Mathematisches Forschungsinstitut Oberwolfach

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# Mathematische Methoden der Geometrischen Datenverarbeitung

October 20th – October 26th, 2002

The conference was organized and directed by Carl de Boor (Madison), Helmut Pottmann (Wien), and Ulrich Reif (Darmstadt). Researchers from various fields of mathematics, including approximation theory, applied differential geometry, computer aided geometric design, and numerical analysis discussed recent advances in geometric computing. Central topics have been

- Partial differential equations for the solution of geometric optimization problems
- Multiscale methods
- Algebraic techniques in geometric design

It has been fascinating to see that communities (Computer Aided Geometric Design and Approximation Theory, Computational Differential Geometry, Partial Differential Equations), which so far did not have much interaction, are addressing closely related problems with partially different techniques or from different points of view. This has been a source for fruitful discussions and the initialization of new cooperations. The addressed applications go far beyond geometric design and include areas such as Image Processing, Computer Vision and Robotics.

Despite a larger number of volunteers, only a moderate number of talks had been scheduled in order to leave sufficient time for discussion and joint work during the conference. The participants acknowledged this opportunity and made extensive use of it. On behalf of all participants we would like to thank director Prof. Dr. G.-M. Greuel and his staff for their hospitality and friendly support, which helped greatly to make this conference a success.

# Abstracts

### A Clifford Algebra Approach to Pythagorean Hodograph Curves

H.I. Choi

(joint work with N.-S. Wee)

The Pythagorean hodograph (PH) curves are characterized by certain Pythagorean $n$-tuple identities that involve the derivatives of the curve coordinate functions in the polynomial ring. It was originally pioneered by Farouki and Sakkalis, and later extended and studied from many different perspectives by Pottmann, Peternell, Dietz, Hoschek, Jüttler, and many others.

Such curves have advantageous properties in computer aided geometric design, the main attraction being the rationality of many of the important geometric quantities. The state of affairs was that each different context in 2- or 3-dimensional Euclidean and Minkowski space gives rise to different combinations of polynomials, each of which again necessitates a different setup and methodology.

With D.S. Lee and H.P. Moon, utilizing the Clifford algebra formalism, we were able to unify the known incarnations of PH curves into a single coherent framework. More specifically, we were able to extend the spin representation to a map, which we called the PH representation map, defined on a suitable subspace of the Clifford algebra. (At a formal level, it is identical to the Kustaanheimo-Stiefel transform in physics.)

In this lecture, we will briefly outline our approach to the PH curves as a way of introduction, and then discuss about mathematical as well as practical aspects of this approach. Finally, we will present our recent result with Chang Yong Han on the Euler-Rodriques frame on the spatial PH curves, which will be useful in designing sweep surfaces.

### Adaptive finite element schemes – how good can a-posteriori information be?

Wolfgang Dahmen

(joint work with P. Binev and R. DeVore)

This talk is concerned with adaptive finite element schemes where successive local mesh refinements are based on a-posteriori error indicators. While a specific variant has only recently been shown by Morin, Nochetto and Siebert to converge at all, there have been no *convergence rates* relating the achieved target accuracy to the adaptively generated number of degrees of freedom and the corresponding computational work. It can be shown that a certain modification of the above scheme, involving an additional coarsening step, exhibits asymptotically optimal convergence rates comparable to those of best $N$-term approximation in the underlying finite element setting. Some ingredients of the analysis are outlined, namely adaptive tree approximation and bounding the complexity of certain refinement and completion strategies for triangulations based on newest vertex bisection.

## Numerical methods for geometric flow problems

Gerhard Dziuk
(joint work with K. Deckelnick and R. Rusu)

Evolutionary geometric differential operators appear to be crucial for the treatment of problems in mathematics and applications such as free boundary problems and image processing.

The main geometric flow problems are mean curvature flow

$$V = H,$$

and Willmore flow

$$V = -\Delta_\Gamma H + H(2K - \frac{1}{2}H^2).$$

Here $\Gamma$ is the moving surface with normal velocity $V$, mean curvature $H$, Gauss curvature $K$, and $\Delta_\Gamma$ denotes the Laplace Beltrami operator on $\Gamma$.

