Mathematisches Forschungsinstitut Oberwolfach

Report No. 39/2002

Mathematical Methods in Tomography

August 11th – August 17th, 2002

The conference was organized by F. Alberto Grünbaum (Berkeley), Alfred K. Louis (Saarbrücken) and Frank Natterer (Münster). Fifty-one mathematicians from universities, research institutes and industry in Brazil, Canada, France, Germany, Great Britain, Holland, Ireland, Israel, Italy, Sweden and USA participated in the conference. Thirty-one participants presented their recent results obtained in the mathematical study of different tomographic applications either for improving current techniques or for future impact. The presented talks dealt with X-ray Computerized Tomography (dynamic and stationary), Vector Tomography, Wave Scattering, Optical Tomography, Positron Emission Tomography, Phase Contrast Tomography, Electron Microscope Imaging, among other things.
Special thanks go to the MFO staff for their hospitality, efficiency in organizing the meeting and running the institute.

Abstracts

Approximations to the Boltzmann Equation and Applications in Optical Tomography

Simon R. Arridge

The steady state Boltzmann Equation for time-harmonic sources is written

$$(\hat{s}\cdot\nabla + \mu_a + \mu_s + \frac{i\omega}{c})\phi(\underline{r},\hat{\underline{s}};\omega) = \mu_s \int_{S^2} \Theta(\hat{s},\hat{s}')\phi(\hat{s}')d\hat{\underline{s}}' + q(\underline{r},\hat{\underline{s}};\omega)$$

with $\underline{r} \in R^3$, $\hat{s} \in S^2$. In Optical Tomography if $\mu_a \ll \mu_s$ the solution ϕ is well approximated by the first two terms in its expansion in spherical harmonics

$$\phi(\underline{r},\hat{\underline{s}}) \approx \alpha\Phi(r) + \beta\hat{\underline{s}}\cdot\underline{J}$$

with

$$\Phi = \int_{S^2} \phi(\underline{r},\hat{s})\, d\hat{s} \qquad \underline{J} = \int_{S^2} \hat{\underline{s}}\ \phi(\underline{r},\hat{\underline{s}})\, d\hat{s}$$

and the resultant Diffusion Equation

$$-\nabla\cdot\nu_s\nabla\Phi + (\mu_a + \frac{i\omega}{c})\Phi = Q \qquad Q = \int_{S^2} q\, d\hat{s}\,.$$

In cases where $\mu_a \not\ll \mu_s$ the diffusion equation is not valid and higher order approximations can be obtained by retaining further terms in the spherical harmonic expansion of ϕ. This talk presents a general formulation for these so called P_N approximations and shows examples of the differ in solutions obtained for cases of low scattering and high absorption.

Discrete 2-D and 3-D Fourier Based Radon Transform

Amir Averbuch

(joint work with R. Coifman, D. Doncho, M. Israeli, Y. Shokilinsky)

We define a notion of Radon transform for $n \times n$ data. It is based on summation along lines of absolute slope less than 1 with values at non-cartesian locations defined using trigonometric interpolations on a zero-padded grid. The definition is geometrically faithful, the lines exhibit no 'wraparound effects'. For a special set of lines we describe an exact algorithm which uses $\mathcal{O}(N \log N)$ flops, where $N = n^2$. This relies on a discrete projection-slice theorem relating this Radon transform to what we call the Pseudopolar Fourier Transform. The Pseudopolar FT evaluates the 2-D Fourier transform on a non-cartesian pointset, which we call the Pseudopolar grid. Fast Pseudopolar FT - the process of rapid exact evaluation of the 2-D Fourier transform at these non-cartesian grid points - is possible using chirp-Z transforms. This Radon transform is one-to-one and hence invertible on its range; it is rapidly invertible to any degree of desired accuracy using a preconditioned conjugate gradient solver. We also describe a 3-D version of the transform and the 3-D X-ray transform.

