \times \mathbf{S}^1$ . In this talk we present recent results on the asymptotics of  $\tilde{\lambda}$  as  $\varepsilon \rightarrow 0$  for small friction  $\beta \rightarrow \varepsilon 2\beta$  and small noise  $\sigma \rightarrow \varepsilon\sigma$ . The calculations depend significantly on the relative sizes of the two coefficients  $a$  and  $b$  in the cubic restoring term  $ax^3$  and the cubic dissipation term  $bx^2\dot{x}$ . These methods show that both cases  $\tilde{\lambda} > 0$  and  $\tilde{\lambda} < 0$  are possible for the original system.

## Freezing waves and spirals in reaction diffusion systems

WOLF-JÜRGEN BEYN

The longtime behaviour of reaction diffusion systems

$$u_t = \Delta u + f(u) \quad x \in \Omega \subset \mathbb{R}^d, \quad t \geq 0, \quad u(x, t) \in \mathbb{R}^n$$

on an unbounded domain  $\Omega$  may differ significantly from that on a bounded domain. For example, a travelling wave will persist on the infinite line while it will usually die out on any finite domain when it reaches the boundary. In this talk we present a method for freezing such solutions in a finite domain. By introducing extra Lagrange parameters and phase conditions for the frozen solution we set up a differential algebraic PDE that may be used for time integration as well as for the bifurcation analysis of certain patterns. Moreover, such systems suggest an efficient way of constructing time-dependent asymptotic boundary

conditions on the finite domain. We show how the method generalizes to infinite dimensional dynamical systems that are equivariant under the action of a Lie group. Advantages and disadvantages of such an approach are discussed for the case when one tries to prevent a plane spiral from rotating.

### **Some Ideas on Almost Invariant Sets for Systems with Bounded Noise**

FRITZ COLONIUS

Using the well known relations between Markov diffusion processes and control systems, a concept for almost invariance is proposed and discussed. It is based on the loss of invariance for control sets under increasing control ranges.

### **Noise-assisted high gain stabilization: Almost surely or in second mean**

HANS CRAUEL

(joint work with I. Matsikis & S. Townley)

We consider a control system with multiplicative noise

$$\begin{aligned} (1) \quad & dx = Ax dt + Bu dt + \sigma A_1 x \circ dW(t) \\ (2) \quad & y = C^\top x \end{aligned}$$

with  $x \in \mathbb{R}^n$ , single input  $u \in \mathbb{R}$  and single output  $y \in \mathbb{R}$ , and  $A, A_1, B, C$  with appropriate dimensions. Assuming  $A, B, C$  and  $A_1$  to be unknown, and  $C^\top B > 0$ , we show a particular case with  $n = 2$ , where the Lyapunov exponent  $\lambda = \lambda_{k,\sigma}$  converges to  $a_{22} - \frac{\sigma^2}{2}$  for  $k \rightarrow \infty$ , and the square mean growth rate converges to  $a_{22} + \frac{\sigma^2}{2}$  for  $k \rightarrow \infty$  (where  $a_{22}$  is the right bottom entry of the drift matrix  $A$ ). Thus noise increases almost sure stabilizability, but worsens square mean stabilizability by high gain in this case.

### **Stochastic Oscillations and Intermittency in Populations Dynamics**

MICHAEL DIMENTBERG

Classical Lotka-Volterra (L-V) model describes periodic oscillations in populations sizes of two interacting species of predator-prey type. This model is conservative, thus cannot withstand environmental random variations. Thus, a generalized model is considered which accounts for interspecies competition and white-noise variations of the preys' reproduction factor. An exact solution to the Fokker-Planck equation is obtained, indicating both population sizes to be stationary gamma-distributed processes. Increasing intensity of external variations may lead to intermittency, with violent short outbreaks in one or both populations and very low level of response for most of the time. The intermittency in predators is especially pronounced in the vicinity of bifurcation point corresponding to predators' extinction. Analytical description is developed for intermittent processes using theory of excursion of random functions.