There are several mathematical models for the geometric object which moves under the respective geometric law of motion: parametric model, level set model and phase field model. Depending on the type of the chosen model, numerical methods have been developed. The main focus of the lecture is the parametric model, for which an efficient method based on a finite element method on surfaces. For mean curvature flow we also discuss the numerical approximation of viscosity solutions of the level set model.

This is joint work with K. Deckelnick (Magdeburg) and R. Rusu (Freiburg).

## Top–Down View–Dependent Terrain Triangulation using the Octagon Metric

Thomas Gerstner

In this talk, we introduce the octagon metric as a very useful distance metric for the interactive visualization of large–scale terrain data. Based on recursive bisection triangle meshes, this metric automatically ensures valid triangular meshes without cracks or T–junctions. We will show the octagon metric can be used for view–dependent refinement at little computational cost and with no additional storage requirements. It can easily be combined with a suitable geometric error metric to extract and render adaptive view–dependent terrain meshes in an output–sensitive way. We will show the performance of the whole system, which is straightforward to implement, in several examples.

## Two Multisided Bernstein/Bezier-Barycentric Schemes: S-Patches and Toric Bezier Patches

Ron Goldman

Three sided and four sided parametric patches are typically used for freeform design, but multisided patches are often required when it is necessary to fill an $n$-sided hole. Three sided and four sided Bezier patches are currently standard tools in approximation theory and computer aided geometric design. In this talk, we show how to extend the standard three sided and four sided Bezier constructions to rational $n$-sided patches.

To construct three sided and four sided Bezier patches, we need three sided and four sided arrays of control points and barycentric coordinate functions for the triangle and the rectangle. Similarly, to construct multisided Bezier patches, we require multisided arrays of control points and barycentric coordinate functions for multisided polygonal domains.

But what exactly are multisided arrays of control points and how precisely do we construct barycentric coordinates for multisided polygons? There is no single answer to either of these questions: different answers lead to different types of multisided Bezier schemes. Based on two different answers to these questions, we develop two distinct, but related, types of multisided Bezier patches: generalized S-patches, which extend the original S-patch construction of Loop and DeRose, and toric Bezier patches, which have recently been introduced by Krasauskas and have their origins in the theory of toric varieties from algebraic geometry. Three common threads tie these schemes together: discrete convolution, Minkowski sum, and a general version the de Casteljau pyramid algorithm. This talk focuses on basic properties – affine invariance, the convex hull property, interpolation of boundary curves – and fundamental algorithms – evaluation, differentiation, blossoming – for these multisided Bezier patches. We also compare and contrast the relative benefits and limitations of these two $n$-sided Bezier schemes.

### Optimization problems for curvatures

K. Grosse-Brauckmann

The goal of my talk was to present some real-world problems for interfaces governed by curvature properties, and indicate how they can be modeled by discrete surfaces.

The problems I explained are the interfaces of satellite fuel tanks, the intermaterial dividing surfaces of di-block copolymers, and amphiphilic monolayers.

To analyse such interfaces, I noted that surfaces are critical for area under a given volume constraint iff they have constant mean curvature $H = \frac{1}{2}(\kappa_1 + \kappa_2)$. Constant mean curvature can also arise for the following reason: The area element of a parallel surface at distance $t$ grows like $(1 + tH)dA_0$ in highest order. Hence wedge-shaped molecules which form a monolayer will have constant ("spontaneous") mean curvature. The same growth is observed for the line element in each principal curvature direction; hence for unequal principal curvatures the cross-section of the molecule in the monolayer undergoes some distortion. Such a distortion will cost energy, and so the energy functional for a monolayer usually contains $(\kappa_1 - \kappa_2)^2 = 4H^2 - 4K$, where $K = \kappa_1\kappa_2$ is Gauss curvature. Since the topology of the interfaces is usually not prescribed, we cannot use the Gauss-Bonnet theorem to ignore the contribution of $\int K$ to the energy. Thus we are lead to consider rather general energies like $\int a(H - c)^2 + bK$ (with $a, b, c$ constant).

The computation of constant mean curvature surfaces is usually straightforward and efficient using the gradient flow of area. However, to compute the critical points of the bending energy $\int H^2$ or other second order integrals is more complicated, and often leads to numerical instabilities.