Novikov's inversion formula for the attenuated Radon transform

Jan Boman

(joint work with J.-O. Strömberg)

The attenuated Radon transform R_ρ is defined by $R_\rho f(\theta, p) = \int_{x\cdot\theta=p} f(x)\rho(x,\theta)\, d\theta$, where

$$\rho(x,\theta) = \exp(-\int_0^\infty \mu(x + t\theta^\perp)\, dt)\,,\ x \in R^2, \theta \in S^1, \tag{1}$$

and $\mu(x)$ is a given function with compact support. An explicit inversion formula for R_ρ was recently given by R.G. Novikov (Ark. Mat. 2002). A simplified proof of Novikov's formula was later given by F. Natterer. We prove a formula similar to Novikov's under the following condition on $\rho(x,\theta)$ on a bounded open set Ω containing the support of f:

(2) $\left\{\begin{array}{l}\text{there exists a complex-valued function } \tau(x,\theta) \neq 0 \text{ which} \\ \text{is constant on all lines } x\cdot\theta = p, \text{ such that } S^1 \ni \theta \longmapsto \\ \rho(x,\theta)\tau(x,\theta) \text{ for every } x \in \Omega \text{ is the boundary value of an} \\ \text{analytic function on the disk } |\theta_1 + i\theta_2| < 1.\end{array}\right.$

This condition is satisfied by the weight function (1). On the other hand, there exist functions ρ satisfying (2) which are not of the form (1), and in fact cannot be written $\rho = \rho_0\rho_1\rho_2$, where ρ_0 is of the form (1), ρ_1 depends only on x and ρ_2 is constant on lines $x \cdot \theta = p$.

Phase-Contrast Tomography

Andrei V. Bronnikov

Phase-contrast x-ray computed tomography (CT) is an emerging imaging technique that can be implemented at third-generation synchrotron radiation sources or by using a microfocus x-ray source. Promising results have recently been obtained in materials science and medicine. At the same time, the lack of a mathematical theory comparable with that of conventional CT limits the progress in this field. Such a theory is now suggested, establishing a fundamental relation between the three-dimensional Radon transform of the object function and the two dimensional Radon transform of the phase-contrast projection. A reconstruction algorithm is derived in the form of a filtered backprojection. The filter function is given in the space and spatial-frequency domais. The theory suggested enables one to quantitatively determine the refractive index of a weakly absorbing medium from x-ray intensity data measured in the near-field region. The results of computer simulations are discussed.

Adaptive Estimation in Tomography

Laurent Cavalier

The principle of tomography is to reconstruct a multidimensional function from observations of its integrals over hyperplanes. We consider here a model of stochastic tomography where we observe the Radon transform Rf of the function f with a stochastic error. Then we construct a "data-driven" estimator which does not depend on any a priori smoothness assumptions on the function f. Considering pointwise mean-squared error, we prove that it has (up to a log) the same asymptotic properties as an oracle. We give an example of

Sobolev classes of functions where our estimator converges to $f(x)$ with the optimal rate of convergence up to a log factor.

Image Reconstruction form Truncated Projections

Rolf Clockdoyle

A common practical problem in cone-beam tomography is that the detector is not large enough to view the entire object, so the cone-beam projections are truncated. Truncated projections have always caused some difficulty in image reconstruction theory, and in 2D the approach has generally been to build large enough detector to avoid truncation. In cone-beam tomography where the typical object (a human being) is too large for reasonable detectors, the truncation problem must be addressed. Since there is always some redundancy when a vertex path satisfies Tuy's condition, there is some hope that the lost information can be compensated by the redundancy. However the general problem is not easily resolved because the condition is not known on which rays must be measured through a 3D object for tomographic completeness of a region-of-interest (ROI).
In 2D, it is well known that all rays through the object must be measured in order to stably reconstruct the object. It has been widely assumed that all such rays are also required even if only a ROI is to be reconstructed. Recent results have shown this statement to be false, and opens up the general ROI question in 2D also. These results appeared in a publication by Frédéric Noo et al [Phys. Med. Biol. 47(14), 2525-2548 (July 2002)] and are based on an old formula $Hp(\phi, s) = g_H(v, \phi)$ (where v satifies $v_1 \cos\phi + v_2 \sin\phi = s$) linking the Hilbert transform of parallel projection data $p(\phi, s) = \int f(sn + t\theta)dt$ and fanbeam data $g(v, \phi) = \int_0^\infty f(v + t\theta)dt$. Here $n = (\cos\phi, \sin\phi)$, $\theta = (-\sin\phi, \cos\phi)$,

