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Komplexe Analysis

August 25th – August 31st, 2002

Die Tagung stand unter der Leitung von J.-P. Demailly (Grenoble), K. Hulek (Hannover) und T. Peternell (Bayreuth). Die Teilnehmer kamen aus einer Vielzahl europäischer und außereuropäischer Länder. In den Vorträgen, bei denen insbesondere auch jüngere Mathematikerinnen und Mathematiker die Möglichkeit erhielten, über ihre Forschungsergebnisse zu berichten, stellten sich folgende Themenschwerpunkte heraus: Calabi-Yau Mannigfaltigkeiten, Modulräume, Flächen von allgemeinem Typ, Blätterungen, Klassifikation spezieller Varietäten, Effektivität von Divisoren, singuläre Metriken, Zyklenräume. Neben dem Vortragsprogramm gab es eine intensive wissenschaftliche Zusammenarbeit vieler Teilnehmer.

Abstracts

Higher dimensional Zariski decompositions

Sébastien Boucksom

Let X be a compact complex manifold. For any pseudo-effective class $\alpha \in H^{1,1}_{\partial\bar{\partial}}(X, \mathbf{R})$, we define pointwise minimal multiplicities $\nu(\alpha, x)$, $x \in X$, which are the local obstructions to the numerical effectivity of α. We can thus distinguish a non-nef locus of α, and define its divisorial Zariski decomposition by removing from α the divisorial part of its non-nef locus, counting multiplicities in order to end up with a class which is nef in codimension one.

Uniformisation of holomorphic foliations by curves

Marco Brunella

We discuss the following result:

Theorem 1. *Let X be a compact connected Kähler manifold and let $\mathcal{F}$ be a (possibly singular) holomorphic foliation by curves on X. Assume that at least one leaf of $\mathcal{F}$ is hyperbolic. Then the Poincare metric on the leaves of $\mathcal{F}$ defines on its canonical bundle $K_{\mathcal{F}}$ a singular hermitian metric whose curvature is positive (in the sense of currents).*

The proof consists in establishing a set of holomorphic convexity for the 'foliated' universal covering of $(X, \mathcal{F})$, in order to apply classical results by Nishino and Yamaguchi about fibrations on Stein manifolds.
A consequence of the theorem is the pseudoeffectivity of $K_{\mathcal{F}}$. In this sense, it can be considered as an explicit metricised version of some results by Miyaoka, Shepherd-Barron, McQuillan, Bogomolov.

Deformation, symplectic and Q.E.D.-equivalence for surfaces of general type

Fabrizio Catanese

Friedman and Morgan's speculation in the '80's was that orientedly diffeomorphic minimal surfaces of general type should be deformation equivalent ($\sim_{def}$). We observe that $X \sim_{def} Y \Rightarrow \exists$ a diffeom. $\phi : X \to Y$ with $(*) : \phi^*(K_Y) = K_X$, K_X the class of the canonical bundle.
Witten's theorem (extended by Taubes for symplec. 4-manifolds) says that $\phi : S \to S'$ a diffeom. $\Rightarrow \phi^*(K_{S'}) = \pm K_S$. Up to date there are 3 types of counterexamples to the Friedman-Morgan Conjecture, namely

i) Manetti('98) used $(\mathbf{Z}/2)^r$ covers of $\mathbf{P}^1 \times \mathbf{P}^1$; his surfaces have $b_1 = 0$ but are not 1-connected,

ii) Kharlamov-Kulikov ('01) used a variant of Hirzebruch's ball quotient, so S is a $K(\pi, 1)$,

iii) I used ('01) surfaces $S = C_1 \times C_2/G$ quotients of product of curves of genus ≥ 2.

For $ii), iii)$ one takes $S' = \bar{S}$, so tat it is the same C^∞ manifold but $\phi^*(K_{\bar{S}}) = -K_S$. The heart of the proof is to show $\nexists \psi : S \to S$ a diffeomorphism such that $\psi^*(K_S) = -K_S$.

iv) there are 1-connected candidates, but the difficulty is to show diffeomorphism.

