Mathematisches Forschungsinstitut Oberwolfach

Report No. 55/2002

Algorithmische Graphentheorie

December 8th – December 14th, 2002

The aim of this meeting was to bring together experts in various fields of Graph Theory and researchers who design algorithms that exploit graph structures. This merger of traditional mathematicians and theoretical computer scientists is becoming increasingly popular and important, as reflected by the growing number of young people entering the field and the calibre of the results being achieved. The broad algorithmic areas covered at the workshop were recognition, approximation and optimization with focus on graph colourings, cycle and path structure of graphs and closure concepts. Well prepared talks and a high attendance rate at the talks made it a fruitful meeting.

Two additional survey talks about very recent developments, the "Strong Perfect Graph Theorem" and "Efficient Primality Testing" were presented by Annegret Wagler and Juraj Hromkovic, respectively. In the two problem sessions a number of challenging problems were posed, generating lively discussion, often carrying on well into the night.

The meeting was organized by Derek Corneil (Toronto), Klaus Jansen (Kiel) and Ingo Schiermeyer (Freiberg); the 46 participants came from 12 countries, many of them being at Oberwolfach for the first time. The abstracts of the talks and the posed problems are presented below. The excellent working conditions at Oberwolfach and the inspiring atmosphere made this a very successful meeting for researchers in different fields of Algorithmic Graph Theory. Many new collaborations were formed.

Abstracts

Preprocessing for treewidth

Hans Bodlaender

Preprocessing is an important but often underestimated technique when solving problems on graphs or other combinatorial problems. In this talk, we look to simplification or divide and conquer strategies for preprocessing graphs when we want to compute their treewidth. Two techniques are considered: reduction and safe separators. Reductions locally rewrite the graphs using certain 'safe' rules, either taken from the work of Arnborg and Proskurowski on recognizing graphs of treewidth three, or generalizations of these rules. A safe separator allows to split the graph in smaller parts, such that the optimal treewidth can be obtained from the optimal treewidth of the parts. Experiments on instances obtained from probabilistic networks (used in some decision support systems) are reported.

Properties of expanding graphs

Stephan Brandt

(joint work with Hajo Broersma, Reinhard Diestel, Matthias Kriesell (undirected case) and Jørgen Bang-Jensen (directed case))

A lot of research was performed on the construction of expanding graphs and on the evaluation of their expansion properties. Their graph theoretical properties seem to have been rarely studied.

We investigate mainly those properties related to cycles in expanding graphs and digraphs using an expansion concept that is particularly suitable to measure large expansion. We show that a quadratic expansion function is sufficient to imply a hamiltonian cycle while we needed an exponential function to imply hamiltonicity in digraphs. In both cases probably a suitable linear function is sufficient. Moreover we show that linear expansion implies linear length cycles, as well as a 2-factor and a cycle factor for a suitable linear function. For a fairly general class of digraphs we prove a tight expansion bound for hamiltonicity. With the use of our results for undirected graphs we can, e.g., prove some graph theoretical properties of so-called Ramanujan graphs.

Graph Classes of Bounded and Unbounded Clique-Width

Andreas Brandstädt

Recently, the concept of clique-width of graphs attracted much attention since it extends the concept of treewidth of graphs and has similar consequences for the efficient solution of problems definable in Monadic Second Order Logic. It is known that cographs (i.e. P_4-free graphs) are exactly the graphs of clique-width at most two. We classify all graph classes defined by forbidden induced one-vertex extensions of the P_4 with respect to their clique-width. This improves and extends some recently published papers.

Lexicographic Breadth First Search (LBFS) recognition algorithms for various classes of graphs

Derek Corneil

LBFS was introduced in 1976 by Rose, Tarjan and Lueker in their seminal paper on the recognition of chordal graphs. Recently it has been shown that using LBFS, one can get linear time easily implementable algorithms for such problems as dominating pairs in AT-free graphs, colouring in various families of perfect graphs, distance approximation in various families of graphs and recognition algorithms (for interval and bipartite AT-free graphs). In this talk new simple recognition algorithms as described for unit interval graphs and cographs. It seems as though these algorithms are the simplest known.

Graph Detachments

Keith Edwards

A detachment of a graph G is a graph which is obtained from G by splitting some or all of its vertices into 2 or more subvertices. Any edges which are incident with an original vertex are shared out among its subvertices. We will consider the problem of deciding, for two given graphs G and H, whether or not H is a detachment of G, and we will describe the computational complexity of various cases of the problem, and state some open problems.

Sum Colouring Interval Graphs

Magnús M. Halldórsson

Given a vertex colouring with the natural numbers, its chromatic sum is the sum of the colours of the vertices. The sum colouring problem is to find a colouring of a given graph with a minimum chromatic sum. We present an algorithm that approximates the sum colouring problem in interval graphs within a factor of 2, improving on the best previous known factor of 2 due to Nicoloso, Sarrafzadeh and Song. Our algorithm applies to any class of graph for which the maximum induced k-colourable subgraph problem is polynomially solvable, including comparability and co-comparability graphs. The technique used can be seen as an extension of a randomized solution of an online number guessing problem. This is joint work with Guy Kortsarz and Hadas Shachnai.

Ramsey numbers relative to graph sequences

Heiko Harborth

The classical Ramsey number $r(G, H)$ asks for the smallest number n such that every 2-colouring of the edges of the complete graph K_n contains given subgraphs G or H of the first or second colour, respectively. Instead of the sequence of the complete graphs K_n we consider other sequences of graphs H_n as host graphs. Then $R(G, H)$ is the smallest n such that every 2-colouring of the edges of H_n contains subgraphs G or H of the first or second colour, respectively. First results are presented for the sequences of complete bipartite graphs, of cube graphs, of octahedron graphs, and of different types of gameboards.