$$Hp(\phi, s) = \int \frac{p(\phi, s')}{\pi(s - s')} ds' \text{ and } g_H(v, \phi) = \int_{-\pi}^{\pi} \frac{g(v, \phi')}{\pi \sin(\phi - \phi')} d\phi'.$$

Basically, values of Hp can be calculated using properly chosen fanbeam projections, and this process partially releases the requirement of measuring uninteresting line-integrals, those not passing through the ROI.

On the inverse problem of optical tomography

Thomas Dierkes

Optical tomography in time and frequency domain is governed by an inverse problem for a parabolic equation and an elliptic equation, respectively. The corresponding boundary to boundary data maps are described in terms of a Lippmann-Schwinger integral equation. We will make use of the singular value decomposition of the integral operator for solving the non linear and ill-posed inverse problem iteratively. Numerical examples in 3D are given both for synthethic and experimental data.

A level set method for shape reconstruction in medical and geophysical imaging

Oliver Dorn

Recently, the level set method for describing propagating fronts has become quite popular in the application of medical or geophysical tomography. The goal in these applications is to reconstruct unknown objects inside a given domain from a finite set of boundary data. Mathematically, these problems define nonlinear inverse problems, where usually iterative solution strategies are required. Starting out from some initial guess for the unknown obstacles, successive corrections to this initial shape are calculated such that the so evolving shapes eventually converge to a shape which satifies the collected data. Since the hidden objects can have a complicated topological structure which is not known a priori, the shapes usually undergo several topology changes during this evolution before converging to the final solution. Therefore, a powerful and flexible tool for the numerical description of these propagating shapes is essential for the success of the inversion method of choice. In the talk, we present a recently developed two-step shape reconstruction method which uses a level set representation of the shapes for this purpose. Numerical results will be presented for three different practically relevant examples: cross-borehole electromagnetic tomography using a 2D Helmholtz model, surface to borehole 3D electromagnetic induction tomography (EMIT) using a model based on the full 3D system of Maxwell's equations, and diffuse optical tomography (DOT) for medical imaging using a model based on the linear transport equation in 2D.

Sampling Theory and Parallel-Beam Tomography

Adel Faridani

Applications of sampling theory in tomography include the identification of efficient sampling schemes; a qualitative understanding of some artifacts; and numerical analysis of reconstruction algorithms. The focus of this talk is the investigation of artifacts caused by undersampling in the angular variable in 2D parallel-beam tomography. A new error analysis for the filtered backprojection algorithm is presented which reveals both the location and the relative strength of these artifacts, depending on the sampling lattice. While the undersampling artifacts are quite small for the standard lattice, they are much more severe for the theoretically more efficient interlaced lattice. The two lattices also differ with respect to the location of the artifacts. If the object is concentrated near a point x_0, then for the standard lattice the artifacts appear first near the boundary of the reconstruction region opposite of x_0. For the interlaced lattice we distinguish two cases. If the detector spacing d assumes its maximal value of $d = 2\pi/b$, then the artifacts appear first on the whole boundary of the reconstruction region. If d is smaller, then they appear first on the part of the boundary closest to x_0. A preprint is available at www.oregonstate.edu/ faridana.