## The Web-Method

K. Höllig
(joint work with U. Reif and J. Wipper)

The web-method is a new finite element technique which uses weighted extended B-splines (web-splines) as basis functions. It combines the computational advantages of uniform splines on regular grids standard triangular finite elements:

- No grid generation is required.
- Boundary conditions are represented exactly.
- Smoothness and order can be chosen arbitrarily.
- Highly accurate approximations are possible with relatively few parameters.
- Hierarchical bases permit adaptive refinement.
- Multigrid algorithms yield solution times proportional to the number of unknowns.

We discuss the basic features of the method, applications to typical boundary value problems, and, in more detail, the implementation of a multigrid scheme.

## Geometry of wavelets: local and global features

Palle Jorgensen

The setting is compactly supported orthogonal wavelets in one or several dimension. The multivariate case, is associated with a given expansive integral matrix. If the support is fixed, two cases are considered, a variety consisting of all the orthogonal wavelets, containing a lower dimensional set of singular points corresponding to wavelets which are only tight-frame wavelets, but not orthonormal. The points of non-uniqueness of the dominant eigenvalue of the transfer operator is identified as the set of singular points within the variety. An index theorem [1] is presented for the class of wavelets with Lipschitz filters, in which the winding number of a unitary matrix function identifies connected components. The wavelet variety carries a many-valued "Gauss map", a map which for scaling number $N = 2$ takes values in a finite set of points on the two-sphere. This map, together with spectrum of the transfer operator determines the wavelets , and their local properties. The spectrum refers to a space of Lipschitz functions. A wavelet subdivision algorithm is presented and illustrated. A quantum version of the wavelet algorithm is presented. Using spectral theory and subdivision, it is proved that the wavelets depend continuously on the masking coefficients: Specifically, if the reduced spectral radius is $< 1$, then the difference between two wavelets, measures in the $L^2(R)$-norm, is less than a constant times the difference between the wavelet filters measured in the Lipschitz-norm. The constant blows up when the reduced spectral radius goes to 1.

[1] O. Bratteli and P. Jorgensen, *Wavelets through a looking glass: The world of the spectrum.* Birkhauser/Springer 2002.

## Approximate algebraic methods for CAGD

BERT JÜTTLER
(joint work with P. Chalmovianský, A. Felis, J. Gahleitner, J. Schicho, and M. Shalaby)

So far, the implicit form of curves and surfaces has not found widespread use in CAGD. This is mainly due to the fact that the conversion processes between implicit and parametric form (implicitization and parameterization) have its problems. For instance, implicitization may produce large data volumes (e.g., a bicubic patch has algebraic order 18, and is described by a polynomial with 1330 coefficients), or unwanted branches of the curve may pass through the region of interest. On the other hand, many computational problems, such as surface–surface–intersection, get simpler if both representations are available.

Approximate algebraic methods may help to avoid the difficulties associated with the conversion processes. A method for approximate implicitization has been proposed by Dokken [1] in his PhD thesis. This talk describes two other methods for approximate implicitization. The first one is based on surface fitting by simultaneously approximating points (scattered data) and associated normals [2]. It can also be used for reconstructing surfaces from point clouds in reverse engineering. The second one uses a direct construction to generate a continuous or $C^1$ spline implicitization of a quadratic spline curve [3]. In addition we discuss variational design and approximate parameterization of algebraic curve segments.

This talk is based on joint work with P. Chalmovianský, A. Felis, J. Gahleitner, J. Schicho, and M. Shalaby. The financial support by the Austrian Science foundation (FWF) through project 15 of SFB F013 "Numerical and Symbolic Scientific Computing" is gratefully acknowledged.

[1] T. Dokken, *Approximate implicitization*, in: Mathematical Methods in CAGD (eds. T. Lyche, L. L. Schumaker), Vanderbilt University Press, Nashville & London 2001.

[2] B. Jüttler and A. Felis, *Least–squares fitting of algebraic spline surfaces*, Advances in Computational Mathematics **17** (2002), 135–152.

[3] B.Jüttler, J. Schicho and M. Shalaby, *Spline implicitization of planar curves*, submitted.