Cyclic chromatic number of plane graphs

Mirko Horňák

The *cyclic chromatic number* of a graph G embedded in a surface, in symbols $\chi_c(G)$, is the smallest number of colours in such a vertex colouring of G that any two vertices sharing a common face receive different colours. Plummer and Toft conjectured in 1987 (PTC) that any 3-connected plane graph G satisfies $\chi_c(G) \leq \Delta^*(G) + 2$ where $\Delta^*(G)$ is the maximum face degree of G. They were able to prove that $\chi_c(G) \leq \Delta^*(G) + 9$. PTC has been proved if $\Delta^*(G) \leq 4$ or $\Delta^*(G) \geq 22$. The best presently known upper bound related to PTC is $\Delta^*(G) + 5$. To prove the result the Discharging Method together with an appropriate set of reducible configurations is used. To illustrate it a special configuration is shown to be reducible (it cannot appear in a minimal counterexample to the inequality $\chi_c(G) \leq \Delta^*(G) + 5$ for 3-connected plane graphs G).

Linear Time Approximation Algorithms for the Matching Problem

Stefan Hougardy

(joint work with D. Drake)

Currently the fastest algorithms for solving the maximum matching problem or the maximum weighted matching problem in graphs have running time $O(n^1/2m)$ respectively $O(nm + n^2 logn)$. For many applications such running times are not affordable. This motivates the study of more efficient approximation algorithms for these problems. We present linear time approximation algorithms with an approximation ratio of 3/4 in the unweighted case and 1/2 in the weighted case.

Stability of approximation algorithms

Juraj Hromkovic

The classical approaches of classifying the hardness of computing tasks are not completely satisfiable because of the definition of complexity as the worst case complexity. Thus, a few hard instances are sufficient to declare a problem to be hard even when the typical problem instances in applications are easily solvable. The concept of stability of approximation suggests to classify the problem instances with respect to their hardness and so to specify the border of practical solvability of optimization problems rather on the level of problem instances than on the level of problems.

Efficient primality testing

Juraj Hromkovic

The aim of this talk was to present the development of ideas for efficient primality testing starting from the method of abundance of witnesses for designing randomized algorithms and finishing with the use of witness concept to obtain a polynomial-time deterministic algorithm for primality testing.

Approximation algorithms for the fractional covering problem

Klaus Jansen

We generalize a method by Grigoriadis et al. to compute an approximate solution of the fractional covering (and max-min resource sharing) problem with M nonnegative linear (concave) constraints f_m on a convex set B to the case with general approximate block solvers (i.e. with only constant, logarithmic, or even worse approximation ratios). The algorithm is based on a Lagrangian decomposition which uses a modified logarithmic potential function and on several other ideas. We show that the algorithm runs in $O(M\epsilon^{-2}\ln(M\epsilon^{-1}))$ iterations (or block optimization steps) for any fixed relative accuracy $\epsilon \in (0,1)$. Furthermore, we show how to apply this method for the fractional weighted graph colouring problem.

Steiner problems for tournament-like digraphs

Joergen Bang-Jensen

(joint work with Gregory Gutin and Anders Yeo)

We consider the so-called directed Steiner problem. Here we are given a strongly connected digraph D and subset X of its vertices and the goal is to find a strong subdigraph which covers X and has a few arcs as possible. This problem is NP-hard for general digraphs as it generalizes the hamiltonian cycle problem. We describe polynomial algorithms for solving the problem in the case of digraphs that are either locally semicomplete or extended semicomplete. Finally we discuss the related problem of finding in a strong digraph with arbitrary real-valued costs on the vertices a strong subdigraph of minimum cost.

Circular chromatic numbers of certain planar graphs and small graphs

Arnfried Kemnitz

(joint work with Peter Wellmann)

A (k,d)-colouring ($k,d \in \mathbb{N}, k \geq 2d$) of a graph G is an assignment c of colours $\{0,1,\ldots,k-1\}$ to the vertices of G such that $d \leq |c(v_i)-c(v_j)| \leq k-d$ whenever two vertices v_i and v_j are adjacent. The circular chromatic number $\chi_c(G)$ (sometimes also called star chromatic number) is defined by $\chi_c(G) = \inf\{k/d : G \text{ has a } (k,d)\text{-colouring}\}$.

Since a $(k,1)$-colouring of G is a k-colouring of G, the circular chromatic number is a refinement of the chromatic number and therefore contains more information about the structure of the graph G.

We determine $\chi_c(G)$ for Platonic solid graphs, Archimedean solid graphs, Archimedean prism graphs, outerplanar graphs and for all graphs of order at most 7.

On $L(d,1)-$ labellings of graphs

Anja Kohl

Given a graph $G = (V, E)$ and nonnegative integers d and k, an $L(d,k)-$labelling of G is a function $f : V(G) \to \{0, 1, ...\}$ such that for any two vertices x and y

1. $\mid f(x) - f(y) \mid \geq d$ if $d(x,y) = 1$ and
2. $\mid f(x) - f(y) \mid \geq k$ if $d(x,y) = 2$.