Now we show the following (easy if K_S is ample):

Theorem 2. *S of general type $\Rightarrow \exists!$ (up to symplectomorphism) symplectic structure ω with $[\omega] = K_S$.*

Theorem 3. *The canonical symplectic structure is invariant for deformation and degeneration to normal surfaces yielding smoothings, $\forall x \in Sing(X_0)$ in the same component of $\mathrm{Def}(X_0, x)$, X_0 the central fibre.*

A corollary of these is

Theorem 4. *Manetti's surfaces yield examples $(S\omega)$, (S', ω'), which are symplectomorphic but not deformation equivalent.*

After sketching the related theory of Donaldson's quasi-holomorphic maps on symplectic 4-manifolds and the Auroux-Katzarkov-Donaldson-Yotov invariants of symplectic 4-manifolds, I finally introduced a very large equivalence relation: Q.E.D. is generated by

i) birational equivalence
ii) flat deformations with fibres with canonical singularities
iii) morphisms $f : X \to Y$ unramified in $cod = 1$.

Siu's theorem $\Rightarrow$ $X \sim_{Q.E.D.} Y$ then $kod(X) = kod(Y)$.

Theorem 5. *For curves and surfaces of special type $kod(X) = kod(Y) \Rightarrow X \sim_{Q.E.D.} Y$.*

Characterizing curves by their theta-characteristic

LUCIA CAPORASO

The geometry of the moduli space of stable spin curves is studied, with particular emphasis on its combinatorial properties. Our moduli theoretic results are applied to a problem in classical projective geometry: we show that a general canonical curve is uniquely determined by the configuration of hyperplanes cutting theta-characteristics on it.

On surfaces of class VII_0^+ with curves

GEORGES DLOUSSKY

A minimal compact complex surface S belongs to class VII_0^+ if its Betti numbers satisfy $b_1(S) = 1$ and $b_2(S) > 0$. It is well known that such surfaces have at most $b_2(S)$ rational curves.

Theorem 6 (G.D., K. Oeljeklaus, M. Toma)**.** *Let S be in class VII_0^+ then S contains a global spherical shell if and only if S contains $b_2(S)$ rational curves.*

Therefore the main problem is to construct such curves. We have $b_2(S)$ curves if S admits a global vector field. Finally a finite quotient of a surface S with a global spherical shell is after desingularisation of the same type.

Tsuji's numerically trivial fibrations for pseudo-effective line bundles

Thomas Eckl

In this talk two theorems were proven which are related to Tsuji's recently defined intersection numbers of (irreducible) curves C and pseudo-effective line bundles L w.r.t. to a positive singular hermitian metric h on L (suppose $h_{|C} \not\equiv \infty$):

$$(L,h).C := \limsup_{m\to\infty} \frac{1}{m} h^0(\overline{C}, \mathcal{O}_{\overline{C}}(m\pi^* L) \otimes \mathcal{I}(\pi^* h^m)),$$

where $\pi : \overline{C} \to C$ is the normalization of C and $\mathcal{I}(\pi^* h^m) \subset \mathcal{O}_{\overline{C}}$ is the multiplier ideal sheaf associated to the positive singular hermitian metric $\pi^* h^m$.
The first theorem characterizes $(L,h)-$ numerical triviality (i.e. for every curve C with $h_{|C} \not\equiv \infty$ the intersection number $(L,h).C$ is 0.

Theorem 7. *X is $(L,h)-$ numerically trivial* $\Longrightarrow \Theta_h = \sum_i a_i[D_i] + R$, $\nu(R,x) = 0$ *for all* $x \in X$.

This is proven by using $(L,h)-$ **general curves** on which the restricted metric $h_{|C}$ has the same Lelong numbers as h.
The second theorem deals with fibrations whose fibres are numerically trivial:

Theorem 8. $\exists f : X$ ----> Y *dominant rational map with connected fibres such that*

(i) *fibres over points outside a pluripolar set are numerically trivial,*
(ii) *all curves C with* $\dim f(C) = 1$ *through points $x \in X$ outside a pluripolar set satisfy* $(L,h).C > 0$.

f is unique up to birational equivalence.

This is proven by using a lemma which states that a family with numerically trivial fibres and an $(L,h)-$ general numerically trivial section is itself numerically trivial.

Relative Gromov-Witten invariants

Andreas Gathmann

Let X be a smooth complex projective variety, and let $Y \subset X$ be a smooth hypersurface. We want to look for a way to compute the Gromov-Witten invariants of Y (of any genus) from those of X.

Our idea is to degenerate the manifold X to a singular (normal crossing) space with two components. One of the components is X again, and the other is the projective completion P of the normal bundle $N_{Y/X}$. The two components X and P intersect in Y.