## Spline Quasi-interpolants

TOM LYCHE

Many applications of splines make use of some approximation method to produce a spline function from given discrete data. Popular methods include interpolation and least squares approximation. However, both of these methods require solution of a linear system of equations with as many unknowns as the dimension of the spline space, and are therefore not suitable for real-time processing of large streams of data. For this purpose local methods, which determine spline coefficients by using only local information, are more suitable. To ensure good approximation properties it is important that the methods reproduce polynomials and maybe preferably the functions in the given spline space. Classical methods of this kind were published in the seventies by de Boor and Fix using derivative information and extended by Schumaker and the author. In order to reproduce the spline space, the local information of the methods in the latter paper was restricted to lie in one knot interval. This restriction was removed in a recent paper by Lee, Mørken, and the author and a recipe for deriving local spline approximation methods which reproduce the whole spline space was given. The methods are obtained by solving a series of local approximation problems. Examples of specific cubic approximation methods will be given and this leads to interesting methods of possible practical use.

### Enclosing Curved Geometry and Inverse Problems
Jörg Peters

The presentation has four parts:

1. Basics of slefes (subdividable linear efficient function enclosures)
A *slefe* is an explicit two-sided approximation $f^+, f^-$ of a map $f$ so that $f^- \leq f \leq f^+$ over the domain of interest.
2. Sleves (subdividable linear efficient variety enclosures)
A *sleve* is a pair of linear approximations that sandwich the surface. In particular, we are interested in efficiently constructing two triangulations, so that matched triangle pairs enclose a piece of the curved surface. The width of the enclosure, i.e. the distance between inner and outer hull, can be easily measured, because it is taken on at a vertex. Enclosures are therefore approximate implicitizations with known error.
3. Midpaths and midpatches and duality with the curve
A *midpath* or *midpatch* is the average of a sleve. It yields a better surface approximation than sampling the surface.
4. Inverse Channel and Cover problems.
The *Channel problem for functions*: given two locally non–intersecting input polygons $\underline{c} < \overline{c}$, construct a spline function $\mathbf{b}$ that stays between $\underline{c}$ and $\overline{c}$ and consists of a small number of pieces.
The *Cover problem* is a 1-sided fitting problem. Both problems are solved by fitting a sleve into the channel and thus reducing a hard continuous optimization problem to a linear program.

Possible applications are conversion between representations, collision detection, root finding, best piecewise linear approximation for rendering, and fitting and layout.

### Spectral Theory for the Convergence of the Subdivision Operator
Amos Ron

The current theory for the convergence of the iteration of the stationary subdivision operator is based on the notion of joint spectral radius. Unfortunately there exists no viable understanding of this notion in analytic terms.

We provide instead a new approach, based on the notion of quasi-interpolation, that reduces the problem to

(1) the smoothness properties of the limit surface,
(2) the spectral properties of a finite-rank linear operator.

### Geometric evolutions problems in image and surface processing
Martin Rumpf

Morphological images processing and general surface processing are closely related topics. Thereby, the geometry of images is represented by the set of level sets. Methods based on partial differential equations turn out to be flexible and powerful tools in both areas. The talk outlines PDE based nonlinear filtering, subdivision and restauration methods. The noise reduction filter technique consist of a local classification and a geometric evolution problem steered by this classification. Based on image processing methodology and the theory of geometric evolution problems novel multi scale methods for surfaces, textured surfaces and 3D level sets are presented. The aim is the fairing of the noisy surfaces while preserving features such as edges and corners. In case of textured surfaces an appropriate coupling of the fairing processes for the surface geometry and the texture is presented. Here, one can especially take advantage of the frequently present strong correlations between edge features in the texture and on the surface edges. As an alternative approach a method based on crystalline curvature motion combined with a classification based on the zero moment of the surfaces is considered. Furthermore it is discussed how a certain class of subdivision methods can be understood as a cascadic multi grid method for a fully nonlinear time step of a geometric evolution problem. Finally, a method for the restauration of surfaces based on Willmore flow is presented.