The $L(d,k)-$number of G, denoted by $\lambda_{d,k}(G)$, is the smallest number m such that G has a $L(d,k)-$labelling with $max\{f(x) : x \in V(G)\} = m$.
We will present some known bounds for $\lambda_{d,1}(G)$ for general graphs, and some exact values of $\lambda_{d,1}(G)$ for special classes of graphs. Moreover, we will determine $\lambda_{d,1}(G)$ for the three regular tilings of the plane and for the rth power of paths and cycles.

On Generalizations of k-ordered hamiltonian graphs

Linda Lesniak

A graph G is k*-ordered hamiltonian* if for every sequence $v_1, v_2, ..., v_k$ of k vertices of G there is a hamiltonian cycle in G that encounters these vertices in this order. Two generalizations of k-ordered hamiltonian graphs are discussed.

On the chromatic index of linear hypergraphs

Marian Margraf

(joint work with Hauke Klein)

The celebrated Erdös, Faber and Lovász Conjecture may be stated as follows: Any linear hypergraph on v points has chromatic index at most v. First we show that the conjecture is equivalent to the following assumption: For any graph $\chi(G) \leq v(G)$, where $v(G)$ denotes the linear intersection number of G. Moreover, $|V| \leq v(G)+v(\overline{G})$ for any graph $G = (V, E)$. It follows that at least G or $\overline{G}$ fulfills the assumption.

Circular chromatic number of digraphs

Bojan Mohar

The notion of the circular chromatic number is a natural refinement of the usual chromatic number. A generalization of this concept to graphs with edge-weights has been introduced recently and there are natural links of this extension of the chromatic graph theory to several other, seemingly unrelated areas, e.g., the travelling salesman problem. An interesting special case is also a new notion of the chromatic number of a digraph whose main properties have been presented in some more depth.

AND/OR-Graphs

ROLF H. MÖHRING
(joint work with Martin Skutella und Frederik Stork)

Partial orders on a set V can be generalized by adding to the usual precedence constraints "$v \in V$ is above *all* elements from $W \subset V$" (AND constraint) disjunctive constraints of the form "v is above *at least one* element from W (OR constraint).

Similar to the representation of partial orders by acyclic directed graphs, these more general precedence constraints can be represented by so-called AND/OR networks, which need no longer be acyclic. Besides in scheduling, such networks have applications in games on graphs (mean payoff games), logic (Horn clauses) and AI (theorem proving).

This lecture will describe some of these applications and investigate basic algorithmic tasks on AND/OR networks. These include testing feasibility of a system of AND/OR conditions, computing the transitive closure and the transitive reduction, and – in the context of scheduling – computing earliest start times. While most of these tasks are shown to be solvable efficiently, we do not know a polynomial time algorithm for the general case of earliest start times, although the corresponding decision problem is in NP $\cap$ coNP.

Station Placement for Multi-hop Routing

MANUELA MONTANGERO
(joint work with Clemente Galdi and Christos Kaklamanis)

Consider the following multicast problem: we are given a population P and a bidirectional tree $T = (V, E)$ where vertices are network nodes and each user $u \in P$ resides at some leaf of the tree. Let $s \in V$ be a node (source) in T willing to broadcast a message (series of) to the users in P. Each node of the tree charges a known fee to duplicate messages, given by function $p : V \setminus P \to \mathcal{R}^+$; every edge of the tree has a known and fixed length $l : E \to \mathcal{R}^+$; every user has a maximum utility, given by function $u : P \to \mathcal{R}^+$, which represents how much it is willing to pay to receive the message form s. The cost of a broadcast is given by three factors: the sum of the lengths of the edges used by messages, where the length is counted once for every time an edge is used; the sum of the fees asked by internal nodes duplicating messages; the opposite of the sum of the utilities of the users reached by the broadcast. The aim is to minimize the cost of the broadcast.

Traditionally, there are mainly two forms of routing for broadcast: In the *unicast* routing each message sent from the source is delivered to a single destination. A message that has to be sent to different destinations is sent in separate copies to each destination, with the consequence that many identical messages traverse the links close to the source wasting bandwidth. In the *multicast* routing the source sends only one message per out-going edge. Whenever a message reaches an internal node it is duplicated and sent over each downstream link. In this way each link is traversed only by one message but every internal node in the tree duplicates messages, incurring in high cost.

We propose to use an intermediate solution, the *multi-hop routing* in which only some nodes on each source-destination path duplicate messages. Whenever a message is duplicated by an internal node, this is directly delivered to other duplicating nodes and/or to some destinations along a path, usually composed by more than one edge. In particular, we consider the case in which at most a fixed number, say k, of duplications per message can be done before the message reaches its destination. The problem is, now, to determine the best choice of nodes that have to duplicate messages.

We show centralized and distributed algorithms to efficiently find the optimal solution for the problem, under the hypothesis that a node duplicating a message becomes a new source for the subtree rooted in the node.

If we consider a general graph G instead of a tree T, we prove that the problem is NP-hard even if $k = 1$, length on edges are constant and equal to one and both p and u are the constant function zero. Under the same hypothesis for functions p and u, we give a centralized approximation algorithm, based on boundend depth Steiner trees, to find an approximate solution on general graphs that is $\log |P|$ far away from optimum, when k is constant.