Thermoacoustic Tomography Motivates a Generalized Radon Inversion Problem

David Finch and Sarah Patch
(joint work with Rakesh)

Thermoacoustic tomography as developed by Kruger et al. measures the generalized Radon transform of integration over spheres $Rf(p,t) = \int_{\theta\in S^2} f(p+t\theta)\, d\theta$. Here the support of f is assumed to lie in a ball and the centers lie on the boundary of the ball. In the physical problem the centers are constrained to lie in a hemisphere. We solve the inversion problem assuming centers on the full sphere, by a formula of filtered back projection type. This is equivalent to an energy identity for solutions of the wave equations which we can establish in all odd dimensions. We also have uniqueness and indirect inversion methods for other center sets, using wave equation methods.

Domain visualization with the factorization method

Natalia Grinberg

We study the Helmholtz equation $\delta u + k^2 u = 0$ in exterior of unknown (to reconstruct) obstacle $\mathcal{D}$ with C^2-boundary Γ. The boundary condition can be a Dirichlet, Newton, Robin or mixed type. The data for domain visualization is the far field operator (FFO) $F : L_2(S^2) \to L_2(S^2)$ (in R^3)

$$(Fg)(\eta) = \int \vartheta_\infty(\eta,\theta) g(\theta)\, d\theta,\ \ k \text{ is fixed}\,,$$

where $\vartheta_\infty(\eta,\theta)$ is the far field pattern of the scattered plane wave $\vartheta^S(\cdot,\theta)$ corresponding to $\vartheta^i(x,\theta) = \exp(ikx\cdot\theta)$. The reconstruction is given pointwise: for each $z \in R^3$ holds: the function $Q_z(\eta) = e^{-ik\eta\cdot z} \in L_2(S^2)$ belongs to the range of the 'data-to-pattern' operator G if and only if $z \in \mathcal{D}$. To find $R(G)$ we use the 'square root characterization':

$$\mathcal{R}(G) = \mathcal{R}([F^*F]^{1/4}) = \mathcal{R}(|F|^{1/2}) \quad \text{(Dirichlet or Neumann case)}$$

resp. $\mathcal{R}(G) = R([|ReF| + ImF]^{1/2})$(impedance case),
or the F-characterization:

$$z \in \mathcal{D} \Longleftrightarrow \inf\{|\langle F\psi,\psi\rangle| : \langle Q_z,\psi\rangle = 1, \psi \in L_2(S^2)\} > 0\,.$$

To handle the mixed boundary problem case (Dirichlet on one part of the boundary Γ_1 and Robin/Neumann on the rest Γ_2) we can use, in principle, the F-characterization with test functions ψ subject to special constraint. If two reference domains $\mathcal{D}'_{1,2}$ with $\bar{\mathcal{D}}_j \subset \mathcal{D}'_i\,, j = 1,2$ are a-priori known, then the test functions for visualization of $\mathcal{D}_1$ should be small on $\mathcal{D}'_2$ and v.v. If no a-priori geometrical information is available, we restrict ourselves to the test sequence $\{\psi_k^{(\sigma)},\ k = l,\dots\infty\}$ concentrating at curve σ, which connects the point z with infinity (it is convenient to take rays). With the F-characterization it is possible to detect the intersection points of σ and Γ.

Dynamic X-ray Computed Tomography

Pierre Grangeat
(joint work with S. Bonnet, A. Koenig, T. Rodet, S. Roux, P. Hugonnard, R. Guillemaud)