## **Analysis of subharmonic motion of a floating crane**

KATRIN ELLERMANN

Subharmonic motion of a periodically excited moored floating crane has been observed in experiments as well as in numerical investigations. Knowledge about the operating conditions which lead to subharmonic motion and coexisting attractors is important for the safe operation of the system.

The model of a floating crane takes into account two rigid bodies and the characteristics of the mooring lines represented by a third order polynomial. The description of the fluid-structure-interaction is based on potential theory. The radiation forces are described in two different ways: As a simple approximation the added masses and dampings derived from the radiation potentials are directly used in the equations of motion. As a second approach the radiation forces are described by an integral formulation as proposed by Cummins in 1962. Two different techniques are applied in the analysis of the system: The multiple-scales method allows for the investigation of the oscillating system in frequency domain and yields an analytical approximation. Secondly, bifurcation analysis reveals the dependence of periodic motion on different parameters numerically. The algorithm extends path following techniques to also approximate implicit surfaces.

## **Statistically Optimal Almost-Invariant Sets**

GARY FROYLAND

Chaotic dynamical systems are often topologically transitive, although this transitivity is sometimes very weak. It is of interest to divide the phase space into large regions, between which there is relatively little communication of trajectories. Fast, simple algorithms are presented to find such divisions. We focus on a statistical description of transitivity that takes into account the fact that trajectories tend to visit different regions of phase space with different frequencies. We will take advantage of theoretical properties of reversible Markov chains. A new adaptive algorithm, with a convergence result, is put forward to efficiently deal with situations where the boundaries of the weakly communicating regions are complicated.

## **Long time numerics for controlled and perturbed systems**

LARS GRÜNE

We consider the problem of numerical approximation of the long time behavior for controlled and perturbed systems. Based on ideas from nonlinear control theory we develop a suitable concept for measuring the influence of (deterministic) perturbations on asymptotic stability, called input-to-state dynamical stability (ISDS). Using this ISDS-concept we derive necessary and sufficient conditions for the convergence of numerically asymptotically stable sets to asymptotically stable sets for the approximated system, and we analyze and construct algorithms for the numerical computation of domains of attraction. Finally, we present some first results about how these techniques can be extended to stochastic systems.

## Stochastic Resonance

PETER IMKELLER

We provide a mathematical underpinning of the physically widely known phenomenon of *stochastic resonance*, i.e. the optimal noise-induced increase of a dynamical system's sensitivity and ability to amplify small periodic signals. The effect was first discovered in energy-balance models designed for a qualitative understanding of global glacial cycles. More recently, stochastic resonance has been rediscovered in more subtle and realistic simulations interpreting paleoclimatic data: the Dansgaard-Oeschger and Heinrich events. The underlying mathematical model is a diffusion in a periodically changing potential landscape with large forcing period. We study 'optimal tuning of the diffusion trajectories with the deterministic input forcing by means of the *spectral power amplification* measure. Our results contain a surprise: due to small fluctuations in the potential valley bottoms the diffusion — contrary to physical folklore — does not show tuning patterns corresponding to continuous time Markov chains which describe the reduced motion on the metastable states. This discrepancy can only be avoided for more robust notions of tuning, e.g. spectral amplification after elimination of the small fluctuations.

### A set oriented approach to optimal control

OLIVER JUNGE

(joint work with Hinke Osinga)

We develop a set oriented method for approximating the optimal value function and approximate optimal trajectories of time discrete optimal control problems.

The idea of the method is to employ a set oriented approach in order to explicitly construct a weighted directed graph that is a finite state model of the original continuous space control system. On this graph, standard graph theoretic methods for computing (all source, single destination) *shortest paths* can be applied in order to compute the desired objects. We show that the approximate optimal value function converges pointwise to the true one as the discretization parameter goes to zero.

As an example we approximate the optimal value function and optimal trajectories for a single resp. a double controlled inverted pendulum.