An $O(2^{n/6.15})$-algorithm for Exact 3-Satisfiability using the Concept of Formula Graphs

BERT RANDERATH

(joint work with Ewald Speckenmeyer and Stefan Porschen)

Let $F = C_1 \wedge \cdots \wedge C_m$ be a Boolean formula in conjunctive normal form over a set V of n propositional variables, s.t. each clause C_i contains at most three literals l over V. Solving the problem exact 3-satisfiability ($X3SAT$) for F means to decide whether there is a truth assignment setting exactly one literal in each clause of F to true. Schaefer proved 1978 a dichotomy result on generalized satisfiability problems. On part of this result, classifying whether a given generalized satisfiability problem is in P or is NP-complete, is the statement that $X3SAT$ is NP-complete. By exploiting the concept of graph formulas and an accompanying perfect matching reduction we prove that $X3SAT$ is deterministically decidable in time $O(2^{0.18674n})$. Thereby we improve a result of Drori and Peleg stating $X3SAT \in O(2^{0.2072n})$ and a bound of $O(2^{0.200002n})$ for the corresponding enumeration problem $\#X3SAT$ stated in a preprint from Dahlöf and Jonson. After that by a more involved deterministic case analysis we are able to show that $X3SAT \in O(2^{n/6.15})$.

Contractible subgraphs, cycle properties in cubic graphs and hamiltonian properties of line graphs

ZDENĚK RYJÁČEK

The contractibility technique was developed recently as an extension of the well-known Catlin's reduction technique for hamiltonian properties of line graphs (it turns out that - roughly speaking - a graph F is contractible if and only if the circumference of $L(G)$ equals the circumference of $L(G|_F)$ for any graph G containing F). The technique yields a new powerful closure concept for line graphs and could be a potential tool for attacking some long-standing open problems, e.g. the dominating cycle conjecture (every essentially 4-edge-connected cubic graph G has a cycle C such that $G - C$ is edgeless).

Forbidden subgraphs and 3-colourability

INGO SCHIERMEYER
(joint work with Bert Randerath and Meike Tewes)

The 3-colourability problem is a well-known NP-complete problem. It remains NP-complete for triangle-free graphs of maximum degree 4 and for claw-free graphs.
Sumner has shown that triangle-free and P_5-free or triangle-free, P_6-free and C_6-free graphs are 3-colourable.
We present polynomial time algorithms to colour a (K_3, P_5)-free graph with three colours and a (K_3, P_6)-free graph with four colours. Furthermore we show that (after suitable reductions) every 4-chromatic (K_3, P_6)-free graph G contains the Mycielski-Grötzsch graph as an induced subgraph and is a subgraph of the Clebsch graph.

Using small dominating sets we show that 3-colourability can be decided and a corresponding 3-colouring can be determined in polynomial time for the class of P_6-free graphs.

3-colourability can be also decided and a corresponding 3-colouring can be determined in polynomial time for the class of claw-free and hourglass-free graphs $(K_{1,3}, K_1 + 2K_2)$ and claw-free and t-spider-free graphs (a $K_{1,t}$ with each edge subdivided).

Approximation Algorithms on Weighted Graphs with Sharpened Triangle Inequality

SEBASTIAN SEIBERT
(joint work with Hans-Joachim Bckenhauer, Dirk Bongartz, Juraj Hromkovič, Ralf Klasing, Guido Proietti, and Walter Unger)

We say that a weighted, complete undirected graph obeys the β-triangle inequality (Δ_β-inequality) if for the given $\beta \in \mathbf{R}^{\geq 1/2}$ the weight function c satisfies

$$c(u, v) \leq \beta \cdot (c(u, w) + c(w, v))$$

for all $u, v, w \in V$. In case $\beta > 1$, we speak of the relaxed, in case $\frac{1}{2} \leq \beta < 1$ of the sharpened Δ_β-inequality.

As a first problem, we look at the traveling salesman problem (TSP). In recent research, algorithms for the relaxed case have been developed, whose approximation ratio depends only on β, and which therefore are called stable. This extends continously from the well known $\frac{3}{2}$ ratio for the classical triangle inequality (case $\beta = 1$).

Here, we extend this research to the sharpened case. First, we show APX-hardness even if β gets arbitrarily close to $\frac{1}{2}$. This contrasts to another specialization of the Δ-inequality, the Euclidian case, where a PTAS exists.

Then we show how existing algorithms can be adopted for the sharpened Δ_β-inequality, and we develop a new one which performs better for $\beta < \frac{2}{3}$.

Other problems that suggest a similar investigation are: search for a minimal k-edge-connected, or 2-vertex-connected spanning subgraph (Min$k$ECSS, Min2VCSS), or augment a given subgraph into a spanning 2-edge-connected one at minimal additional costs (Min2ECA). The best known approximation ratios for these problems were 2 in general, and $\frac{3}{2}$ for Min2ECSS, Min2VCSS under Δ-inequality.

First, we transfer the lower bound from Δ_β-TSP to these problems. Then, we develop new approximation algorithms for these problems under sharpened Δ_β-inequality. The new algorithms give better approximation ratios (depending on β) at least for $\beta < \frac{2}{3}$, and in some cases beyond.

Complexity of the k-colour problem on a fixed surface

Carsten Thomassen

The following general problem is discussed. Fix a surface S and natural numbers k,q. Does there exist a polynomial time algorithm for deciding whether a graph of girth q embedded on S can be k-coloured. The problem is trivial for k less than 3 and NP-complete for k=q=3. For (k,q)=(3,4) or (4,3) we do not know if the problem is NP-complete, and we do not know a polynomial time algorithm except for the sphere and (when (k,q)=(3,4)) the projective plane. In all other cases a polynomial time algorithm is known. The most difficult cases (apart from the algorithm for 4-colouring a planar graph) are the case k=5, q=3 which was settled a few years ago, and the case k=3,q=5 which was settled recently.