Dynamic Computed Tomography (CT) imaging aims at reconstructing image sequences where the dynamic nature of the living human body is of primary interest. Concerned applications are radiotherapy planning, image-guided interventional procedures, functional studies and cardiac imaging. The introduction of ultra-fast rotating gantries along with multirow detectors and in near future area detectors allows a huge progress towards the imaging of moving organs with low-contrast resolution. In order to compensate for time evolution and motion artefacts, we propose to use a dynamic particle model to describe the object evolution. One main interest is to process data acquisition on several half-turns in order to reduce the dose delivered per rotation while keeping the same signal to noise ratio for every frame. The proposed algorithm is based on an adaptive motion-compensated temporal prediction along the particle trajectories within the backprojection computation. We describe the dynamic particle model and its approximations, the dynamic cone-beam CT acquisition model and the dynamic cone-beam reconstruction algorithm associated with a cone-beam to fan-parallel beam rebinning approach. Such an algorithm provides 4D image sequences with accurate spatio-temporal information. Results have been illustrated on simulated data.
Reference: P. Grangeat, A. Koenig, T. Rodet, S. Bonnet, "Theoretical framework for a dynamic cone-beam reconstruction algorithm based on a dynamic particle model", Phys. Med. Biol., August 2002 issue.

Nonlinear network tomography

F. Alberto Grünbaum
(joint work with S. Patch, L. Matusevich)

Consider a Multiterminal Network, i.e. a directed graph with 3 kinds of nodes: incoming (sources), outgoing (sinks) and hidden (interior). For each directed edge there is an unknown transition probability between the nodes determining the edge in question. For any edge the 'time' for a transition is one.
The problem is to recover as much as possible of the unknown probability matrix from the 'moments' of the 'time of flight' for any source-sink pair.
For certain networks inspired by diffuse tomography one can do an optimal job, i.e. determine exactly all the unknowns except for a 'natural gauge'. The dimension of the gauge equal the number of hidden states.
In some small cases this inversion can be done by means of explicit formulas.

An exact FBP-type inversion algorithm for spiral cone-beam CT

ALEXANDER KATSEVICH

We discuss a theoretically exact formula for inversion of data obtained by a spiral CT scan with a 2-D detector array. The formula can be implemented in a filtered back-projection fashion, in which the filtering step is shift-invariant. Some properties of the formula are studied. We find the dependence of the back-projection coefficients on the x-ray source-voxel distance. We show that if f is z-independent or if the pitch of the spiral goes to zero, the formula transforms to a familiar in the 2-D case. Results of numerical experiments with the formula are presented.

Inversion of the exponential ray transform in 3-D using parallel-beam data measured from closed orbits

LEONID KUNYANSKY

We derive an explicit inversion formula for the 3-D exponential ray transform in the case of parallel-beam measurements made from closed 1-D orbits, valid for the case of constant attenuation. As we show, this problem can be reduced to inversion of a series of the exponential Radon transforms in 2-D with complex-valued angle-dependant attenuation. We present an inversion formula for such 2-D transforms, thus solving the original 3D problem. An interesting property of the present approach is that the reconstruction is possible even when the detector's trajectory does not satisfy the well-known Tuy condition — in the both attenuated and unattenuated cases.

3D CT: Tomosynthesis and phase contrast

ALFRED K. LOUIS
(joint work with R. Müller, P. Jonas)

In this talk the approximate inverse is presented as a general tool to derive reconstruction formulae. It is a regularization technique that allows to incorporate invariance properties of the operator in the reconstruction kernel. As a first example the derivation of inversion formulae for 3D CT is considered reconstructions and movies from real data are presented. The next example is tomosynthesis, a 3D CT technique with a special scanning geometry. Here a hybrid approach is used. The first step is the classical backprojection. In the second step a reconstruction kernel is used that is numerically determined as inverse of R^*R. Reconstructions from a realistic phantom show the applicability of the method and the typical limitations in the resolution.
As third example a holographic imaging technique is presented which leads to phase contrast tomography. This technique is applied if the object under consideration does not vary to much, and where wave phenomena bear the information. A mathematical model relating the data $|(u^i + u^s)(x)|^2$ with the complex-valued object is presented and nonlinear inversion schemes are derived. Reconstructions and movies from real data are shown also here.