Following a construction of Jun Li, we define Gromov-Witten invariants of this singular space. These invariants can be computed as a certain product of invariants of the two components X and P. More precisely, we need the so-called relative invariants of X and P relative Y. These invariants can be interpreted as numbers of curves in X (resp. P) with given multiplicity conditions to Y. We sketch the construction of the corresponding compact moduli spaces of relative stable maps.

Finally, we show how a relation between the relative invariants of P and the absolute invariants of Y could lead to a formula for the Gromov-Witten invariants of Y in terms of those of X. In genus 0, this program has already been completed and leads to the famous mirror formula.

Effective Algebraic Schottky Problem

Samuel Grushevsky

Schottky problem, the question of characterizing Jacobians of Riemann surfaces among principally polarized abelian varieties, has been solved by Shioda. He completed the proof of Novikov's conjecture, which states that an abelian variety is a Jacobian if and only if a certain modification of its associated theta function satisfies the Kadomtsev-Petviashvili (KP) differential equation. However, this solution is not effective and not algebraic in terms of theta constants. An effective algebraic solution in the spirit of Schottky and Jung's original approach has not yet been obtained.

We obtain formulas for degrees of the image of the Jacobian locus and of the moduli of principally polarized abelian varieties in the projective space, under the embedding by level two theta constants map, in terms of some intersection numbers on the moduli spaces. These degrees are then computed numerically in low genera — the results agree with the known numbers in genera up to three, and are new for genus four and higher. Using Hodge index theorem and previous work on Weil-Petersson volumes, we further obtain an explicit upper bound for the degree of the Jacobian locus in terms of the genus.

Using this bound and effective Nullstellensatz, we then show that the KP equation for the theta function of an abelian variety is equivalent to an effectively constructed system of algebraic equations for theta constants. We thus effectively obtain an algebraic solution to the Schottky problem.

Some recent effective results in algebraic geometry

Gordon Heier

As a result towards the freeness part of the Fujita Conjecture, it is proven that, if X is a smooth compact complex manifold of dimension n, L is an ample line bundle and K_X is the canonical line bundle, then $K_X + mL$ is base point free for any integer

$$m \geq (e + \frac{1}{2})n^{\frac{4}{3}} + \frac{1}{2}n^{\frac{2}{3}} + 1,$$

where $e \approx 2.71$ is Euler's constant.

Furthermore, as an interesting instance of (effective) 'boundedness and rigidity'-type theorems, an estimate on the cardinality of certain finite sets of surjective maps between polarized manifolds of arbitrary dimension is proven, generalizing (effectively) a theorem of de Franchis-Severi for curves to arbitrary dimension. Then, based on the same philosophy, uniform effective versions of the Shafarevich Conjecture over function fields (Theorem of Parshin-Arakelov) and the Mordell Conjecture over function fields (Theorem of Manin) are proven. The proofs rest on a number of new algebraic geometric results that should be of independent interest.

Characterisation of cycle domains by Schubert calculus and Kobayashi

Alan Huckleberry

A real form G of a complex semisimple Lie group $G^{\mathbf{C}}$ has only finitely many orbits in any given $G^{\mathbf{C}}$ flag manifold $Z = G^{\mathbf{C}}/Q$. The complex geometry of these orbits is of interest, e.g. for the associated representation theory. The open orbits D generally possess only the constant holomorphic functions and the relevant associated geometric objects are certain positive dimensional compact complex submanifolds of D which, with certain well-understood exceptions, are parametrized by the Wolf cycle domains $\Omega_W(D)$ in $\Omega := G^{\mathbf{C}}/K^{\mathbf{C}}$, where K is a maximal compact subgroup. Thus, for the various domains D in the various ambient spaces Z, it is possible to compare the cycle spaces $\Omega_W(D)$.

The main result discussed in the lecture is that, with the few exceptions mentioned above, for a fixed real form G all of the cycle spaces $\Omega_W(D)$ are the same. They are equal to a universal domain Ω_{AG} which is natural from the point of view of group actions and which in essence, can be explicitly computed.

The inclusion $\Omega_{AG} \subset \Omega_W(D)$ follows from a Schubert intersection theory, which allows us to construct supporting incidence hypersurfaces at every boundary point of $\Omega_W(D)$, along with the identification of Ω_{AG} with a domain of cycles which is defined by all possible incidence hypersurfaces (joint work with J.A. Wolf together with recent results of the lecturer).