Search problems with guaranteed solution

Zsolt Tuza

(joint work with C. Bazgan and M. Santha)

We study the approximability of search problems where the existence of a solution is guaranteed by some "structural" property. In particular, we consider

(1) Pigeonhole Subset Sums: Given a set of natural numbers $a_1, \ldots, a_n$ with $a_1 + \cdots + a_n < 2^n - 1$, find two subsets with the same sum.

(2) Second Hamiltonian Cycle: Given a 3-regular graph G and a Hamiltonian cycle $H \subset G$, find another Hamiltonian cycle of G.

We design a FPTAS finding two disjoint partial sums whose ratio is nearly 1, and an EPTAS finding a cycle $H' \subset G$, $H' \neq H$, whose length is nearly n (the number of vertices).

It remains an open problem to find exact solutions for (1) and (2) in polynomial time.

Partial list colourings of graphs

Margit Voigt

Let G be a graph with vertex set V, $|V| = n$, edge set E and chromatic number $\chi(G)$. Furthermore let $L(v)$ be a list of allowed colours assigned to each vertex $v \in V(G)$. The collection of all lists is called a list assignment and denoted by $\mathcal{L}$.

The graph G is called *$\mathcal{L}$-list colourable* if there is a colouring c of the vertices of G such that $c(v) \neq c(w)$ for all $vw \in E(G)$ and $c(v) \in L(v)$ for all $v \in V(G)$. Furthermore, G is *k-choosable* if it is $\mathcal{L}$-list colourable for every list assignment $\mathcal{L}$ with $|L(v)| = k$ for all $v \in V(G)$. The *list chromatic number* $\chi_\ell(G)$ is the smallest number k such that G is $k - choosable$.

Furthermore let $\lambda_\mathcal{L}$ be the maximum number of vertices of G which are colourable with respect to the list assignment $\mathcal{L}$. Define $\lambda_t := min \lambda_\mathcal{L}$ where the minimum is taken over all list assignments $\mathcal{L}$ with $|L(v)| = t$ for all $v \in V$.

Clearly, if $t \geq \chi_\ell(G)$ then $\lambda_t = n$. Thus it is interesting to ask about λ_t if $t < \chi_\ell(G)$.

Recent results and algorithmic aspects concerning this question are discussed in the talk.

Perfectness is an Elusive Graph Property

ANNEGRET WAGLER

A graph property is called elusive (or evasive) if every algorithm for testing this property has to read in the worst case $\binom{n}{2}$ entries of the adjacency matrix of the given graph. Several graph properties have been shown to be elusive, e.g. planarity (Best et al 1974), k-colourability (Bollobas 1978), 2-connectivity (Triesch 1982), or the membership in any minor closed family (Chakrabarti, Khot, Shi 2002) . A famous conjecture of Karp (1973) says that every non-trivial monotone graph property is elusive. We prove that a non-monotone but hereditary graph property is elusive: perfectness.

A framework for network reliability problems on graphs of bounded treewidth

THOMAS WOLLE

We consider problems related to the network reliability problem, restricted to graphs of bounded treewidth. We look at undirected simple graphs with each vertex and edge a number in $[0, 1]$ associated. These graphs model networks in which sites and links can fail, with a given probability, independently of whether other sites or links fail or not. The number in $[0, 1]$ associated to each element is the probability that this element does not fail. In addition, there are distinguished sets of vertices: a set S of servers, and a set L of clients.

We present a dynamic programming framework for graphs of bounded treewidth for computing for a large number of different properties Y whether Y holds for the graph formed by the nodes and edges that did not fail. For instance, it is shown that one can compute in linear time the probability that all clients are connected to at least one server, assuming the treewidth of the input graph is bounded. The classical S-terminal reliability problem can be solved in linear time as well using this framework. The method is applicable to a large number of related questions. Depending on the particular problem, the algorithm obtained by the method uses linear, polynomial, or exponential time.

On some special packings of trees

MARIUSZ WOŹNIAK

A packing of a tree $T = (V, E)$ of order n is a a permutation $\sigma : V \to V$ such that if $xy \in E$, then $\sigma(x)\sigma(y) \notin E$. It is known that all non-star trees are packable. Two examples of special situation were considered.

A) A cyclic permutation allows to get a packing of given non-star tree T such that the graph $T \oplus \sigma(T)$ is planar. This fact was proved by A.Garcia, C.Hernando, F.Hurtado, M.Noy and J.Tejel (*Packing Trees into Planar Graphs*, JGT **40** (2002), 172-181). The presented proof is easier and shorter than the original one.

B) Distinct length labelling (DLL) of a tree of size t in K_n (n-odd) is very useful tool related to some packing or decomposition problems. For instance, Ragonal graphs, Prepr. ser. - Univ. Ljubl. Inst. Math., 2001, vol. 39, no. 776.

Problems

Open problems involving sum multicolouring

Magnús M. Halldórsson

Given a graph $G = (V, E)$ and a length function $x : V \to \mathbf{N}$, a proper *multicolouring* is a function $\psi : V \to 2^{\mathbf{N}}$ assigning sets of colours to the vertices such that each vertex v receives $x(v)$ colours, $|\Psi(v)| = x(v)$, and adjacent vertices receive non-overlapping sets of colours, $\psi(v) \cap \psi(u) = \emptyset$ for $u, v \in V$. In the *non-preemptive* version, the colours assigned to a vertex must form a contiguous sequence, while in the *preemptive* version, any sequence of colours is valid. The sum multicolouring problem is to find a multicolouring of a given graph such that the sum $\sum_{v \in V} \max_{c \in \psi(v)} c$ of the largest colours assigned to each vertex is minimized.

