Mathematisches Forschungsinstitut Oberwolfach

Report No. 5/2003

Miniworkshop: Dynamics of Nonlinear Waves

January 19th – January 25th, 2003

Introduction.

Nonlinear waves act as information carriers in models of systems arising in areas ranging from neurobiology and optical communications to fluids and smart materials. A fundamental issue in assessing the capacity of a model to support nonlinear wave motion is whether such waves are stable to perturbations in initial conditions. By the same token an understanding of the mechanisms causing instability can reveal blocks to information propagation and lead to the development of controls to facilitate the guiding of waves.

Although the study of wave stability is a well-studied subject in applied mathematics, the classical analysis has been largely restricted to very simple waves, such as constant states or plane waves. The area has gained renewed impetus through advances in analysis and dynamical systems that have afforded stability analyses of waves with non-trivial internal structure. This recent work has led to stability techniques for waves in complicated systems of partial differential equations, those with non-trivial spatial dependence and even higher-dimensional structure.

A key point in the philosophy underlying these advances is that the wave itself contains information that can reveal its stability character. That information may lie in the way the wave is constructed as an intersection of a stable and unstable manifold, the geometry of the ambient phase space or some aspect of a functional analytic approach to its construction. The mediation between this "hidden" information and the stability result is achieved by invoking a number of different mathematical techniques. The goal was to bring various researchers together for this mini-workshop who are experts in these different methods and develop a new agenda for taking these stability investigations to the next level.

Focus of the mini workshop.

Within the general field of wave dynamics there are a number of specific subjects at which various of the above sketched developments come together in such a way that important novel insights can be expected to occur soon. The mini workshop focused on these areas:

Wave interactions and stabilization. A large part of the theory of nonlinear waves is devoted to waves that are stationary in a certain co-moving frame. However, this covers only a fraction of the rich dynamics of nonlinear waves. Multi-pulse and multi-front patterns are in general not stationary within any frame, the various structures interact in general.

Pulses and fronts can repel or attract each other. An unstable pulse or front might stabilize by such an interaction into a stable structure. These interactions might even be the origin of the creation or annihilation of new pulses/fronts. Recent work has shown that various methods that have been developed in the context of solitary waves, such as centre manifold reductions, renormalization techniques, singular perturbation theory, can be applied and extended to cover wave interactions. Moreover, the methods and concepts of the theory of modulation equations are also valid in this context. However, these are only first steps in the direction of understanding these truly complex phenomena.

Systems in spatial dimension > 1. The stability theory for nonlinear waves has been developed mostly in the context of one-dimensional systems. In recent years, the Evans function method has taken a central position within the one-dimensional theory. A stability theory for waves in more than one spatial dimension can be built on the foundation of a higher dimensional Evans function. At the moment, various promising steps in that direction are in the early stages of development.

The interactions of continuous and point spectrum. The study of the destabilization of trivial, homogeneous, 'background' states with respect to continuous spectra has a long and rich history. The multiple phenomena associated with this type of bifurcation have been put in a rigorous mathematical framework in the last 10 years. Bifurcations of localized solutions with respect to discrete eigenvalues can be studied through centre manifold reductions by finite dimensional methods. A significant new challenge to the field is understanding more general situations in which nontrivial patterns are destabilized by either essential or point spectrum, or an interplay between the two. The existence and stability of modulated fronts and pulses is a crucial aspect of this analysis.

Abstracts

Symplectic structure of the Evans function for Hamiltonian PDEs and its implications

Tom Bridges

The aim of this talk was to give an informal introduction to the linear stability analysis of solitary wave solutions of Hamiltonian PDEs using the Evans function. The backbone of the analysis is the multi-symplectic representation of Hamiltonian PDEs. The talk began with an introduction to multisymplecticity. Then, the implications of this structure for the existence and linear stability problem was presented. The Evans function was shown to have a geometric representation, in terms of the determinant of a matrix of restricted symplectic forms. A geometric instability condition was presented, and the theory illustrated by application to a 2-parameter family of solitary waves of the Boussinesq equation. An introduction to the implications of multisymplecticity for two (and more) space dimensions was also given. An application of the 2+1-dimensional case is to the transverse instability of solitary waves. Other applications such as symplectic structure of soliton interaction (for example in the Zakharov-Kuznetsov equation), analyticity of the symplectic Evans function, the potential role of the Maslov index, extension to infinite dimensions of the symplectic Evans function, solitary waves of Hamiltonian PDEs on manifolds and other potential open problems were also briefly discussed.