Nonlinear multiscale smoothing and an application in MEG imaging

Peter Maass

The MEG group at the neurophysiological department at the University of Tübingen research 168 channels of MEG data in an endeavour to determine the risk of premature birth. The image processing task is to seperate the different structures in the MEG data (heart mother, heart baby, contraction, etc.).
In the second part of the talk we compare different concepts of multiscale smoothing and analyse the relations between there concepts. As a theoretical result we determine a PDE which is equivalent to Donoho shrinkage (Wavelet smoothing).
Finally we present results for different applications, including the separation of the contractions from the MEG data. This rests on a careful, adaptive choice of the truncation levels.

A practical Sobolev space for Compterized Tomography

Prabhat Munshi

An inverse theorem has been developed to predict the error in CT images obtained by the convolution backprojection algorithm. The numerical verification and experimental validation has also been carried out. The main advantage (of these Sobolev space based error estimates) is the prediction of reconstruction error in 'real' CT images where the actual cross-section is not known.

The role of microlocal analysis in high-frequency imaging

Clifford Nolan

Geometrical Optics and its generalisations using microlocal analysis represent an ideal tool for representing the propagation and scattering of waves.
Starting with the acoustic wave equation, we review how to approximate singly-scattered waves. Such waves are viewed as perturbed pressure waves about a known background field. They rise from an unknown singular (non-smooth) perturbation in sound speed about a known reference speed. The data is measured for pairs of sources and receivers on a hypersurface and can be shown to be a 'generalised Radon Transform' of the sound speed perturbation. We then show how singularities are mapped from the sound speed to the data; defining the 'scattering relation', and explain how, when the relation is $\geq 2:1$ (many-to-one), artifacts will be generated in the backprojected image. We have an explicit criterion of how to avoid artifacts based on where/how data is collected and the reference sound speed (which is variable in general).
Finally, we present a recent result of G. Uhlmann and C. Nolan, where a geometrical optics expansion is developed for the Green's Tensor in a generic EM (Maxwell's Eq.) or linear elastic system.

Dynamic Emission Tomography

Dominikus Noll

Dynmaic Single Photon Emission Tomography (dSPECT) visualizes the changing distribution of a radiopharmacentical in the human body. This requires new reconstruction methods, which in the case presented are based on nonlinear optimization methods. Clinical applications are discussed.

Ray methods in diffraction tomography

Victor Palamodov

Any novel technique for solving the inverse scattering problem for the acoustic equation is described. Any beam like solution of the Helmholtz equation can be approximated by an exact solution which decreased fast out of the given ray in the configuration plane. Substituting this solution together with a beam like solution of a perturbed Helmholtz equation into the Green integral skew form gives the sharpest possible localization of the perturbation of the refraction coefficient in the phase space in terms of boundary measurements of both solutions. We can scan the support of the perturbation by variation of two rays.

Linear Methods in Microwave Tomography

Michele Piana

Microwave Tomography (MT) is a nonlinear inverse scattering problem which is difficult to solve because it is ill-posed and generically non-linear. Possible approaches involve Newton-type optimization schemes (accurate but typically computationelly heavy), linearized methods (not applicable in the case of resonance frequencies) and linear sampling methods (not yet experimentally validated). Here I describe a Japanese microwave tomograph where the input signal is a chirp compactly supported signal and the geometrical setup is similar to the X-ray tomography parallel beam one. In particular I introduce a linear model showing that, in space invariant conditions, the image formation processing can be described by means of the convolution product of the device response function and the Radon projection of the outcast function. This model naturally suggests an image restoration algorithm based on two steps: regularized deconvolution of the chirp sinogram and application of FBP to the regularized Radon projection.

Tomography plus the wave equation

Eric Todd Quinto

(joint work with M. Agranousky, O. Öktem, U. Skoglund, Sidec, L. Quinto)

The author spoke about two topics. First, he discussed a problem in electron microscopy, the reconstruction of objects from unevenly spaced limited tomographic data. He showed a reconstruction of an RNA molecule using a version of Lambda CT he developed for this problem. This is work in collaboration with Ulf Skoglund and Ozan Oktem of SIDEC Technologies in Sweden.