(1) *Is there a polynomial algorithm for preemptive sum multicolouring of paths?* For weights up to nearly logarithmic, the problem is solvable in polynomial time even on trees, while in general there exists a polynomial time approximation scheme that applies also to partial k-trees [1]. Recently, Marx [3] showed that the problem is NP-hard for trees. His reduction holds also for polynomial lengths and binary trees. The problem of determining the solvability of paths has eluded considerable effort.

(2) *Is there a polynomial algorithm for non-preemptive sum multicolouring of outerplanar graphs?* A $O(n^2)$ time algorithm is known for this problem on trees and a fully polynomial-time approximation scheme for partial k-trees [2]. The question is whether there exists a more general class of graphs than trees for which the problem is polynomially solvable.

References

[1] M. M. Halldórsson and G. Kortsarz. Tools for Multicolouring with Applications to Planar Graphs and Partial k-Trees. *Journal of Algorithms*, **42**(2), 334-366, February 2002.

[2] M. M. Halldórsson, G. Kortsarz, A. Proskurowski, R. Salman, H. Shachnai, and J. A. Telle. Multi-Colouring Trees. In *Proceedings 5th Intl. Computing and Combinatorics Conf. (COCOON)*, Tokyo, Japan, LNCS Vol. 1627, Springer-Verlag, July 1999. To appear in *Information and Computation*.

[3] D. Marx. The Complexity of Tree Multicolourings. In Proc. 27th Intl. Symp. Math. Found. Comput. Sci. (MFCS), LNCS, 2002.

Forbidden subgraphs and 3-colourability

Bert Randerath and Ingo Schiermeyer

The 3-colourability problem is a well-known NP-complete problem. Using small dominating sets we have shown recently that 3-colourability can be decided and a corresponding 3-colouring can be determined in polynomial time for the class of P_6-free graphs (graphs containing no induced P_6).

Question: Is 3-colourability solvable in polynomial time for the class of P_7-free graphs? Does there exists $k \geq 7$ such that 3-colourability remains NP-complete for P_k-free graphs?

Weakly pancyclic graphs

Zdeněk Ryjáček

Let G be a finite simple undirected graph and let $g(G)$ and $c(G)$ be the girth and the circumference of G (i.e. the length of a shortest cycle of G and the length of a longest cycle of G), respectively. We say that G is *weakly pancyclic* if G contains cycles of all lengths ℓ for $g(G) \leq \ell \leq c(G)$. The graph G is *locally connected* if the neighborhood of every vertex of G induces a connected graph.

Conjecture: Every connected locally connected graph is weakly pancyclic.

Hexagonal graphs are induced subgraphs of the triangular lattice

Janez Žerovnik

Problem I: n-$[k]$colouring is an assignment of k subsets of $\{0, 1, \ldots, n-1\}$ to vertices

of G. It is known that every triangle-free hexagonal graph is 5-[2]colourable and there is a distributed algorithm for 5-[2]colouring [2]. It is also known that every triangle-free hexagonal graph is 7-[3]colourable [1]. *Question:* Is every triangle-free hexagonal graph is 9-[4]colourable? (The affirmative answer would imply the conjecture of Reed and McDiarmid [4].) *Subproblem:* find algorithmic solutions.

Problem II: A graph G is H-colourable, if there is a homomorphism from G to H. We know that every triangle-free hexagonal graph is C_5-colourable [9]. There are examples of triangle-free hexagonal graphs which are not C_9-colourable. (See Fig. 1.) *Question:* Is it true that every triangle-free hexagonal graph is C_7-colourable?

Problem III: Weighted hexagonal graph has a weighting function $d : V(G) \to N$ and $d(v)$ is refered to as the demand of V. The weighted clique number of G, $\tilde{\omega}(G)$, is just the maximum of the sums of vertex demands over all cliques of G. The weighted chromatic number of G, $\tilde{\chi}(G)$, is the minimum number of colours needed for an assignment such that each vertex is assigned $d(v)$ colours and the sets of colours assigned to adjacent vertices are disjoint. It is known [4, 6, 7, 8] that for weighted hexagonal graphs

$$\tilde{\omega}(G) \leq \tilde{\chi}(G) \leq \left\lfloor \frac{4\tilde{\omega}(G)+1}{3} \right\rfloor$$

McDiarmid and Reed conjectured that for triangle free hexagonal graphs $\tilde{\chi}(G) \leq \left\lfloor \frac{9\tilde{\omega}(G)}{8} \right\rfloor$. According to Havet $\tilde{\chi}(G) \leq \lceil \frac{7D}{3} \rceil = \lceil \frac{7\tilde{\omega}(G)}{6} \rceil$ for triangle free hexagonal graphs with uniform demand D [1]. Klostermeyer and Zhang have proved the following: for any $\varepsilon > 0$ there exists an integer M such that if there is no odd cycle of length $\leq M$ in G, $\tilde{\chi}(G) \leq (1+\varepsilon)\tilde{\omega}(G)$ [3]. Havet conjectures that $\tilde{\chi}(G) \leq \frac{(2p+1)\tilde{\omega}(G)}{2p}$ for triangular lattice graphs with no induced odd cycles of size less than $2p+1$. *Problem:* Prove or disprove the two conjectures. *Subproblem:* If true, find algorithmic solutions.

