Mathematisches Forschungsinstitut Oberwolfach

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Reelle Methoden der Komplexen Analysis

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Introduction

The conference was organized by K. Diederich (Wuppertal), T. Ohsawa (Nagoya), and E. L. Stout (Seattle). It was dedicated to Complex Analysis with emphasis on methods from the theory of partial differential equations. It has found large interest and was attended by 38 researchers from China, France, Germany, Japan, Slovenia, Poland, Russia, Sweden,and the United States of America. In 15 morning sessions of 50 minutes and 11 afternoon sessions of 40 minutes they reported on their recent results, giving an important impression of the progress that has been made in the different areas of research and proving that complex analysis is a lively field worldwide.

In the lectures topics from the following areas were discussed:

- Automorphism groups of complex manifolds
- Complex analysis in Banach spaces
- Existence and non-existence of Levi flat hypersurfaces in symmetric spaces
- Bergman theory
- Hartogs phenomena
- The Gromov-Eliashberg theory of Stein manifolds
- CR-geometry and CR mappings
- Analytic continuation of holomorphic and CR mappings
- Integral kernels for the Cauchy-Riemann operator
- The Oka principle
- Subellipticity and compactness in the $\bar{\partial}$-Neumann problem
- Complex dynamics
- Calabi-Yau surfaces and Kähler - Einstein manifolds
- Geometry of weakly pseudoconvex domains of finite type

The abstracts of the talks follow below in alphabetical order.

Abstracts

The Bergman metric and the pluricomplex Green function

Zbigniew Błocki

We start with the following result:

Theorem 1: *If* Ω *is a bounded pseudoconvex domain in* $\mathbb{C}^n$, *and* $w, \widetilde{w} \in \Omega$ *are such that* $\{g_{\Omega,w} < -1\} \cap \{g_{\Omega,\widetilde{w}} < -1\} = \emptyset$, *then* $\mathrm{dist}_\Omega(w, \widetilde{w}) \geq c_n > 0$.

Here, $g_{\Omega,w}$ *is the pluricomplex Green function with pole at* w *and* dist_Ω *denotes the distance w.r.t. the Bergman metric of* Ω.

Improving methods from a paper of Herbort we obtain the following estimate:

Theorem 2: *If* Ω *is a bounded pseudoconvex domain in* $\mathbb{C}^n$ *with* C^2*-boundary, then there exist constants* $a, C > 0$, *such that, for* $w \in \Omega$ *sufficiently close to* $\partial\Omega$ *we have*

$$\{g_{\Omega,w} < -1\} \subset \{\frac{1}{C} r |\log r|^{-1/a} \leq \delta_\Omega \leq C r\, |\log r|^{n/a}\},$$

where $r = \delta_\Omega(w)$, *(the euclidean distance from* w *to* $\partial\Omega$*).*

If $\partial\Omega \in C^2$, *then, combining theorems 1 and 2, we can improve an estimate due to Diederich and Ohsawa as follows:*

Theorem 3: If Ω *is as in theorem 2, then there exists a constant* $C > 0$ *such that, for a fixed* $z_0 \in \Omega$ *and* $z \in \Omega$ *sufficiently close to* $\partial\Omega$, *we have*

$$\mathrm{dist}_\Omega(z, z_0) \geq \frac{\log \frac{1}{\delta_\Omega(z)}}{C \log \log \frac{1}{\delta_\Omega(z)}}$$

Nonexistence of higher codimensional Levi-flat CR manifolds in compact symmetric spaces

Judith Brinkschulte

An old problem in complex dynamics asks about the existence of minimal sets of holomorphic foliations on $\mathbb{CP}^n$. The problem is closely related to the existence of Levi-flat CR manifolds in complex projective spaces. Siu proved recently that in $\mathbb{CP}^n$, $(n \geq 2)$, there exist no smooth Levi-flat real hypersurfaces.

We discuss the generalization of this result to higher codimensional CR manifolds. We have presented the following theorem:

Theorem: *Let* X *be an irreducible compact Hermitian symmetric space of complex dimension* n *whose bisectional curvature is* $(s-1)$*-nondegenerate. Then in* X *there exists no smooth Levi-flat CR manifold* M *of real codimension* $n-s$ *and complex dimension* $s \geq 2$, *such that* $\det(N^{1,0}_{M,X})$ *is smoothly trivial.*

Adjoint line bundle convexity and lower bound estimates on holomorphic sections

Bo - Yong Chen

We give differential geometric conditions that imply line bundle convexity of certain complex manifolds and also give a lower bound estimate for the Bergman kernel function of the line bundles.