The opposite direction is based on the following result (joint with G. Fels): If $\hat{\Omega}$ is a G-invariant Stein domain in Ω which contains Ω_{AG} and is Kobayashi hyperbolic, then $\hat{\Omega} = \Omega_{AG}$. The inclusion then follows from the fact that $\Omega_W(D)$ is Kobayashi hyperbolic. This is in turn proved by embedding it in projective space minus the appropriate number of hyperplanes in general position.

Degree of Fano 4-folds

Jun-Muk Hwang

We show that the anti-canonical degree of a 4-dimensional Fano manifold of Picard number 1 is bounded by 625 and when the degree is exactly 625 the Fano 4-fold is $\mathbf{P}^4$. The proof uses the geometry of standard rational curves on the Fano 4-folds in combination with Nadel's product lemma.

MMP and derived categories

Yjiro Kawamata

We discuss the K-equivalence and D-equivalence of smooth projective varieties.

Theorem 9. *Assume X, Y are smooth projective of dimension n and $D^b(CohX) \simeq D^b(CohY)$. If $\kappa(X) = n$, then X and Y are birational and K-equivalent.*

Theorem 10. *Let X, Y be $\mathbf{Q}$-factorial terminal projective 3-folds which are K-equivalent. Then $D^b(Coh\mathcal{X}) \simeq D^b(Coh\mathcal{Y})$, where $\mathcal{X}$ and $\mathcal{Y}$ are canonical covering stacks of X and Y. (An example shows that $D^b(CohX)$ and $D^b(CohY)$ are not necessarily equivalent).*

Meromorphic functions on cycle spaces defined by integration

Jörn Magnusson

After a brief introduction to the theory of integration of meromorphic cohomology classes on analytic families we state and explain the following theorems:

Theorem 11 (J.M., D. Barlet). *Let Z be a compact manifold, Y an ample l.c.i. of codim $n+1$ in Z and $(X_s)_{s\in S}$ an analytic family of n-cycles in Z with S compact. For every irreducible component Σ of the incidence divisor of Y and $(X_s)_{s\in S}$ there exists a rational number $\kappa \leq 1$ such that*

i) *for every ξ of order ν in $H^{n+1}_{[Y]}(Z, \Omega^n_Z)$ we have*

$$\text{order of } \rho^{\circ}(\xi) \text{ along } \Sigma \leq \lceil \nu\kappa \rceil,$$

ii) *there exist an arbitrarily big integers ν and ξ_ν of order ν in $H^{n+1}_{[Y]}(Z, \Omega^n_Z)$ such that*

$$\text{order of } \rho^{\circ}(\xi) \text{ along } \Sigma = \nu\kappa.$$

Theorem 12 (J.M., D. Barlet). *With the same hypothesis as in the previous theorem, let Σ' be the union of the irreducible components of the incidence divisor having the biggest κ and let L be the line bundle associated to the incidence divisor. There exists a Zariski open dense subset U of Σ' such that*

i) *For every s in U there exist j and z in $H^0(S, L^j)$ with $z(s) \neq 0$.*
ii) *For every s, s' in U with $|Y \cap X_s \cap X_{s'}| = \emptyset$ there exist j and z in $H^0(S, L^j)$ such that $z(s) = 0$ and $z(s') \neq 0$.*

Finally we give an application concerning algebraic dimension of compact manifolds.

From Severi varieties to exceptional groups

Laurent Manivel

Severi varieties were classified $\sim$ 1980 by F. Zak, and more generally Serca varieties a few years later. The first part of the talk presents two theorems of P.C.Chaput which allow to simplify greatly the classification:

Theorem 13. *Any Severi variety is homogeneous.*

Theorem 14. *The ambient space of any Serca variety is the projectivisation of a simple Jordan algebra.*

These Jordan algebras are the algebras $J_n(\mathbf{A})$ of Hermition $n \times n$ matrices with coefficients in a division algebra, including the exceptional $J_3(\mathbf{O})$.

The relation between $J_3(\mathbf{O})$ and the exceptional Lie algebras is classical. Surprising recent results have been obtained by P. Vogel and P. Deligne concerning the representations of exceptional Lie algebras, using methods coming from knot theory. We explain and expand Deligne's results by using a variant of the Tits-Freudenthal construction of exceptional Lie algebras, whose main ingredient is triality.