## Sum rules, ideals and bases

THOMAS SAUER
(joint work with H. M. Möller)

A function $\phi : \mathbb{R}^s \to \mathbb{R}$ is called *refinable* provided there exists a finitely supported sequence $a$, called the *mask*, such that

$$\phi = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\, \phi\,(M \cdot - \alpha)\,,$$

where $M \in \mathbb{Z}^{s \times s}$ is an *expanding matrix*, that is, all its eigenvalues are $> 1$ in modulus. Instead of considering the function $\phi$ itself which is most often only given implicitly as the solution of the above functional equation, one tries to consider properties of the *stationary subdivision operator* $S_a$ defined for any sequence $c$ as

$$S_a c := \sum_{\alpha \in \mathbb{Z}^s} a\,(\cdot - M\alpha)\; c(\alpha).$$

A crucial condition for the function $\phi$ to provide a certain approximation order or to be smooth is (under suitable additional conditions, of course) that $S_a$ maps all polynomial sequences of a certain total degree to a polynomial sequence of (at most) the same total degree.

It is well known how to describe this property in terms of linear identities or in terms of zero conditions for the *symbol* $a^*(z) := \sum a(\alpha)\, z^\alpha$, which is a Laurent polynomial. The talk introduces another description, namely that $a^*$ is contained in the quotient ideal $\left\langle z^M - 1 \right\rangle : \langle z - 1 \rangle$ and explains how the theory of Gröbner- and in particular H–bases can be used to obtain differenced representations of the subdivision operator as well as minimally supported masks of a given order.

[1] H. M. Möller and T. Sauer, *Multivariate refinable functions of high approximation order via quotient ideals of Laurent polynomials*, Advances Comput. Math. (2002), to appear.

[2] T. Sauer, *Gröbner bases, H–bases and interpolation*, Trans. Amer. Math. Soc. **353** (2001), 2293–2308.

[3] ———, *Polynomial interpolation, ideals and approximation order of refinable functions*, Proc. Amer. Math. Soc. **130** (2002), 3335–3347.

## Geometric conditions on free boundaries

Reiner Schätzle

The Stefan problem with Gibbs Thomson law is a model for melting and solidification of materials. The Gibbs Thomson law determines the melting temperature on the free boundary between the liquid and solid phase in geometrical data of the free boundary namely the mean curvature.

Existence of solutions for the Stefan problem with Gibbs Thomson law was proved in [1] by Luckhaus 1991 using a time-discrete approximation and an absolute minimization process at each time step. From the thermodynamical point of view, this absolute minimization is difficult to justify.

In 2002 Matthias Röger, Phd-student of H.W. Alt and mine, succeeded in proving existence of solutions for the Stefan problem with Gibbs-Thomson law using only a local minimization process. The difficulty is that area may be lost when passing to the limit. The limit procedure relies on an identity recently established in [3] which writes the weak mean curvature in the context of geometric measure theory in terms of approximate differentials of the height function.

[1] S. Luckhaus, *The Stefan Problem with Gibbs Thomson law*, Sezione di Analisi Matematica e Probabilitita, Universita di Pisa, **2.75 (591)**, 1991.

[2] R. Schätzle, *Hypersurfaces with mean curvature given an ambient Sobolev function*, Journal of Differential Geometry, **58**, No. 3, 371-420, 2001.

[3] ——— ,*Quadratic tilt-excess decay and strong maximum principle for varifolds*, submitted, 2000.

## Subdivision for Modeling and Simulation

Peter Schröder

Subdivision surfaces are now solidly established as a major modeling primitive for free-form design. As it turns out they also have very favorable qualities when it comes to solving 4th order PDEs such as the thin-shell equations. The latter describe the behavior of thin flexible structures as they appear in all areas of engineering design. In this way subdivision surfaces are highly suited for integrated engineering design, removing the usual troubles associated with converting geometric representations to a form more suitable for finite element analysis.

My talk covers two subjects from this area. In a first part I will describe some recent developments in the construction of subdivision schemes based on repeated averaging. As it turns out, primal/dual mesh averaging operators are sufficient to build large families of classical as well as new subdivision schemes for a variety of possible topological split operators. Among them primal and dual quad schemes and more exotic dual $\sqrt{3}$ schemes (among many others). In the second part I will review some of the work on thin-shell modeling with the *Subdivision Element Method* and discuss strategies for the simple (in terms of data structures) construction of adaptive solvers.