We sketched proof of the following results:

Theorem 1: *Let (X, g) be a complete Kähler manifold such that*

$$-a^2 \leq \text{sectional curvature } \leq b^2$$

and its injectivity radius is bounded from below by some positive constant τ. Let (L, h) be a positive line bundle such that $\Theta(h) \geq c\, g$. Then X is holomorphically convex with respect to $(K_X + mL, g^* h^m)$ for any

$$m > \frac{9n}{c\tau_0^2}(1 + \frac{1}{2}\log 3)$$

where $\tau_0 = \min\{\tau, \pi/2b\}$.

Theorem 2: *Let (X, g) be a complete Kähler manifold such that*

$$\text{sectional curvature } \leq b^2$$

with an injectivity radius that is everywhere $\geq \tau$ for some number $\tau > 0$. Let (L, h) be a positive line bundle as in theorem 1. Then the Bergman kernel of the bundle $K_X + mL$ can be estimated by

$$B_{g^* h^m}(x) \geq \frac{1}{2^8 \pi \tau_0^2}$$

for any

$$m > \frac{9n}{c\tau_0^2}(1 + \frac{1}{2}\log 3)$$

where τ_0 is as in theorem 1.

On boundary measures of subharmonic and holomorphic functions in several complex variables

Evgeni Chirka

(joint work with Claudio Rea)

Some generalizations of the classical F. and M. Riesz theorem for holomorphic functions of several complex variables are proved using the correspondent generalizations for subharmonic functions and the methods from CR - theory.

Theorem 1: *Let Γ be a real hypersurface of class $C^{1+\alpha}$, $(\alpha > 0)$, in a complex manifold and f a measure-type CR-distribution on Γ. Then indeed $f \in L^1_{\text{loc}}(\Gamma)$.*

Theorem 2: Let M be a CR-manifold in $\mathbb{C}^n$ of class $C^{2+\alpha}$, $(\alpha > 0)$, and f a measure-type CR-distribution on M. Then f is in L^1 in a neighbourhood of any minimal point of M.

Theorem 3: Let f be a holomorphic function of class $H^1(W, M)$ in a wedge $W \subset \mathbb{C}^n$ with the edge M. Then f has boundary distribution f_* on M and $f_* \in L^1_{\text{loc}}(M)$. Here $M \in C^\infty$.

A smooth family of holomorphic support functions for lineally convex domains

John E. Fornæss
(joint work with Klas Diederich)

A smoothly bounded domain in $\mathbb{C}^n$ is lineally convex, if the complex tangent plane of any boundary point does not intersect the domain. If $z \in \partial\Omega$, the function $\phi_z(\zeta)$ is a holomorphic support function, if $\{\zeta \in \mathbb{C}^n \mid \phi_z(\zeta) = 0\}$ does not intersect Ω in a neighbourhood of z, and $\phi_z(z) = 0$.

We proved that by perturbing the complex tangent planes and assuming that Ω has finite type, one can find $\phi_z(\zeta)$, $z \in \partial\Omega, \zeta \in \mathbb{C}^n$ with optimal order of tangency with $\partial\Omega$. In particular, the order of tangency is constant in each complex line in each complex tangent plane.

Holomorphic submersion of Stein manifolds to affine spaces

Franc Forstneric

We prove that every Stein manifold of dimension n admits $[\frac{n+1}{2}]$ holomorphic functions with pointwise independent differentials and this number is maximal for every n. Furthermore, every surjective complex vector bundle map from the tangent bundle TX onto the trivial bundle $X \times \mathbb{C}^q$, for $q < \dim X$ is homotopic to the differential of a holomorphic submersion $X \longrightarrow \mathbb{C}^q$ (the so-called "homotopy principle for holomorphic submersions"). This implies that any complex subbundle $E \subset TX$ with a trivial quotient TX/E is homotopic to an integrable subbundle of the form $\ker(df)$, where f is a holomorphic submersion $X \longrightarrow \mathbb{C}^q$, ($q = \dim X - \dim E$).

A technical result of independent interest is a lemma on compositional splitting of biholomorphic maps.

Spectral theory of the $\bar\partial$-Neumann and Kohn Laplacians

Siqi Fu

We discuss the several complex variables version of Mark Kac's question: "Can one "hear" the shape of a drum?".