Holomorphic Vector Fields on Fano Manifolds and Applications to Deformation Rigidity

Ngaiming Mok

In a series of articles Jun-Muk Hwang and the speaker have been developing a program of study on the geometry of uniruled projective manifolds X. Fixing an irreducible component $\mathcal{K}$ of the Chow space of X consisting of minimal rational curves as general members, we consider at a general point x of X its variety of minimal rational tangents (VMRT) $\mathcal{C} \subset \mathbf{P}T_xX$.

In a recent work, motivated in part by the difficult cases of deformation rigidity of rational homogeneous spaces $S = G/P$ of Picard number 1 as projective manifolds, we have studied holomorphic vector fields on X particularly for X of Picard number 1.

As guiding problems we formulate two conjectures: (1) at a general point $x \in X$ there is no non-trivial global holomorphic vector field vanishing at x to the order ≥ 3, (2) $\dim(\mathrm{Aut}(X)) \leq n^2 + 2n$, $n = \dim X$, with equality iff $X \cong \mathbf{P}^n$.

We prove special cases of those conjectures under additional geometric assumptions on the VMRT's. The results and methods of proof are then applied to deformation rigidity. Given a regular family $\pi : \mathcal{X} \to \Delta$ whose general fibre is $S = G/P$ we consider over the central fibre X_0 the Lie algebra $\mathbf{g}_0$ of global holomorphic vector fields belonging to the direct image of the relative tangent bundle.

In the difficult cases including the isotopic Grassmannians $S_{n,k}$ of isotropic k-planes in a $2n$-dimensional symplectic vector space, $1 < k < n$, we show that $\mathbf{g}_0$ is isomorphic to the Lie algebra $\mathbf{g}$ of the simple Lie group G.

The first difficulty is the degeneration of the Lie algebra $\mathbf{g}_t = \mathrm{aut}(\mathrm{X_t})$ as $t \to 0$, for instance the existence of non-trivial holomorphic vector fields on X_0 vanishing to the order ≥ 3 at a given general point $x_0 \in X_0$. We can apply our results and methods on holomorphic vector fields to X_0 by showing that X_0 satisfies the additional geometric assumptions on VMRT's which we imposed. The Lie algebra $\mathbf{g}_0 \cong \mathbf{g}$ is reconstructed from the symbol algebra of leading terms at x_0 of holomorphic vector fields in $\mathbf{g}_0$.

In the analogous case of Hermitian symmetric spaces the Lie algebra structure can be determined by the VMRT and by the Taylor coefficients of the leading terms.

Enhanced gauge symmetry for Calabi–Yau threefolds

Balázs Szendrői

Let Γ be a finite subgroup of $\mathrm{SL}(2, \mathbf{C})$ and let $Y \to \mathbf{C}^2/\Gamma$ be the minimal resolution of the quotient surface with exceptional lines $\{F_i\}$. Let $f : X \to B$ be a fibration over a smooth curve B with all fibres isomorphic to Y. Over the curve B, the lines F_i may or may not undergo monodromy. Accordingly, the exceptional surfaces in X are in one-to-one correspondence with nodes of a Dynkin diagram Δ which is of ADE type in case there is no monodromy, or of quotient type B, C, F or G if there is monodromy.

Let $B(\Delta)$ denote the (generalized) braid group corresponding to the Dynkin diagram Δ; by definition, it is the group generated by

$$\{x_i : i \text{ a node of } \Delta\}$$

subject to relations of the form

$$x_i x_j x_i \ldots = x_j x_i x_j \ldots$$

with m_{ij} terms on both sides, where m_{ij} is the index of the edge ij.

Theorem There is a homomorphism

$$B(\Delta) \to \mathrm{Auteq}(D^b(X))$$

where $\mathrm{D}^b(X)$ is the bounded derived category of coherent sheaves of X and $\mathrm{Auteq}(D^b(X))$ is its autoequivalence group.

The theoretical physics literature discusses a certain connection between Calabi–Yau manifolds and Lie algebras under the name of enhanced gauge symmetry. The manifold X is a local quasiprojective model for the simplest case of enhanced gauge symmetry. The theorem shows that (a cover of) the Weyl group of the appropriate Lie algebra acts on the derived category of X. This fits into, and has connections with, the framework of mirror symmetry, and via Kontsevich' homological mirror symmetry conjecture, also with symplectic geometry (of the mirror of X).