## Approximate Duals of Bernstein polynomials and B-splines for the Construction of Tight Frames

Joachim Stöckler
(joint work with C. Chui, W. He, and K. Jetter)

The dual basis of the B-spline basis consists of functions of full support. We develop new relations for certain quasi-interpolants of B-splines that generalize the quasi-interpolants of Bernstein-Durrmeyer type for the Bernstein polynomials. These relations give rise to the definition of approximate duals of B-splines of order $m$, which are a linear combination of only $m-1$ B-splines and reproduce all polynomials of degree $m-1$. The approximate duals satisfy the same uniform stability estimate that was conjectured by C. de Boor and shown by A. Shadrin for the dual B-splines. They play an important role in the construction of tight frames of compactly supported splines on bounded intervals. Our explicit characterization of spline frames with arbitrary knots includes the cases of multiple non-uniform knots as well.

An extension of the Bernstein-Durrmeyer quasi-interpolants to the multivariate case is also mentioned in this talk. Parts of this work are joint work with C. Chui, W. He, and K. Jetter.

## Curvature Measures for Discrete Surfaces

John M. Sullivan

There is a well-known interpretation of Gaussian curvature for discrete (triangulated) surfaces. It is natural because it preserves the Gauss–Bonnet theorem, which equates the integral of Gaussian curvature to a boundary integral. Less familiar are analogous boundary integral relations for mean curvature, including the equation

$$\int_{\partial D} \eta \, ds = \int_{\partial D} \nu \times d\mathbf{x} = 2 \iint_D H\nu \, dA = 2 \iint_D \mathbf{H} \, dA,$$

which can be understood as a balance of physical forces.

We will show how to use such relations to guide the proper interpretation of mean curvature (and other geometric quantities) for discrete surfaces. This new understanding helps to explain the theory of discrete minimal surfaces. It also elucidates why, in early work on simulations of Willmore energy ($W = \int H^2 \, dA$), certain discretizations were better than others.

Similarly, for space curves, some quantities, like total curvature or even writhe, have natural interpretations for polygons. We examine further cases, like knot energies and ropelength, where a proper consideration of the geometry involved can lead to a natural discretization.

## Distance functions, active curves, and motion design

J. Wallner
(joint work with H. Pottmann)

We assume that a collection of feature points $x_1, \ldots, x_r$ in $\mathbb{R}^n$ depends affinely on a set of control points $b_1, \ldots, b_d$ — an example is provided by evaluating a B-spline curve $b(t)$ at prescribed parameter values $x_i = b(t_i)$ — and that a geometric object $T$ allows computing the distance $d(x, T)$ of a point $x$ from $T$. Then the so-called iterative closest point algorithm may be used to let the collection of feature points converge towards $T$. It works by iteratively finding control points which minimize a certain nonnegative quadratic function determined by $T$'s distance field. In the case that $T$ has smooth boundary, we present an extension of the ICP algorithm which uses refined quadratic approximants and appears to substantially increase the rate of convergence.

An especially important application of this method is the design of near-Euclidean affine spline motions. A one-parameter affine spline motion (i.e., a spline curve $b(t)$ in the affine space $\mathbb{R}^{n\times n+n}$ of affine transformations) is moved towards the Euclidean motion group $T$. An affine transformation's footpoint on $T$ is computed via an SVD-type decomposition of its linear part.

Near-Euclidean near-gliding motions may be designed by moving $b(t)$ towards the configuration manifold of surface-surface contact, which is, in general, a 1-codimensional submanifold of the Euclidean motion group.

## Self-tuning algorithms for surface fitting

Volker Weiss
(joint work with T. Varady)

Fitting parametric surfaces with tight tolerances is a crucial element in the process of reverse engineering complex shapes. The point cloud to be approximated typically contains a large number of noisy data points over an irregular domain. The created surfaces need to be fair and extendable, and must be generated by minimal amount of user assistance.

The majority of classical least-square surface fitting methods is based on fixed configurations of several mathematical entities, such as the knot-vectors, the parametric values associated with the data points, and a "well chosen" smoothness factor. These entities can hardly be set in advance and iterative methods are needed to find their optimum. Self-tuning surface fitting is based only on a tolerance value, everything else is computed by the algorithm itself.

Four related problems are discussed:

(1) Generating valid "B-spline-like" initial parameterization.
(2) Setting automatically smoothness weights.
(3) Handling "weak" control points.
(4) Inserting knots according to a shape dependent strategy.

*Edited by Georg Umlauf*

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