Destabilization of fronts in a class of bi-stable systems

Arjen Doelman

We consider a class of bi-stable reaction-diffusion equations in two components on the real line. We assume that the system is singularly perturbed, i.e. that the ratio of the diffusion coefficients is (asymptotically) small. This class admits front solutions that are asymptotically close to the (stable) front solution of the 'trivial' scalar bi-stable limit system $u_t = u_{xx} + u(1-u^2)$. However, in the system these fronts can become unstable by varying parameters. This destabilization is either caused by the essential spectrum associated to the linearized stability problem, or by an eigenvalue that exists near the essential spectrum. We use the Evans function to study the various bifurcation mechanisms and establish an explicit connection between the character of the destabilization and the possible appearance of saddle-node bifurcations of heteroclinic orbits in the existence problem.

Dynamics of pulse-like localized patterns in R-D systems in 2D

Shin-Ichiro Ei

I propose a new type of Billiard problem in which angles of incidence and reflection are different. This problem comes from the dynamics of pulse-like localized patterns in R-D systems in 2D. In fact, a system of 3-components possesses a moving pulse- like localized solution which moves with constant velocity and constant shape. It is called "a travelling spot solution". The ODE which describes the motion of a travelling spot is given in mathematically rigorous way under some assumptions. Moreover, the case that there exist multi-travelling spot solutions in R^2 are also treated. As the consequence, they interact repulsively each other. Next, we consider the motion of one travelling spot in some region with Neumann boundary conditions. Then, a travelling spot reflect from the boundary.

In the reflection, the angle of incidence is larger than the angle of reflection, which gives a new type of Billiard problem. If the region is a square, there exists a stable limit cycle of orbit. On the other hand, if we consider a rectangle with the ratio L/M and change it, some chaotic orbit which almost fills the region appears. We also give a picture of the global structure of the orbits with respect to the ratio by numerics.

Stability of modulated fronts

THIERRY GALLAY

(joint work with Guido Schneider, Hannes Uecker)

When the steady states at infinity become unstable through a pattern forming bifurcation, a travelling wave may bifurcate into a modulated front which is time-periodic in a moving frame. This scenario has been studied by B. Sandstede and A. Scheel for a class of reaction-diffusion systems on the real line. Under general assumptions, they showed that the modulated fronts exist and are spectrally stable near the bifurcation point. I present here a model problem for which one can prove the nonlinear stability of these solutions with respect to small localized perturbations. This result does not follow from the spectral stability, because the linearized operator around the modulated front has essential spectrum up to the imaginary axis.

Mathematical modelling and analysis for radiation enhanced flame fronts in gaseous mixtures

JOOST HULSHOF

The classical model for subsonic flame fronts in diffusive gaseous mixtures under the assumption of simple chemistry is formulated as a free boundary problem for temperature and fuel concentration. The reaction takes place at the free boundary which is moving into the fresh zone leaving the burnt region behind. Mass flux going into the flame is balanced by heat flux coming out of the flame and is given by a temperature dependent reaction rate.

If the mixture contains dust, a radiation effect has to be taken into account which has a nonlocal effect on the temperature equation. Modelling the radiative flux by the Eddington equation of astrophysics, we find that the flame front temperature is enhanced to values well above the adiabatic temperature. The presence of dust makes the medium more flammable.

This model has been suggested by both Joulin and Buckmaster. Our goal is to obtain a complete bifurcation and stability picture of travelling waves in terms of the temperature and fuel concentration far away ahead of the flame, and the relevant parameters in the model. These are, after some normalisations, the Lewis number, the opacity, the Boltzmann number and the Zel'dovich number.

A Morse Index Theorem for Elliptic Problems

Chris Jones
(joint work with Jian Deng)

It has been a long standing issue to relate the structure of a solution of an elliptic boundary value problem with its degree of instability, i.e. its Morse Index. In one dimension, this is easily achieved using Sturm-Liouville theory which affords a relationship between the number of sign-changes of the derivative of the solution with its Morse Index. A generalization of this idea to systems of elliptic equations with a gradient nonlinearity is known and based on the idea that the linearization of the system generates a flow on the space of Lagrangian planes. This is closely related to the Morse Index Theorem which was originally formulated for geodesics on a Riemannian manifold. We prove a result of a similar structure but for solutions of elliptic problems in arbitrary space dimensions, as long as the underlying domain is start-shaped. The shrinking of the domain supplies the parameter that renders a curve of Lagrangian subspaces from the linearization. The key is to view these subspaces as lying in a Fredholm Lagrangian of infinite-dimensional subspaces and develop a Maslov Index theory. The result generalizes earlier work of Smale to now cover more general boundary conditions and explains why Smale's result only worked for Dirichlet conditions.