### An Averaging Principle Revisited: An Asymptotic Expansion Approach

RAFAIL KHASHMINSKII

We consider a diffusion process with fast and slow components. It is known that under suitable assumptions the slow component can be approximated (in distribution) by the diffusion process with averaged in some sense characteristics. We construct and justify an asymptotic for the solution of suitable Kolmogorov backward equation. Some new probabilistic results can be obtained using the expansion. It is the joint work with G. Yin.

## Higher order Taylor schemes for nonlinear affine control systems

PETER KLOEDEN

It was explained how the Platen-Wager formalism for Taylor schemes for stochastic differential equations could be adapted to give Taylor schemes for arbitrary order for deterministic nonlinear affine control system, ie. of the form  $\frac{dx}{dt} = f(t, x) + g(t, x)u(t)$  with measurable controls  $u(t)$  taking values in a compact set  $U$ .

## Risk-Sensitive Criteria for Stochastic Oscillations

AGNESSA KOVALOVA

The theory of risk-sensitive control investigates the manner in which modifications of the cost structure affects the associated optimal policies. In stochastic control, the well known Linear-Quadratic-Gaussian (LQG) problem is a notable instance. The LQG problem leads to a linear feedback control policy that is insensitive to additive white-noise input disturbances. It has been proposed to consider instead a Linear Exponential Quadratic Gaussian (LEQG) problem, for which the linear feedback optimal control becomes sensitive to noise intensity. The recent trend of research applies the large deviations approach to risk-sensitive control problems. In this approach, nonlinear models with small noise are studied. It has been proposed to transform the traditional control criteria into the exponential form. This transformation shifts the attention toward paths with rare but large deviations from the unperturbed state. In the paper, we extend risk sensitive control theory to typical nonlinear oscillatory systems with random perturbations. The control task is to counteract large deviations from a prescribed mode of bounded oscillations. The paper develops an asymptotic procedure for approximate solution of a relevant HJB equation. It is shown that the averaged HJB equation is reduced to a first order PDE with coefficients dependent on the noise intensity in the leading order term, though this intensity tends to zero in the original system. The leading order nearly optimal control is constructed as a nonlinear stationary feedback with parameters dependent on the noise intensity.

## A numerical scheme for stochastic PDEs with Gevrey regularity

GABRIEL LORD

We consider strong approximations to parabolic stochastic PDEs with additive noise. We take this noise to be in a Gevrey space of analytic functions (the smallest space in which the deterministic part of the PDE is known to have solutions) and to be white noise in time. We introduce in a simple way Gevrey regularity and show that our numerical scheme has solutions in a discrete equivalent of the Gevrey space. This can then be exploited to improve the standard strong error estimate.

We present numerical results for stochastic PDEs with Ginzburg-Landau and FitzHugh-Nagumo type nonlinearities in 1 and 2 dimensions. We compare the scheme to a more standard numerical scheme and show the effect of taking both smooth (Gevrey) and non-smooth noise on traveling fronts and spirals.

## On time discretizations of rate-independent hysteresis models

ALEXANDER MIELKE

(joint work with Florian Theil)

Motivated by the sliding under dry friction of a glass on a rocking tray and by applications of hysteresis effects in inelastic material behavior (e.g. elastoplasticity, shape memory alloys, ferro-magnetism and superconductivity) we consider the following abstract energetic model. Given is an energy-storage functional  $\mathcal{I}(t, \cdot) : z \in Z \subset X \rightarrow [0, \infty]$  and a dissipation norm  $\Delta$  on the Banach space  $X$ . The solutions are described by imposing a stability condition (S) and an energy inequality (E):

$$(S) \quad \mathcal{I}(t, z(t)) \leq \mathcal{I}(t, a) + \Delta(a - z(t)) \quad \text{for all } a \in Z,$$

$$(E) \quad \mathcal{I}(t_1, z(t_1)) + \int_0^T \Delta(\dot{z}(t)) dt \leq \mathcal{I}(t_0, z_0) - \int_{t_0}^{t_1} \partial_t \mathcal{I}(t, z(t)) dt.$$

This energetic formulation replaces the stronger formulations using variational inequalities or flow rules. Its importance arises from the fact that (S) & (E) do not involve derivatives of  $\mathcal{I}(t, \cdot)$ ,  $\Delta$  and the sought solution  $z$ . In particular, solutions  $z : [0, T] \rightarrow X$  may be discontinuous.