Problem IV: The hexagonal cells naturally arise from the optimal sphere (ball) packing on the plane. It may be interesting to consider the corresponding problem on the 3D cellular system. *Question:* What ratio $\tilde{\chi}(G)/\tilde{\omega}(G)$ can we obtain by generalization of the red-blue-green-(purple) algorithms? The problem may also be of practical interest, because when designing a network in the urban environment with very high buildings the 3-dimensional model is much more natural than the 2-dimensional model.

For additional references and related problems see [5].

References

[1] F.Havet, Channel Assignment and Multicolouring of the Induced Subgraphs of the Triangular Lattice, *Discrete Mathematics*, 233 (2001), 219-232.

[2] F.Havet and J. Žerovnik, Finding a five bicolouring of a triangle-free subgraph of the triangular lattice, *Discrete Mathematics* 244 (2002) 103-108.

[3] W.Klostermeyer and C.Q.Zhang, $(2+\epsilon)$ -Colouring of Planar Graphs with Large Odd-Girth, *Journal of graph theory*, 33 (2000), 109-119.

[4] C.McDiarmid and B.Reed, Channel Assignment and Weighted Colouring, *Networks Suppl.*, 36 (2000), 114-117.

[5] C.McDiarmid, Discrete Mathematics and Radio Channel Assignment, manuscript 2001 (45 pages).

[6] L.Narayanan and S.Shende, Static Frequency Assignment in Cellular Networks, *Algorithmica*, 29 (2001), 396-409.

[7] L.Narayanan and S.Shende, Corrigendum to Static Frequency Assignment in Cellular Networks, http://www.crab.rutgers.edu/shende/Algorithmica/errata/errata.html.

[8] P.Šparl, S.Ubeda and J. Žerovnik, A note on upper bounds for the span of the frequency planning in cellular networks, Prepr. ser. - Univ. Ljubl. Inst. Math., 2001, vol. 39, no. 738. (To appear in International journal of Pure and Applied Mathematics.)

[9] P.Šparl and J. Žerovnik, Homomorphisms of hexagonal graphs to odd cycles (submitted manuscript).

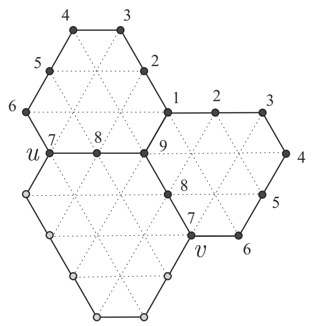

FIGURE 1. An example of a hexagonal graph, which is not C_9-colourable.

Edited by Marian Margraf

Participants

Prof. Dr. Joergen Bang-Jensen
jbj@imada.sdu.dk
Institut for Matematik og Datalogi
Syddansk Universitet
Campusvej 55
DK-5230 Odense M

Franziska Berger
berger@mathematik.tu-muenchen.de
Zentrum Mathematik
TU München
Boltzmannstr. 3
D–85747 Garching bei München

Dr. Hans Bodlaender
hansb@cs.uu.nl
Department of Computer Science
Utrecht University
P. O. Box 80. 089
Padualaan 14
NL-3508 TB Utrecht

Dr. Thomas Böhme
tboehme@theoinf.tu-ilmenau.de
Institut f. Mathematik
Technische Universität Ilmenau
Weimarer Str. 25
D–98693 Ilmenau

Prof. Dr. Andreas Brandstädt
ab@informatik.uni-rostock.de
Institut für Informatik
Universität Rostock
D–18051 Rostock

Dr. Stephan Brandt
sbrandt@mathematik.tu-ilmenau.de
Institut f. Mathematik
Technische Universität Ilmenau
Weimarer Str. 25
D–98693 Ilmenau

Prof. Dr. Derek G. Corneil
dgc@cs.utoronto.ca
Dept. of Computer Science
University of Toronto
10 King's College Road
Toronto Ontario M5S 3G4 - Canada

Dr. Keith Edwards
K.j.edwards@dundee.ac.uk
Department of Applied Computing
University of Dundee
GB-Dundee DD1 4HN

Prof. Dr. Ralph Faudree
Dept. of Mathematical Sciences
University of Memphis
Memphis, TN 38152 - USA

Dr. Gasper Fijavz
gasper.fijavz@fmf.uni-lj.si
FRI
University of Ljubljana
Trzaska 25
1000 Ljubljana - Slovenia

Olga Gerber
oge@informatik.uni-kiel.de
Institut für Informatik und
Praktische Mathematik
Christian-Albrechts-Universität
Olshausenstr. 40
D–24118 Kiel

Prof. Dr. Magnus M. Halldorsson
mmh@hi.is
Taeknigardur
University of Iceland
IS-107 Reykjavik

Prof. Dr. Heiko Harborth
h.harborth@tu-bs.de
Diskrete Mathematik
TU Braunschweig
Pockelsstr. 14
D–38106 Braunschweig

Prof. Dr. Chinh T. Hoang
choang@wlo.ca
Department of Physics and Computing
Wilfrid Laurier University
Waterloo ONT N2L 3C5 - Canada

Dr. Mirko Hornak
hornak@science.upjs.sk
P.J. Safarik University
Institute of Mathematics
041 54 Kosice - Slovakia

Dr. Stefan Hougardy
hougardy@informatik.hu-berlin.de
Institut für Informatik
HU-Berlin
Unter den Linden 6
D–10099 Berlin

Prof. Dr. Juray Hromkovic
Lehrstuhl für Informatik I
RWTH Aachen
Ahornstr. 55
D–52074 Aachen