Second, the author presented new results with Mark Agranovsky on stationary sets for the Dirichlet problem for the wave equation on crystallographic domains Ω in R^n. A stationary set is the subset of Ω where the solution to the wave equation initial boundary value problem stays stationary for all time. We completely characterize stationary sets when the domain is a fundamental domain of a crystallographic group generated by reflections in the boundary hyperplanes of Ω. We assume the initial position is $u(x, 0) = 0$ and the initial velocity, $u_t(x, 0) = f(x)$ is compactly supported in the interior of Ω. Stationary sets, if not empty, consist only of cross-sections of the domain by hyperplanes union lower dimensional sets. The hyperplanes have well defined symmetry.

Dynamical systems method for solving linear and nonlinear ill-posed and well- posed problems

Alexander G. Ramm

Consider an operator equation $F(u) = 0$ in a Hilbert space. The problem of solving this equation is ill-posed if the operator $F'(u)$ is not boundedly invertible. A general method for solving linear and nonlinear ill-posed problems in a Hilbert space is presented. This method consists of the construction of a nonlinear dynamical system, that is, a Cauchy problem, which has the following properties

(1) it has global solution
(2) this solution tends to a limit as time tends to infinity
(3) the limit solves the original linear or non-linear problem.

Examples of the applications of this approach are given. The method works for a wide range of well-posed problems as well.

Mathematical Methods in Tomography

Andreas Rieder

The filtered backprojection algorithm is probably the most often used reconstruction algorithm in 2D-computerized tomgraphy. For the parallel scanning geometry we prove optimal L_2-convergence rates for density distribution in Sobolev spaces. The key to success is a new representation of the filtered backprojection which enables us to apply techniques from approximation theory. Our analysis provides further a modification of the Shepp-Logan reconstruction filter with an improved convergence behavior. Numerical experiments reproduce the theoretical predictions.

Micro-CT: Special Challenges and Solutions

Erik L. Ritman

With the increasing interest in drug discovery and micro-architectural features of intact organs, there is much interest in high resolution tomographic imaging of small rodents. While to some extent this involves building 'mini' versions of clinical whole-body scanners with spatial resolution in the $0.5 - 1.0\text{mm}^3$ range, the greatest interest is in increasing the spatial resolution to the micrometer level. This latter requires 'micro' scanners, which involves both physics and precision requirements and oppertunities not considered in the clinical or mini scanners. While hardware solutions to many of these special needs can be found, the limits of the hardware can be extended by judicious use of novel scanning and reconstruction algorithms.
Examples of new developments in, and speculations about, meeting these challanges are presented.

Defect convection in vector field tomography using boundary element methods

Thomas Schuster

The problem of 2D vector field tomography consists of reconstructing a vector field $f \in L^2(\Omega)^2$ from the Doppler transform

$$Df(\omega, s) = \int_{\langle x,\omega\rangle = s} \langle \omega^\perp, f(x)\rangle \, dl(x)$$

of f for finitely many given directions $\omega_k \in S^1$ and distances from the origin $s_l \in [-1, 1]$. An approximate inverse of f is computed by

$$(f_{\text{app}})_j(x) := S^j_{u,\gamma} D_n f(x) = \sum_{k,l} Df(\omega_k, s_l)\Phi^j_\gamma(\omega_k, s_l - (x, \omega_k)) \, ,$$

which is a reconstruction formula of filtered backprojection type. Using the Helmholtz decomposition of f_{app}

$$f_{\text{app}} = f^s + \nabla p \, ,$$

which becomes unique postulating $\langle n, f^s\rangle = 0$ on $\partial\Omega$, we obtain

$$\|f - f^s\|_{L^2} < \|f - f_{\text{app}}\|_{L^2} \, .$$

The potential part ∇p is determined solving the Neumann boundary problem

$$(*) \quad \begin{cases} \Delta p &= \nabla \cdot f_{\text{app}} \text{ in } \Omega \, , \\ \frac{\partial p}{\partial n} &= \langle n, f_{\text{app}}\rangle \text{ on } \partial\Omega \end{cases}$$