The most natural time-discretization is obtained by solving the time-incremental minimization problem

$$z_{k+1} \in \arg \min \{ \mathcal{I}(t_{k+1}, z) + \Delta(z - z_k) \mid z \in Z \},$$

which provides natural discretized versions of (S) & (E), and hence a-priori estimates. We discuss conditions on  $\mathcal{I}$ ,  $Z$  and  $\Delta$  which guarantee the convergence of this time discretization. By the Banach space version of Helly's selection principle we obtain pointwise convergence in  $X$  of a subsequence to a solution  $z \in \text{BV}([0, T], X)$ , if we assume that the sublevels  $\{ (t, z) \mid \mathcal{I}(t, z) \leq M \}$  are compact. Uniform strong convergence can be obtained under the assumption of uniform convexity of  $\mathcal{I}(t, \cdot)$ ,  $Z$  being the full Banach space and technical smoothness conditions:  $\|z_k - z(t_k)\|_X \leq C \left( \max_{1 \leq j \leq k} t_j - t_{j-1} \right)^{1/2}$ .

## The PI-Observer for Estimating Nonlinear Characteristics in Nonlinear Dynamical Systems

PETER C. MÜLLER

Dynamical systems are often influenced by troublesome effects such Coulomb friction, hysteresis or backlash. An extended Luensberger observer, i.e. the PI-observer, is able to estimate the states of the dynamical systems and the nonlinear effects as well. Some sufficient criteria for the asymptotic stability of the estimation errors are given. Additionally, some applications are shown in the fields of fault detection of cracks in turbo rotors and of high accurate position control of robots. In the latter case the disturbance rejection control method is applied to counteract the nonlinearities by the estimated signals.



## **Numerical studies of a stochastic model for ship motion**

GUNTER OCHS

We model the roll model of ship as a nonlinear oscillator with one degree of freedom and a stochastic forcing term. In this model capsizing means the crossing of a potential wall. We are interested in regions of stability (where no capsizing occurs), local attractors, bifurcations, and invariant measures. In this talk we discuss two examples. First we show numerically, that a bifurcation scenario, which shows up in a system with purely periodic forcing, is destroyed by adding small random noise. Furthermore, we show numerical approximations calculated by set-oriented methods of an invariant random set of a system with additive white noise.

## **Influence of turbulence on the fluidelastic instability of tube bundles and prismatic bodies**

KARL POPP

Different mechanisms are known that excite vibrations in heat exchanger tube bundles, the most dangerous one results in the fluidelastic instability. The corresponding large amplitudes can cause short term failures. The lecture presents experimental and theoretical investigations of the influence of turbulence on the stability behaviour of tube bundles and prismatic single bodies in cross flow. In wind tunnel experiments a surprising stabilizing effect of stochastic flow velocity perturbations generated by turbulence grids was found for tube bundles. For a better understanding of the stabilization some numerical investigations on the stability behaviour have been performed on the basis of a simple wavy wall channel model. The numerical results are in good agreement with the experimental ones. Furthermore, ongoing research on the stability behaviour of a single prismatic body has been reported. Here, still discrepancies exist between wind tunnel experiments and numerical simulations based on a quasi static theory.

## **Stochastic Phenomena of Railway Vehicles**

WERNER SCHIEHLEN

Travelling at higher speeds leads to structural vibrations requiring an analysis of the middle and higher frequency vibrations. The eigenbehaviour of a railway bogie with flexible frame, rigid wheelsets and a rigid car-body is analyzed using the method of elastic multibody systems. The excitation of structural vibrations is restricted to track irregularities. Realizations of track irregularities in space and time domain are obtained from power spectral densities published in literature and compared to measurement data. Forced vibrations resulting in stress and fatigue of the bogie frame are investigated by simulations. A quasi-static stress distribution is shown, regions of high stress concentrations are noticed.