A counterexample to the Hodge conjecture for Kähler varieties

Claire Voisin

The Hodge conjecture concerns Hodge classes, namely degree $2p$ rational cohomology classes which are of Hodge type (p,p), on projective complex varieties. It asks whether such classes are generated over $\mathbf{Q}$ by classes of algebraic subsets of codimension p.

For a Kähler compact manifold X, one has the notion of Hodge class, but there are known examples where the Hodge classes are not generated by classes of analytic subsets. The reason is that there is a more general construction of Hodge classes, which consists in defining the Chern classes of analytic coherent sheaves.

We show that still the last classes do not necessarily generate the set of Hodge classes in the Kähler case. We also show that the Chern classes of holomorphic vector bundles do not necessarily generate the same set of Hodge classes as those of coherent analytic sheaves. A consequence is that coherent analytic sheaves on compact Kähler manifolds do not necessarily admit a locally free resolution.

Modularity questions of Calabi-Yau varieties

Noriko Yui

The main theme of this talk is the modularity of Calabi–Yau varieties defined over number fields (e.g.,$\mathbf{Q}$) in dimensions $d = 1, 2$ and 3. Here by the modularity, we mean a Galois representation $\rho : Gal(\bar{\mathbf{Q}}/\mathbf{Q}) \to GL(H^d_{et}(\bar{X}, \mathbf{Q}_\ell)$ associated to a Calabi-Yau variety X of dimension d is modular. We define the L-series of a Calabi-Yau variety X over $\mathbf{Q}$ of dimension d by $L(X,s) = L(H^d_{et}(\bar{X}, \mathbf{Q}_\ell), s)$.

For dimension 1 Calabi-Yau varieties (namely, elliptic curves) over $\mathbf{Q}$, the modularity conjecture has been established by the celebrated efforts of Wiles et. al. *For any elliptic curve X over $\mathbf{Q}$, there exists a cusp form f of weight 2 on some $\Gamma_0(N)$ such that $L(X,s) = L(f,s)$.*

For dimension 2, the modularity has been known for extremal (=singular) $K3$ surfaces by Shioda and Inose, and Livné. *Let X be an extremal K3 surface over $\mathbf{Q}$, and assume that the 20 algebraic cycles generating the Neron-Severi group $NS(X)$ are all defined over $\mathbf{Q}$. Then $L(X,s) = \zeta(s-1)^{20} L(g,s)$ where $\zeta(s)$ is the Riemann zeta-function and g is a cusp form of weight 3 on some congruence subgroup of $PSL_2(\mathbf{Z})$.*

From here on, my talk is focused on the modularity question of Calabi-Yau threefolds defined over **Q**. We classify Calabi-Yau threefolds into two classes. A Calabi-Yau threefold X is *rigid* if $h^{2,1}(X) = 0$ (so $B_3(X) = 2$), *non-rigid* otherwise.

For rigid Calabi-Yau threefolds over **Q**, there is a well-formulated conjecture that *every rigid Calabi–Yau threefold X over* **Q** *is modular*, that is, *there exists a cusp form f of weight 4 on some $\Gamma_0(N)$ such that $L(X,s) = L(f,s)$. Here N is divisible only by primes of bad reduction.* This conjecture is a special case of the conjecture of Serre, and that of Fontaine and Mazur.

Theorem 15. *Up to date, there are at least 30 rigid Calabi-Yau threefolds over* **Q** *for which the modularity conjecture has been established. (Some of these 30 rigid Calabi-Yau threefolds may be birationally equivalent over* **Q**.*)*

The modularity question of non-rigid Calabi-Yau threefolds is also addressed. We consider non-rigid Calabi-Yau threefolds fibred by semi-stable K3 surfaces. The existence of these non-rigid Calabi-Yau threefolds is proved in the paper of Sun, Tan and Zuo entitled *Families of K3 surfaces over curves satisfying the equality of Arakelov-Yau type and modularity.* However, we are not able to give explicit formulation of the modularity conjecture nor its proof for these non-rigid Calabi-Yau threefolds. As Galois representations associated to these non-rigid Calabi-Yau threefolds are highly reducible, it is our hope that the modularity may be established sooner or later for these non-rigid Calabi-Yau threefolds.

Edited by Michael Lönne

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