An approximate solution of $(*)$ ist presented using boundary element methods, which avoids the differentiation of f_{app}. Furthermore we get a stable and fast approximation to the arising Newton potential using the special structure of $S^j_{n,\gamma}$ and the filter Φ^j_γ. Several numerical results are shown.

Moment based tools for Doppler Vector Tomography

Gunnar Sparr

Given a vector field in some region in 2D or 3D. By Doppler Vector Tomography we mean situations where known data constist of, for 'each' line, the distribution (or histogram) of the velocity components of the vector field along the line. Such data can be achieved e.g. by Doppler ultrasound measurements. The problem is to reconstruct the vector field. For this inverse problem uniqueness can not be expected, as shown by simple examples of vector fields having the same velocity distribution for every line. In the planar case, by considering moments of different orders of such velocity distributions, a new transform is defined, the M-transform. It is shown that for each order, the M-transform can be expressed as a differential operator acting on the field components. Recursion formulas for M-transforms of different orders are derived, with the implication that the vector field has to fulfil one Poisson equation, one Monge-Ampere equation, and an infinite set of algebraic equations, with coefficients described by M-transforms of different orders. In particular, at each point the number of solutions has an upper limit of six. (Reference: Fredrik Andersson, Lic.Thesis, Lund 2002.)

Evaluation of Radical Functions by Fast Fourier Transforms at Nonequispaced Knotes

Gabriele Steidl

We develop a new algorithm for the fast evaluation of radial functions

$$f(y_j) = \sum_{k=1}^{N} \alpha_K K(\|y_j - x_K\|)$$

at knots $y_j \in R^2$ $(j = 1, \dots M)$ based on the recently developed fast Fourier transform at nonequispaced knots. Our algorithm is simply structured so that it can easily be adapted to different kernels. This holds in particular for our regularization procedure which is necessary for singular kernels. For these kernels our algorithm needs $\mathcal{O}(N \log \sqrt{N} + N + M)$ arithmetic operations, where the constant of proportionality depends only on the precision required. The complexity reduces further to $\mathcal{O}(N + M)$ arithmetic operations for smooth kernels. We prove error estimats to obtain clues about the choice of the involved parameters and present numerical examples for various frequently applied singular and smooth kernels in two dimensions.

On Novikov's inversion formula for the attenuated Radon transform

Jan-Olav Strömberg

(joint work with J. Boman)

The inversion formula holds for weighted Radon transforms for attenuated weights $\rho(\theta, x) = \exp(-\int_0^\infty \mu(x + t\theta^\perp)\, dt)$. The formula can be proved by rather elementary arguments: using polar coordinates, a change of the order of integration and the calculation of three simple residue integrals. With a sligt change in the formula it will also hold for a class of weights which is somewhat larger than the class of attenuated weights. We also observe that with a careful formulation of the inversion formula it will hold also when the attenuation function $\mu(x)$ is complex valued.

Three-Dimensional Reconstruction by Chahine's Method from Projections Corrupted by Electron Microscope Aberrations

J. P. Zubelli

(joint work with R.Marabini, C.O.S.Sorzano and G.T.Herman)

A projection image obtained by an electron microscope can be conceived of as an "ideal" projection subjected to a contrast transfer function (CTF), which eliminates some frequencies and reverses the phase of others. The aberration caused by the CTF makes the problem of reconstruction from such data difficult. We reformulate the problem so that Chahine's method becomes applicable to it. We substantiate our results with ample numerical evidence using both simulated and actual electron microscopy data.

Edited by Peter Jonas

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