## **Invariant/inertial manifolds for infinite dimensional random dynamical systems**

BJÖRN SCHMALFUSS

We consider a parabolic differential equation with random coefficients. Under particular assumptions we can formulate a random graph transform. This transform defines a 'lifted' random dynamical system. Random fixed points of this system are the graphs of invariant manifolds. Applications to systems with fast and slow variables are discussed.

## Extracting Macroscopic Stochastic Dynamics of Biomolecules

CHRISTOF SCHÜTTE

The function of many important biomolecules comes from their dynamic properties and their ability to make statistically rare switches between different conformations. Recent investigations demonstrated that (a) these conformations can be understood as metastable or almost invariant sets of certain Markov chains related with the dynamical behavior of the molecular system and that (b) these sets can efficiently be computed via the dominant eigenvectors of an appropriately defined transfer operator. Hence, one can reformulate the effective dynamical behavior: in a certain sense it just is a simple low-dimensional finite state space Markov chain that describes stochastic hops between the metastable conformation sets of the system. This Markov chain can explicitly be derived even for “real” biomolecules like peptides and RNA systems by means of novel algorithmic techniques.

## Calculation of Inertial Manifolds for fluid conveying tubes

ALOIS STEINDL

In order to reduce the dimension of the discretized PDEs governing the motion of a planar fluid conveying tube different inertial manifold approximations and basis functions for the dominating modes are investigated. Due to internal material damping approximately half of the spectrum accumulates at a finite value leading to a large number of modes, which have to be included in the dominant set.

## Applications of Nonlinear Dynamic Systems Theory in Engineering

JÖRG WALLASCHEK

First the problem of brake squeal is addressed. Basic modelling assumptions are discussed and it is pointed out that many engineering disciplines are involved in the study of brake noise generation. The present state of knowledge is characterized by a sound understanding of the basic principles of noise excitation, like e.g. stick-slip, negative slope of the friction coefficient, mode-coupling and binary flutter. It is however very difficult to identify the contribution of different parameters, like e.g. contact stiffness, structural damping and friction force generation, within these models. Some new results on the importance of the Third Body Friction Layer and its modelling are given.

The second part of the contribution is dedicated to the application of set-oriented methods to the study of railway dynamics. The “Railcab”-system of the University of Paderborn is chosen as an example and it is shown, how limit cycles, observation and absorption probabilities can be computed using the GAIO-software. The benefit of the set-oriented methods in comparison to modal analysis, quasi-linearization and bifurcation analysis is discussed. Limits of the method resulting from high computational load are discussed and the evaluation of a 7 DOF nonlinear system under random disturbances (track irregularity) shows that medium-sized problems in engineering can already be studied using set-oriented methods.

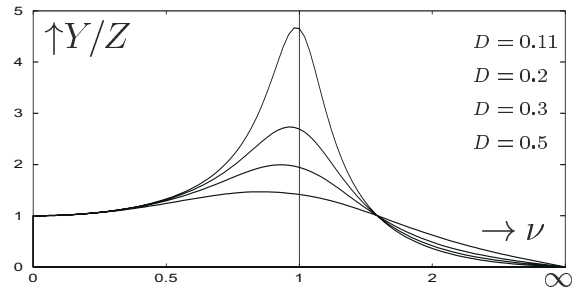
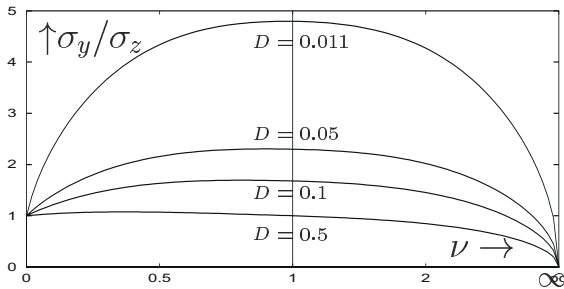
# Nonlinear Stochastic Road-Vehicle Systems