Semi-strong interaction of pulses

Tasso Kaper

Pulse-pulse interactions play central roles in a variety of pattern formation phenomena, including self-replication. In this talk, we present theory for the semi-strong interaction of pulses in a class of singularly perturbed, coupled reaction-diffusion equations that includes the (generalized) Gierer-Meinhardt, Gray-Scott, Schnakenberg, and Thomas models, among others. Geometric conditions are determined on the reaction kinetics for whether the pulses in a two-pulse solution attract or repel, and ordinary differential equations are derived for the time-dependent separation distance between their centers and for their wave speeds. In addition, conditions for the existence of stationary two-pulse solutions are identified, and the interactions between stationary and dynamically-evolving two-pulse solutions are studied. The theoretical results are illustrated on a series of examples. In two of these, which are related to the classical Gierer-Meinhardt equation, we find that the pulse amplitudes blow up in finite time. Moreover, the blowup of stationary one-pulse solutions and of dynamically-varying two-pulse solutions occurs precisely at the parameter values for which the theory we develop predicts that these solutions should cease to exist. Finally, for one of these examples, we discover a new type of codimension two point in which the bifurcation curve of pulse-splitting and the bifurcation curve of blowup intersect.

Evans' function for infinite dimensional systems

Shunsaku Nii

The aim of the talk is to illustrate the construction of Evans' function in infinite-dimensional setting.

The classical Evans' function was defined to be the determinant of solutions which decay in plus or minus infinity. Several way of defining Evans' function have been used which depend on some finite dimensionality of each infinite dimensional system.

In this talk, a new definition of Evans' function is introduced, which depends on Fredholm property of the system rather than infinite dimensionality. The function is defined to be an analytic section to the pull back of the determinant bundle over the space of Fredholm operators. The multiplicity lemma and winding number argument can also be shown in general framework.

Scattering dynamics in dissipative systems

Yasumasa Nishiura

Scattering of particle-like patterns in dissipative systems is studied, especially we focus on the issue how the input-output relation is controlled at a head-on collision in 1D and 2D spaces where travelling pulses and spots collapse. It remains an open problem due to the large deformation of patterns at a colliding point. We found that special type of steady or time-periodic solutions called separators and their stable and unstable manifolds direct the traffic flow of orbits. Such separators are clearly visible at a transition point where the input-output relation is qualitatively changed such as from annihilation to repulsion as parameters vary. Separators are in general highly unstable which causes a variety of input-output relations during the process of scattering.

Coherent Manifolds for Damped Hyperbolic Systems

Keith Promislow

The interaction of light with matter is dominated by diffraction and dispersion, absorption and stimulated emission all combining to produce a noisy background of radiation. Against this background, certain localized structures display a remarkable coherence through a robust balancing of nonlinearity against forcing and dispersion. The emerging technology of modern optics seeks to control these coherent structures for purposes of data propagation, pattern generation, and optical logic.

¿From these coherent localized structures we iteratively build up a coherent manifold: a low-dimensional, invariant manifold which captures the salient dynamics of pulse interaction, bifurcation of fixed points and travelling waves, oscillatory instabilities and modulation of multi-dimensional fronts.

The manifold is a sum of a base Φ_0 describing a collection of localized coherent structures, and two small extensions,

$$\Phi(x;\vec{p}) = \Phi_0 + \Phi_1 + \Phi_2, \tag{1}$$

where Φ_1 lies in a finite dimensional subspace and serves to extend the dimension of the tangent plane of the manifold, and Φ_2, the radiative extension, lies in the large, complementary subspace and is chosen to enforce the invariance or quasi-invariance of the manifold, capturing radiative effects arising from the interplay between forcing and damped dispersion. The manifold coordinates $\vec{p}$ represent slow modes of the system and are typically identified with localized eigenmodes of the linearization $L_{\vec{p}}$ of the vector field about $\Phi_0(\vec{p})$. These eigenmodes arise, for example, from symmetries or self-similar scalings of the PDE, and are the "free" coordinates: position, phase, polarization, which describe the state of the coherent structures.