We explain how one can "hear" the pseudoconvexity, strictly pseudoconvexity, finite type (in the sense of Kohn-D'Angelo) in $\mathbb{C}^2$, and absence of analytic structures for certain domains via the $\bar\partial$-Neumann and Kohn Laplacians.

Estimates for $\bar\partial$ on convex domains depending on Catlin's multitype

Torsten Hefer

Let $D \subset\subset \mathbb{C}^n$ be a smooth convex domain of finite type m and Catlin multitype $M(bD) = (1, m_2, ..., m_n)$. We show that the optimal estimates for the $\bar\partial$ operator on $(0, q)$-forms in L^p and Hölder spaces are determined by the entry $m_{n-q+1} = \Delta_q(bD)$ as follows:

Theorem: *Let* $\lambda_q := m_{n-q+1}(n - q + 1) + 2q$. *Then there are linear continuous solution operators for the* $\bar\partial$ *equation on* D *between the following spaces:*

- *1)* $L^p_{0,q} \longrightarrow L^s_{0,q-1}$, *for* $\frac{1}{s} = \frac{1}{p} - \frac{1}{\lambda_p}$, *if* $1 \leq p < \lambda_q$,
- *2)* $L^p_{0,q} \longrightarrow L^s_{0,q-1}$, *for all* $s < \infty$, *if* $p = \lambda_q$,
- *3)* $L^p_{0,q} \longrightarrow \Lambda^\alpha_{0,q-1}$, *for* $\alpha = \frac{1}{m_{n-q+1}}(1 - \frac{\lambda_q}{p})$, *if* $p > \lambda_q$.

The Hölder-space estimates are optimal; in L^p-spaces, the estimates are optimal for $(0,1)$-forms.

The proof is based on constructions by K. Diederich-J.E.Fornæss-B.Fischer and J. McNeal. A modification of McNeal's ε-extremal bases is introduced.

Localization lemmas in Bergman theory

Gregor Herbort

Let $D \subset\subset \mathbb{C}^n$ be a domain and $w \in D$; let $E = \{T_1, ..., T_k\}$ denote a set of linear constant coefficient differential operators, and $Y \in \mathbb{C}^k$. Assume that

$$A^D(E, Y, w) := \{f \in H^2(D) \mid (T_1 f(w), ..., T_k f(w)) = Y\} \neq \emptyset$$

(where $H^2(D)$ is the space of square-integrable holomorphic functions on D) . Furthermore, let $G_D(\cdot, w)$ denote the pluricomplex Green function on D with pole at w and $A_w = \{z \in D \mid G_D(z, w) < -1\}$. We sketched a proof of the following

Theorem 1: *Assume that* D *is pseudoconvex and* $\zeta \in \partial D$ *a point, such that* $\mathrm{diam}(A_w) \longrightarrow 0$, *when* $w \longrightarrow \zeta$.

a) Then we have for any $R > 0$:

$$\frac{\|M_{D \cap B(\zeta,R)}(\cdot, w)\|}{\|M_D(\cdot, w)\|} \longrightarrow 1, \quad \text{when } w \longrightarrow \zeta$$

b) The concentration of mass property holds at ζ, *i.e.: For any* $s, R > 0$ *there exists* $\delta > 0$, *such that*

$$\|M_D(\cdot, w)\|_{D \cap B(\zeta,R)^c} \leq s\|M_D(\cdot, w)\|_D$$

whenever $w \in D \cap B(\zeta, \delta)$.

Here, $M_D(\cdot, w)$ *denotes a "minimal function", i.e.: a function in* $A^D(E, Y, w)$ *of minimal* L^2*-norm.*

This theorem applies to smooth bounded pseudoconvex regular domains (in the sense of Diederich-Fornæss). We further communicated:

Theorem 2: Let ∂D be C^2-smooth and pseudoconvex and $\zeta \in \partial D$ a point that admits a Hölder-continuous plurisubharmonic peak function. Then $\mathrm{diam}(A_w) \longrightarrow 0$, when w tends to ζ in a non-tangential way.