WALTER WEDIG

The paper investigates road-vehicle systems by resonance diagrams as shown below. In the first figure, the root mean square  $\sigma_y$  of the vertical car vibrations related to the excitation variance  $\sigma_z = \sigma/\sqrt{2\Omega}$  are plotted versus  $\nu = v\Omega/\omega_1$  of the car speed  $v$  times the corner way frequency  $\Omega$  of the stochastic road excitation related to the natural frequency  $\omega_1$  of the vehicle. In the linear case, the road surface  $Z_s$  is modeled in the way domain by the Itô equation  $dZ_s = -\Omega Z_s ds + \sigma dW_s$ . Herein,  $s$  denotes the way coordinate and  $\sigma$  is the intensity of the Wiener increments  $dW_s$  with the normalized mean square  $E(dW_s^2) = ds$ . Accordingly, the transformation into the time domain is performed by  $ds = v dt$  and  $dW_s = \sqrt{v} dW_t$ . Applied to quarter car models with one degree of freedom and a viscous damper  $D$ , the results obtained for the above low-pass excitation  $Z_t$  are compared with those of base excitations by harmonic wave roads  $Z \cos \Omega s$ . Both are noted below. The paper extends these investigations to higher order vehicle systems and to non-linear excitation models which are non-normal and bounded in the amplitude range.

$$\frac{\sigma_y}{\sigma_z} = \sqrt{\frac{2D + (1 + 4D^2)\nu}{2D[1 + (\nu + 2D)\nu]}}$$

$$\frac{Y}{Z} = \sqrt{\frac{1 + (2D\nu)^2}{(1 - \nu^2)^2 + (2D\nu)^2}}$$



## Stabilizing the double oscillator by mean zero noise

VOLKER WIHSTUTZ

By generalizing the methods developed for stabilizing the inverted pendulum by random vibration, it is shown that the double oscillator can be stabilized by the same kind of noise (parametric mean-zero noise) that stabilizes the single oscillator.

## Hasselmann's program and the Coupled Lorenz-Maas Model

YONGHUI WU

The feasibility of Hasselmann's program to simplify the deterministic weather-climate dynamical system to a stochastic climate model is examined with the Lorenz-Maas model. It is shown that for the same initial conditions both the statistical climate models and the stochastic climate models with various invariant measures simulate well the behaviors of the climate variables of the coupled Lorenz-Maas model within the time interval comparable to the timescale ratio of climate to weather. We show further that for the long-time asymptotic behavior study Hasselmann's stochastic climate model is more useful than the statistical climate model in the sense that both the flow patterns of the climate variables and the power spectral density obtained by Hasselmann's stochastic climate model resemble the whole one much better than those of the statistical climate model. Moreover, there exist cases in which the long-time asymptotic behavior of the coupled Lorenz-Maas model can be reproduced by Hasselmann's stochastic climate model while it fails with the statistical climate model. Therefore, we argue here that Hasselmann's program with minor modifications works well in the simplification of the coupled Lorenz-Maas model, and it is expected that Hasselmann's approach could carry over to more general climate models.

*Edited by Edwin Kreuzer*

## Participants

**Prof. Dr. Sunil K. Agrawal**

agrawal@me.udel.edu  
Dept. of Mechanical Engineering  
University of Delaware  
229 Spencer Lab.  
Newark, DE 19716 - USA

**Prof. Dr. Alfons Ams**

ams@imfd.tu-freiberg.de  
Institut für Mechanik und  
Fluiddynamik  
TU Bergakademie Freiberg  
Lampadiusstr. 4  
D-09599 Freiberg

**Prof. Dr. Sinnathamby Ariaratnam**

stariara@civoffice.watstar.uwaterloo.ca  
Solid Mechanics Division  
Faculty of Engineering  
University of Waterloo  
Waterloo, Ontario N2L 3G1 - CANADA