The construction is based upon

(1) Detailed analysis of families of linearized operators by
Evans function and non-perturbative Krein-signature methods

(2) Development of decay estimates for time-dependent semi-groups
via renormalization group methods.

As a pro-typical optical system we consider the optical parametric oscillator (OPO) which has attracted recent attention as a source of broadly-tunable coherent radiation for use in pattern recognition technologies, optical information processing, and infrared spectroscopy. OPOs exploit the quadratic $\chi^{(2)}$-nonlinearity of a non-centrosymmetric crystal such as lithium niobate to convert energy parametrically from one frequency to another. The development of quasi-phase matching (QPM) through periodic poling of the crystal's molecular structure has has greatly increased the viability and simplified the control of these devices while extending their operational range and possible applications. In a continuous pumping limit, the OPO process converts a coherent pump field V into a frequency shifted signal field U, governed by the equations in $1+2$ dimensions,

$$\begin{aligned} U_t &= \tfrac{i}{2}\nabla^2 U + VU^* - (1+i\Delta_1)U, \\ V_t &= \tfrac{i}{2}\rho\nabla^2 V - U^2 - (\alpha + i\Delta_2)V + S, \end{aligned} \tag{2}$$

where Δ_1 and Δ_2 are the cavity detuning parameters and S is a spatially dependent pump strength.

We examine pulse-pulse interactions for the OPO system, and describe a snaking-bifurcation which results in multi-pulses. In the second talk we extend the method to consider oscillatory pulses and the generation of stable periodic solutions and address pattern formation in the context of multi-dimensional fronts.

Mathematical justification of a nonlinear integro-differential equation for the propagation of spherical flames

Jean-Michel Roquejoffre

We study the singular limit of a reaction-diffusion system known, in the engineering literature, as the thermo-diffusive model for flame propagation with one-step chemistry. The unknowns are the temperature and the mass fraction of the reactant. There is a diffusivity ratio - the Lewis number, denoted by Le - which is usually taken equal to unity in the mathematical literature on this system. The parameter that makes the system singular is the inverse of the high activation energy ϵ, which makes the reaction term converge - at least formally - to a measure on the surface where the temperature reaches its maximum value as $\epsilon \to 0$.

In the present context, 3D spherical symmetry is assumed, and the surface of maximal temperature is a sphere whose radius (flame radius) has an evolution law to be determined. We prove, under the assumption $Le < 1$, the existence of a class of initial data such that, for the solution starting from these data:

- the correct time scale is ϵ^{-2},
- the the flame radius, after a rescaling in time, converges as $\epsilon \to 0$ to the solution of a nonlinear integro-differential equation.

Such an equation had been derived by G. Joulin in 1985, but only in the formal style (matched asymptotic expansions). The present paper gives a completely rigorous justification on Joulin's analysis; the method of proof is basically in two steps.

- The construction of an approximate solution to sufficiently high order in ϵ.
- Checking that the true solution is close enough to the approximate one; this involves sharp resolvent and semigroup estimates.

An important open problem is the study of the system on times scales $>> \epsilon$.

Stability of large Ekman boundary layers in rotating fluids

Frederic Rousset

We study the limit of the Navier-Stokes equation for an incompressible three dimensional rotating fluid when the Ekman and Rossby number go to zero. Near horizontal boundaries an Ekman layer appears and its stability can be characterized with a Reynolds number. For large Reynolds numbers, the Ekman layer is known to be linearly unstable whereas for very small Reynolds numbers, the stability can be proved by classical energy estimates. Here, we study the stability for intermediate Reynolds numbers by means of Evans function/Symmetrizer's construction techniques.

Defects in oscillatory media — towards a classification

Björn Sandstede

(joint work with Arnd Scheel)

Among the many patterns that have been observed in experiments and numerical simulations are defects. Defects are modulated waves that are time periodic in an appropriate moving frame and asymptotic in space to wave trains. In this talk, we show that spatial dynamics can be used to analyse defects and their spectral stability properties. It appears as if there are four different types of defects that are robust: sinks, sources, transmission and contact defects. The different defect classes are distinguished by their codimension and the characteristic curves associated with them. Spatial dynamics also allows us to determine the PDE spectrum of defects near the origin in L^2 and exponentially weighted L^2 norms. The analysis of the spectrum of contact defects is particularly involved and requires an extension of the Evans function in situations where the Gap Lemma fails. Open problems include a complete classification of defects, the nonlinear stability of sources, transmission and contact defects, bifurcations from and of defects, and finally their interaction properties.