Newton polyhedra and the Bergman kernel

Joe Kamimoto

We study the singularities of the Bergman kernel at the boundary for pseudoconvex domains of finite type from the viewpoint of the theory of singularities. Under some assumptions on the domain $\Omega = \{(z, w) \in \mathbb{C}^{n+1} \mid \mathrm{Im}\,(w) > F(z)\}$ (where F is a plurisubharmonic smooth function in $\mathbb{C}^n$), the Bergman kernel $B(z)$ of Ω takes near a boundary point p the form:

$$B(z) = \frac{\Phi(z, \rho)}{\rho^{2+\,2/d_F}(\log(1/\rho)\,)^{m_F - 1}},$$

where (w, ρ) denotes some polar coordinates on a non-tangential cone with apex at p and ρ means the distance from the boundary. Here Φ admits some asymptotic expansion with respect to the variables $\rho^{1/m}$ and $\log(1/\rho)$ as $\rho \longrightarrow 0$ in Λ.

The values of $d_F > 0$, $m_F \in \mathbb{Z}^+$ and $m \in \mathbb{N}$ are determined by geometrical properties of the Newton polyhedron of the defining function of the domain, and the limit of Φ, as $\rho \longrightarrow 0$ in Λ, is a positive constant depending only on the Newton principal part of the defining function. Analogous results are obtained in the case of the Szegö kernel.

Fundamental solutions for $\bar{\partial}_b$

Christine Laurent-Thiébaut

Let M be a q-concave CR generic submanifold of $\mathbb{C}^n$ of class C^3 and real codimension k. For each $z_0 \in M$ there exists a neighborhood U_{z_0} of z_0 in M and some kernel R_M, defined on $U_{z_0} \times U_{z_0} \setminus \Delta(U_{z_0})$, such that for $n-k-q+1 \leq r \leq n-k$ one has

$$\bar{\partial}_{b,z} R_M^{n,r-1} + \bar{\partial}_{b,\zeta} R_M^{n,r} = [\Delta(U_{z_0})],$$

where $R_M^{n,r}$ denotes the part of bidegree (n, r) in z of R_M and $[\Delta(U_{z_0})]$ the integration current on the diagonal $\Delta(U_{z_0})$ of $U_{z_0} \times U_{z_0}$.

Moreover, if M is of class $C^{\ell+2}$, the operator

$$\widetilde{R}_M : f \longmapsto \int_{\zeta \in M} f(\zeta) \wedge R_M(z, \zeta)$$

is continuous from the set $(C_c^\ell)^{n,r}(U_{z_0})$, of (n,r)-forms of class C^ℓ with compact support in U_{z_0} into $(C^{\ell+1/2})^{n,r-1}(U_{z_0})$, and this estimate is sharp.

We deduced the following homotopy formula: Let f be a C^1-form with compact support in U_{z_0}, then

$$(-1)^{(k+1)(r+n)} f = \bar{\partial}_{b,z} \widetilde{R}_M f + (-1)^{k+1} \widetilde{R}_M (\bar{\partial}_b f).$$

By exchanging z and ζ, we get the same results for $0 \leq r \leq q-1$.

A relative Oka-Grauert principle on 1-convex spaces

Jürgen Leiterer
(joint work with Viorel Vâjâitu)

Let X be a 1-convex complex space and S the exceptional set of X. The following generalization of the Oka-Grauert principle was proved:

Theorem: *(i) Suppose E, F are two holomorphic vector bundles on X such that there is a continuous isomorphism between E and F which is holomorphic over some neighborhood of S. Then E and F are holomorphically isomorphic.*

(ii) Suppose E is a complex vector bundle on X and there is given a holomorphic structure on $E|U$ for some neighborhood U of S. Then there exists an extension of this holomorphic structure to a holomorphic structure on X.

For $S = \emptyset$, i.e.: X is Stein, this is the well-known theorem of Grauert from 1957. If X is smooth it was obtained in a joint paper with G. M. Henkin (*Math. Ann.* **311**, (1988)).

On open question is whether this theorem can be generalized to Banach space bundles (for $S = \emptyset$ this is possible, as was proved L. Bungart in 1972).

Plurisubharmonic domination

László Lempert

Consider a Banach space X over the complex numbers, that has a so-called unconditional basis, and a pseudoconvex open $\Omega \subset X$.