**Prof. Dr. Ludwig Arnold**

arnold@math.uni-bremen.de  
Institut für Dynamische Systeme  
Universität Bremen  
Fachbereich 3  
Postfach 330440  
D-28334 Bremen

**Prof. Dr. Bala Balachandran**

balab@eng.umd.edu  
Dept. of Mechanical Engineering  
University of Maryland  
College Park, MD 20742-3035 - USA

**Prof. Dr. Peter H. Baxendale**

baxendal@math.usc.edu  
Dept. of Mathematics, DRB 155  
University of Southern California  
1042 W 36th Place  
Los Angeles, CA 90089-1113 - USA

**Prof. Dr. Wolf-Jürgen Beyn**

beyn@mathematik.uni-bielefeld.de  
Fakultät für Mathematik  
Universität Bielefeld  
Postfach 100131  
D-33501 Bielefeld

**Prof. Dr. Fritz Colonius**

colonius@math.uni-augsburg.de  
Institut für Mathematik  
Universität Augsburg  
D-86135 Augsburg

**Dr. Hans Crauel**

hans.crauel@tu-ilmenau.de  
Institut für Mathematik  
Technische Universität Ilmenau  
Weimarer Straße 25  
D-98693 Ilmenau

**Prof. Dr. Michael Dellnitz**

dellnitz@uni-paderborn.de  
FB 17: Mathematik/Informatik  
Universität Paderborn  
D-33095 Paderborn

**Prof. Dr. Peter Deuffhard**

deuffhard@zib.de  
Konrad-Zuse-Zentrum für  
Informationstechnik Berlin (ZIB)  
Takustr. 7  
D-14195 Berlin

**Prof. Dr. Michael F. Dimentberg**

diment@wpi.edu  
Dept. of Mechanical Engineering  
Worcester Polytechnic Institute  
Worcester, MA 01609 - USA

**Katrin Ellermann**

ellermann@tuhh.de  
Arbeitsbereich Mechanik und Meeres-  
technik  
TU Hamburg-Harburg  
Eißendorfer Str. 42  
D-21073 Hamburg

**Prof. Dr. Bernold Fiedler**

fiedler@math.fu-berlin.de  
Institut für Mathematik I (WE 1)  
Freie Universität Berlin  
Arnimallee 2-6  
D-14195 Berlin

**Dr. Gary A. Froyland**

gary.a.froyland@bhpbilliton.com  
BHP Billiton  
GPO Box 86A  
Melbourne, VIC 3001 - AUSTRALIA

**Dr. Lars Grüne**

gruene@math.uni-frankfurt.de  
Fachbereich Mathematik  
Universität Frankfurt  
Robert-Mayer-Str. 6-10  
D-60325 Frankfurt

**Prof. Dr. Peter Hagedorn**

hagedorn@mechanik.tu-darmstadt.de  
Institut für Mechanik  
Technische Hochschule Darmstadt  
Hochschulstr. 1  
D-64289 Darmstadt

**Prof. Dr. Peter Imkeller**

imkeller@mathematik.hu-berlin.de  
Institut für Mathematik  
Humboldt-Universität zu Berlin  
Unter den Linden 6  
D-10099 Berlin

**Dr. Oliver Junge**

junge@uni-paderborn.de  
FB 17: Mathematik/Informatik  
Universität Paderborn  
Warburger Str. 100  
D-33098 Paderborn

**Prof. Dr. Rafail Z. Khasminskii**

rafail@math.wayne.edu  
Department of Mathematics  
Wayne State University  
656 West Kirby Avenue  
Detroit, MI 48202 - USA

**Prof. Dr. Wolfgang H. Kliemann**

lastec@iastate.edu  
Research Administration  
Iowa State University  
2810 Beardshear Hall  
Ames, IA 50011 - USA

**Prof. Dr. Peter E. Kloeden**

kloeden@math.uni-frankfurt.de  
Fachbereich Mathematik  
Universität Frankfurt  
D-60054 Frankfurt