Extending the Evans function beyond the Gap Lemma

Arnd Scheel

(joint work with Björn Sandstede)

Contact defects arise in open classes of reaction diffusion systems. Like other types of defects they are time-periodic in a suitably comoving frame and spatially asymptotic to nonlinear wave trains. Their characteristic feature is that the wavenumbers to the left and to the right of the defect coincide, and that the speed of propagation of the defect is given by the group velocity of the wave trains in the far field. We are interested in stability, interaction, and bifurcation properties of these types of defects. We therefore investigate spectral properties of the linearization at the defect, in a neighbourhood of the origin, $\lambda = 0$. The fact that the speed of propagation is given by the group velocity leads to two difficulties. First, the convergence of the defect to the asymptotic wave trains is only algebraic, $\sim 1/x$, and then $\lambda = 0$ is a branch point in the essential spectrum of the defect. In order to characterize spectral properties, we construct an Evans function for the ill-posed modulated wave equation, using first Galerkin approximations globally in λ, and then centre-manifold and Lyapunov-Schmidt reduction, locally near $\lambda = 0$. For the actual construction of the Evans function, we use geometric blow-up and resonant normal forms for the differential equation on the reduced Grassmannian. As a main result, we show that the Evans function E can be written as $E(\lambda) = \sqrt{\lambda} E_0(\sqrt{\lambda}, \sqrt{\lambda}\log\lambda)$, where $E_0(\cdot,\cdot)$ is C^∞. Moreover, $\partial_2 E_0(0,0) \neq 0$ whenever the decay towards the wavetrains is of the form $1/x + a/x^2 + \dots$ with $a \neq 0$. We also show that a similar construction can be employed to continue the Evans function beyond the Gap Lemma as a meromorphic function. Resonances between the exponential decay of the wave profile and the gap between the leading eigenvalues generate poles in the Evans function.

Hopf bifurcation and exchange of stability in diffusive media

Guido Schneider

(joint work with Thomas Brand, Markus Kunze, Thorsten Seelbach)

We consider solutions bifurcating from a spatially homogeneous equilibrium under the assumption that the associated linearization possesses continuous spectrum up to the imaginary axis, for all values of the bifurcation parameter, and that a pair of imaginary eigenvalues crosses the imaginary axis. For a reaction-diffusion-convection system we investigate the nonlinear stability of the trivial solution with respect to spatially localized perturbations, prove the occurrence of a Hopf bifurcation and the nonlinear stability of the bifurcating time-periodic solutions, again with respect to spatially localized perturbations.

The small viscosity limit for multiD shocks

Mark Williams

Starting with a curved multiD shock solution to a hyperbolic system of conservation laws, we show how to obtain it as a limit as viscosity goes to zero of smooth solutions to an associated parabolic (hyperbolic + viscosity) problem. We first construct the viscous boundary layer on each side of the shock to high order (i.e., an approximate solution). The leading term in the boundary layer expansion is given by a family of travelling waves connecting endstates given by the original shock, and the problem reduces to proving nonlinear stability of this family. This is accomplished by proving L^2 estimates for the parabolic problem linearized about the approximate solution. A key step is to remove x/ϵ dependence from the coefficients of this problem by an argument that combines the Gap Lemma with pseudodifferential operators. One is then in a position to prove linear estimates using Kreiss-type symmetrizers, and these estimates suffice to yield nonlinear stability.

In a final discussion the participants found the following **big issues** for future research:

1) **Interaction of pulses**

- Weak and strong interactions
  - Semi-strong interactions

   - Separators and self-replication
   - Compounding unstable effect
 - Conservative or nearly conservative cases?
 - Unstable invariant sets as organizers of dynamics
 - Role of heteroclinic orbits: why so clean?
   - Clarify initial conditions in terms of separators globally; relation to weak interaction limit (in context of simplified model)
 - Interacting pulses as Free Boundary Value problem