We have presented the following two theorems:

Theorem 1: *Suppose* $u : \Omega \longrightarrow I\!R$ *is continuos. Then*

(a) There is a plurisubharmonic $v : \Omega \longrightarrow I\!R$*, such that* $v \geq u$

(b) There is a Banach space Z *and a holomorphic map* $f : \Omega \longrightarrow Z$*, such that* $\|f(x)\|_Z \geq u(x)$*, for all* $x \in \Omega$*.*

Theorem 2: If $E \longrightarrow \Omega$ *is a holomorphic Banach bundle and* $q \geq 1$*, then* $H^q(\Omega, E) = 0$*.*

L^2-cohomology on some Kähler manifolds

Jeffery D. Mc Neal

Let (M, ω) be a complete Kähler manifold, $\dim M = n$, and suppose $\omega = i\partial\bar{\partial}\lambda$, for some $\lambda \in C^2(M)$. If the complex gradient $\partial\lambda$ of λ grows slower than λ, i.e.:

$$(*) \qquad |\partial\lambda(p) \wedge \bar{\partial}\lambda(p)| \leq (A + B\,\lambda(p)\,)\, i\,\partial\bar{\partial}\lambda(p), \quad \forall p \in M,$$

for some constants A and B, then we show

$$\mathcal{H}^{p,q}_{(2)}(M) = 0, \ \text{ if } \ p + q \neq n$$

where $\mathcal{H}^{p,q}_{(2)}(M)$ denotes the space of L^2-harmonic forms on M. Additionally, if one can choose $B < 1$ in (*), then we obtain the vanishing of $\bar{\partial}$-cohomology, measured in the ω-metric.

This work generalizes earlier results of Gromov. I also presented examples of situations where (*) holds for metrics of interest in complex analysis, e.g. the Bergman metric on some weakly pseudoconvex domains in $\mathbb{C}^n$.

Symmetries of partial differential equations: Application to CR geometry

Joël Merker

Let $n \geq 1, m \geq 1, x = (x_1, ..., x_n) \in I\!K^n, u = (u^1, ..., u^m) \in I\!K^m$, with $I\!K = I\!R$ or $\mathbb{C}$. Consider a completely integrable system of the form

$$(\mathcal{E}) \qquad u^j_{x^\alpha} = F^j_\alpha\left(x, u, (u^i_{x^\beta})^{1\leq i\leq m}_{|\beta|\leq k-1}\right)$$

where $j = 1, ..., m, \alpha \in I\!N^n, |\alpha| = k$.

Theorem: *Assume* $k \geq 2$*. Then the following bounds hold for the dimension of the symmetry group of* $(\mathcal{E})$ *(in the sense of S. Lie):*

- $k = 2$: $\quad \dim_{I\!K}(\,\mathrm{Sym}(\mathcal{E})\,) \leq (n+m)(n+m+2)$ *(A. Sukhov)*
- $k \geq 3$: $\quad \dim_{I\!K}(\,\mathrm{Sym}(\mathcal{E})\,) \leq n^2 + 2n + m^2 + m\frac{(n+k-1)!}{n!(k-1)!}$ *(H. Gaussier, J. Merker)*

Applications to local symmetry groups of real-analytic CR-manifolds were given: In the direction of CR mappings the following was quoted:

Theorem: (*Ann. Inst. Fourier* ,**52** (2002), 1443-1523) *Let $h : M \longrightarrow M'$ be a C^∞-smooth CR mapping between connected real-analytic hypersurfaces in $\mathbb{C}^n$ which are essentially finite at every point. Assume that h is of rank equal to $\dim_{\mathbb{R}}(M)$ in at least one point of M. Then h is real-analytic at every point of M.*

A real-analytic hypersurface $M \subset \mathbb{C}^n$, $n \geq 2$, is called locally algebraizable if it can be represented as a Nash algebraic hypersurface in some holomorphic coordinates system.

Theorem: (H. Gaussier, J. Merker, to appear in Math. Z.). *Let $M \subset \mathbb{C}^n$ be a real-analytic Levi non-degenerate hypersurface whose equation is of the form*

$$\operatorname{Im} w = \sum_{k=1}^{n-1} \varepsilon_k z_k \overline{z_k} + \chi(z, \overline{z})$$

where $\varepsilon_k = \pm 1$ and $\chi(0, \overline{z}) \equiv 0$, and $\chi_{z_k}(0, \overline{z}) \equiv 0$. Assume that M is strongly rigid, namely $\underline{\mathrm{Aut}}_{\mathrm{CR}}(M)$ is of dimension 1, generated by $\frac{\partial}{\partial w} + \frac{\partial}{\partial \overline{w}}$. If M is locally algebraizable, then the derivatives $\chi_{z_k}(z, \overline{z})$ are all algebraic.

The Chern-Moser theory is imcomplete in many respects:

- there is lack of explicit formulas
- it does not provide information on $\mathrm{Aut}_{\mathrm{CR}}(M)$
- it is not suitable for homogeneous models

For sociological reasons its influence has been too wide. Instead, one has to come back to S. Lie and É. Cartan 's original works.