**Prof. Dr. Agnessa S. Kovaleva**

a.kovaleva@ru.net  
Kavkazsky blv. 44-3-17  
115516 Moscow - RUSSIA

**Prof. Dr.-Ing. Edwin Kreuzer**

kreuzer@tu-harburg.de  
Arbeitsbereich Mechanik und  
Meerestechnik  
Techn. Univ. Hamburg-Harburg  
Eißendorfer Str. 42  
D-21073 Hamburg

**Dr. Gabriel Lord**  
gabriel@ma.hw.ac.uk  
Dept. of Mathematics  
Heriot-Watt University  
Riccarton-Currie  
GB-Edinburgh, EH14 4AS

**Prof. Dr. Alexander Mielke**  
mielke@mathematik.uni-stuttgart.de  
Mathematisches Institut A  
Universität Stuttgart  
D-70550 Stuttgart

**Prof. Dr. Peter C. Müller**  
mueller@srm.uni-wuppertal.de  
Sicherheitstechnische Regelungs-  
und Meßtechnik  
Bergische Universität/GH Wuppertal  
D-42097 Wuppertal

**Prof. Dr. N. Sri Namachchivaya**  
sri@nsgsun.aae.uiuc.edu  
Dept. of Aeronautical &  
Astronautical Engineering  
University of Illinois at Urbana-C.  
104 South Wright Street  
Urbana, IL 61801-2935 - USA

**Dr. Gunter Ochs**  
gunter@math.uni-bremen.de  
Institut für Dynamische Systeme  
Universität Bremen  
Fachbereich 3  
Postfach 330440  
D-28334 Bremen

**Kathrin Padberg**  
padberg@math.upb.de  
FB 17: Mathematik/Informatik  
Universität Paderborn  
Warburger Str. 100  
D-33098 Paderborn

**Prof. Dr. Karl Popp**  
popp@ifm.uni-hannover.de  
Institut für Mechanik  
Universität Hannover  
Appelstr. 11  
D-30167 Hannover

**Prof. Dr. Jürgen Scheurle**  
scheurle@mathematik.tu-muenchen.de  
Zentrum Mathematik  
TU München  
D-80290 München

**Prof. Dr. Werner Schiehlen**  
wos@mechb.uni-stuttgart.de  
Institut B für Mechanik  
Universität Stuttgart  
D-70550 Stuttgart

**Dr. Björn Schmalfuß**  
bjoern.schmalfuss@in.fh-merseburg.de  
Fachbereich 1  
Fachhochschule Merseburg  
Geusarer Str. 1  
D-06217 Merseburg

**Prof. Dr. Christof Schütte**  
schuette@math.fu-berlin.de  
Institut für Mathematik II (WE2)  
Freie Universität Berlin  
Arnimallee 2-6  
D-14195 Berlin

**Prof. Dr. Alois Steindl**  
Alois.Steindl@tuwien.ac.at  
alois.steindl+e3252@tuwien.ac.at  
Institut für Mechanik  
Technische Universität Wien  
Wiedner Hauptstr. 8-10  
A-1040 Wien

**Prof. Dr. Jörg Wallaschek**

`jw@hni.upb.de`

Heinz Nixdorf Institut  
Universität Paderborn  
Fürstenallee 11  
D-33102 Paderborn

**Prof. Dr. Volker Wihstutz**

`wihstutz@uncc.edu`

Dept. of Mathematics  
University of North Carolina  
at Charlotte - UNCC  
Charlotte, NC 28223-0001 - USA

**Prof. Dr. Walter Wedig**

`wedig@itm.uni-karlsruhe.de`

Institut für Technische Mechanik  
Fakultät für Maschinenbau  
Universität Karlsruhe  
D-76128 Karlsruhe

**Dr. Yonghui Wu**

`yonghui@mathematik.hu-berlin.de`

c/o Prof. Dr. Peter Imkeller  
Institut fuer Mathematik  
Humboldt-Universität zu Berlin  
Unter den Linden 6  
D-10099 Berlin