2) **Higher dimensional structures**
 - Dynamical systems techniques to understand transitions of multi-d shocks
   - Break up of smooth shock surfaces
   - mechanics of formation of shocks in multi-d
 - Small viscosity limit in multi-d Navier-Stokes shocks, i.e. handle degenerate viscosity
 - When does Evans hypothesis hold? Strong shocks particularly
 - Spherically symmetric advancing fronts
   - When does invasion occur?
 - Perturbations of fronts
   - Role of micro-local analysis
 - Evans function hypothesis =¿ stability?
 - Construction of higher-d Evans function
   - Calculate it!
   - Continuation
 - Identify problems where Evans hypothesis is sharp

3) **Essential spectrum**
 - Beyond the gap lemma
   - Example where it is important
   - Bifurcations, e.g. contact defect -¿ transmission defect
 - Use of Evans function to compute decay rates of Greens function in cases of embedded/non-embedded eigenvalues
 - Edge bifurcations: when and where do they occur?
 - Numerical computation of Evans function
   - Wedgies vs non-wedgies
 - Interactions between point and essential spectrum
   - Bifurcation and stability (when exponential weights don't work)
 - Selection mechanisms for fronts
 - Multi-d defects
 - Interactions of spirals/defects/non-localized patterns
 - Validity of model for strong defects
 - Vanishing viscosity for defect problems?

Edited by Guido Schneider

Participants

Prof. Dr. Thomas J. Bridges
t.bridges@surrey.ac.uk
Department of Mathematics and Statistics
University of Surrey
Guildford
GB-Surrey GU2 7XH

Dr. Arjen Doelman
doelman@science.uva.nl
Korteweg-de Vries Instituut
Faculteit WINS
Universiteit van Amsterdam
Plantage Muidergracht 24
NL-1018 TV Amsterdam

Prof. Dr. Shin-Ichiro Ei
ei2s@yokohama-cu.ac.jp
Graduate School of Integrated Science
Yokohama City University
22-2 Seto, Kanzawa-ku
Yokohama 236-0027 - Japan

Dr. Thierry Gallay
thierry.gallay@ujf-grenoble.fr
Institut Fourier
Universite de Grenoble I
BP 74
F-38402 Saint-Martin d'Heres

Dr. Joost Hulshof
jhulshof@cs.vu.nl
Mathematisch Instituut
Vrije Universiteit
De Boelelaan 1081 a
NL-1081 HV Amsterdam

Prof. Dr. Christopher Jones
ckrtj@cfm.brown.edu
ckrtj@amath.unc.edu
Dept. of Mathematics
University of North Carolina
Phillips Hall
Chapel Hill, NC 27599-3250 - USA

Prof. Dr. Tasso Kaper
tasso@math.bu.edu
Dept. of Mathematics
Boston University
111 Cummington Street
Boston, MA 02215-2411 - USA

Prof. Dr. Shunsaku Nii
snii@math.kyushu-u.ac.jp
Faculty of Mathematics
Kyushu University
Ropponmatsu 4-2-1
Chuo-ku
Fukuoka 810-8560 - Japan

Prof. Dr. Yasumasa Nishiura
nishiura@aurora.es.hokudai.ac.jp
Research Inst. f. Elect. Science
Hokkaido University
Sapporo 060-0812 - Japan

Dr. Keith Promislow
kpromisl@sfu.ca
Dept. of Mathematics
Simon Fraser University
Burnaby, B.C. V5A 1S6 - Canada

Prof. Dr. Jean-Michel Roquejoffre
roque@mip-ups-tlse.fr
Mathématiques
Université Paul Sabatier
118, route de Narbonne
F-31062 Toulouse Cedex 4

Prof. Dr. Frederic Rousset
Frederic.Rousset@math.unice.fr
Laboratoire J.A. Dieudonne
Universite de Nice
Sophia Antipolis
Parc Valrose
F-06108 Nice Cedex 2

Dr. Björn Sandstede
sandsted@math.ohio-state.edu
Department of Mathematics
Ohio State University
231 West 18th Avenue
Columbus, OH 43210-1174 - USA

Prof. Dr. Guido Schneider
guido.schneider@mathematik.uni-karlsruhe.de
Mathematisches Institut I
Universität Karlsruhe
Englerstr. 2
D–76128 Karlsruhe

Prof. Dr. Arnd Scheel
scheel@math.umn.edu
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.
Minneapolis, MN 55455 - USA

Prof. Dr. Mark Williams
williams@email.unc.edu
Dept. of Mathematics
University of North Carolina
Phillips Hall
Chapel Hill, NC 27599-3250 - USA