Prof. Dr. Klaus Jansen
kj@informatik.uni-kiel.de
Institut für Informatik und
Praktische Mathematik
Universität Kiel
Olshausenstr. 40
D–4118 Kiel

Prof. Dr. Arnfried Kemnitz
a.kemnitz@tu-bs.de
Technische Universität Braunschweig
Diskrete Mathematik
D–38023 Braunschweig

Dipl. Math. Anja Kohl
Kohl@math.tu-freiberg.de
Fak. für Mathematik und Informatik
Bergakademie Freiberg
D–09596 Freiberg

Prof. Dr. Guy Kortsarz
guyk@crab.rutgers.edu
Rutgers University
Computer Science Department
640 Batholomew Road
Piscataway NJ 08854-8003 - USA

Prof. Dr. Dieter Kratsch
kratsch@lita.sciences.univ-metz.fr
Université de Metz
LITA
Ile du Saulcy
F-57045 Metz Cedex 01

Dr. Matthias Kriesell
kriesell@math.uni-hannover.de
Institut für Mathematik (A)
Universität Hannover
Welfengarten 1
D–30167 Hannover

Prof. Dr. Linda Lesniak
llesniak@drew.edu
Mathematics and Computer Science
Department
Drew University
Madison NJ 07940 - USA

Dr. Marian Margraf
mma@informatik.uni-kiel.de
Institut für Informatik und
Praktische Mathematik
Christian Albrechts Universität
Olshausenstr. 40
D–24098 Kiel

Prof. Dr. Rolf Möhring
moehring@math.tu-berlin.de
Institut für Mathematik - Fak. II
Technische Universität Berlin
Sekr. MA 6-1
Straße des 17. Juni 136
D–10623 Berlin

Prof. Dr. Bojan Mohar
bojan.mohar@uni-lj.si
FMF - Matematika
University of Ljubljana
Jadranska 19
1111 Ljubljana - Slovenia

Dr. Manuela Montangero
manuela.montangero@iit.cnr.it
Istituto di Informatica e
Telematica - CNR
Via Moruzzi 1
I-56124 Pisa

Dr. Haiko Müller
hm@comp.leeds.ac.uk
The School of Computing
University of Leeds
GB-Leeds LS2 9JT

Prof. Dr. Hartmut Noltemeier
noltemei@informatik.uni-wuerzburg.de
Institut für Informatik
Universität Würzburg
Am Hubland
D–97074 Würzburg

Dr. Bert Randerath
randerath@informatik.uni-koeln.de
Institut für Informatik
Universität zu Köln
Pohligstr. 1
D–50969 Köln

Dr. Dieter Rautenbach
rauten@math2.rwth-aachen.de
Lehrstuhl II für Mathematik
(für Ingenieure)
RWTH Aachen
Wüllnerstr. 5 - 7
D–52062 Aachen

Prof. Dr. Zdenek Ryjacek
ryjacek@kma.zcu.cz
Department of Mathematics
University of West Bohemia
30614 Plzen - Czech Republic

Dipl. Math. Marta Salvador
marta_leeds@yahoo.es
Technische Universität
Bergakademie Freiberg
Graduiertenkolleg Räumliche Statistik
D–09596 Freiberg

Prof. Dr. Ingo Schiermeyer
schierme@math.tu-freiberg.de
Fakultät für Mathematik und
Informatik; Technische Universität
Bergakademie Freiberg
Agricolastr. 1
D–09599 Freiberg

Dr. Sebastian Seibert
seibert@cs.rwth-aachen.de
Lehrstuhl für Informatik I
RWTH Aachen
Ahornstr. 55
D–52074 Aachen

Dr. Martin Sonntag
M.Sonntag@math.tu-freiberg.de
Fakultät für Mathematik und
Informatik; Technische Universität
Bergakademie Freiberg
Agricolastr. 1
D–09599 Freiberg

Dr. Hans-Martin Teichert
teichert@math.mu-Luebeck.de
Institut für Mathematik
Med. Universität zu Lübeck
Wallstr. 40
D–23560 Lübeck

Ralf Thöle
rth@informatik.uni-kiel.de
Institut für Informatik und
Praktische Mathematik
Christian Albrechts Universität
Olshausenstr. 40
D–24098 Kiel

Prof. Dr. Carsten Thomassen
c.thomassen@mat.dtu.dk
Matematisk Institut
Danmarks Tekniske Universitet
Bygning 303
DK-2800 Lyngby

Prof. Dr. Zsolt Tuza
Comp. and Automation Research Inst.
Hungarian Academy of Sciences
LORDS
P. O. Box 63
H-1518 Budapest

Dr. Margit Voigt
voigt@mathematik.tu-ilmenau.de
Institut f. Mathematik
Technische Universität Ilmenau
Weimarer Str. 25
D–98693 Ilmenau

Dr. Annegret Wagler
wagler@zib.de
Konrad-Zuse-Zentrum für
Informationstechnik Berlin (ZIB)
Takustr. 7
D–14195 Berlin

Thomas Wolle
thomasw@cs.uu.nl
Department of Computer Science
Utrecht University
P. O. Box 80. 089
Padualaan 14
NL-3508 TB Utrecht

Prof. Dr. Mariusz Wozniak
mwozniak@uci.agh.edu.pl
Department of Applied Mathematics
Technical University AGH
Al. Mickiewicza 30
30-059 Krakow - Poland

Prof. Dr. Janez Zerovnik
janez.zerovnik@uni-lj.si or
janez.zerovnik@uni-mb.si
University of Maribor
FS
Smetanova 17
2000 Maribor - Slovenien