Envelopes of holomorphy of immersed real surfaces in complex surfaces

Stefan Nemirowski

The generalized adjunction inequality

$$[\Sigma] \cdot [\Sigma] - 2(\varkappa_+ - \varkappa_-^{\mathrm{ess}}) + |c_1(X) \cdot [\Sigma]| \leq 2g(\Sigma) - 2$$

is established for an immersed real surface $\Sigma \looparrowright X$ in a Stein complex surface X which is not a homotopically trivial 2-sphere.

This result is then used to study analytic continuation from immersed real surfaces and to show that the sufficient conditions for the existence of Stein neighborhoods (due to Kharlamov-Eliashberg-Forstneric) of real surfaces are in fact necessary for the existence of topologically small Stein neighborhoods.

Optimal Lipschitz estimates for the $\bar{\partial}$-equation on a class of convex domains

Nguyen Viêt Anh

(joint work with El Hassan Youssfi)

In this talk we considered the Cauchy-Riemann equation $\bar{\partial} u = f$ in a new class of convex domains Ω_N in $\mathbb{C}^n$. The domains Ω_N are defined as follows:

For any $N = (n_1, ..., n_m) \in \mathbb{N}^m$, (with $1 \leq n_1 \leq n_2 \leq ... \leq n_m$) let

$$\Omega_N := \{z = (z_{(1)}, ..., z_{(m)}) \in \mathbb{C}^{n_1} \times ... \times \mathbb{C}^{n_m} \mid \sum_{j=1}^{m} \left(|z_{(j)} \bullet z_{(j)}| + |z_{(j)}|^2 \right) < 1\}$$

where $z \bullet w = \sum_{\ell=1}^{n} z_\ell w_\ell$, for $z, w \in \mathbb{C}^n$, and $|z_{(j)}|$ is the euclidean norm of $z_{(j)} \in \mathbb{C}^{n_j}$.

We prove that, given L^p-data f, we can choose a solution in the Lipschitz space Λ_α, where α is an optimal positive number, that can be given explicitly in terms of p:

$$\alpha = \begin{cases} \frac{1}{2} - \frac{|N|+m-\ell+1}{p} & \text{, if} \quad N \neq (2,2,...,2), \text{ and } p > 2(|N|+m-\ell+1) \\ \frac{1}{2} - \frac{3m}{2} & \text{, if} \quad N = (2,2,...,2), \text{ and } p > 6m \end{cases}$$

Here ℓ denotes the minimal integer with $n_{\ell+1} > 1$.

The Mabuchi energy functional and stability

Duong H. Phong

A well-known conjecture of Yau is that the existence of Kähler-Einstein metrics should be equivalent to stability in the sense of geometric invariant theory. In the variational approach, it is natural to try to link the energy functional for Kähler-Einstein metrics to the orbit of the Chow vector. We describe joint work with J. Sturm providing such a link. A new key feature emerges, which is a current term associated to the singular locus of the Chow variety. We discuss several applications and related developments, including the Lu-Yotav formulas for the Futaki invariant of complete intersections, the corresponding Futaki and Mabuchi functionals, and asymptotics for the Mabuchi functional for curves.

Regularity of CR mappings

Sergey Pinchuk

(joint work with Klas Diederich)

In my talk I discussed the main ideas of the proof of the following joint result with K. Diederich

Theorem: *Let $M, M' \subset \mathbb{C}^n$ be real-analytic smooth hypersurfaces of finite type and $f : M \longrightarrow M'$ a continuous CR map. Then f extends holomorphically to a neighborhood of M.*

This result was not previously known even under the additional conditions that M and M' are pseudoconvex and $f \in C^\infty(M)$.

On the Hartogs phenomenon on nowhere strictly pseudoconvex hypersurfaces of class C^2

Egmont Porten

We proved the following

Theorem: *Let $M \longrightarrow \mathbb{C}^n$ be a nowhere strictly pseudoconvex C^2-hypersurface, given as the graph*

$$M = \{y_n = h(z', x_n) \mid (z', x_n) \in \widetilde{M}\}$$

where $\widetilde{M} \subset \mathbb{C}^{n-1} \times I\!R$ is a domain such that $\widetilde{M} \times (i\, I\!R)$ is pseudoconvex. For any compact $K \subset M$ with $M \setminus K$ connected the functions $f \in \mathcal{O}(M \setminus K)$ extend simultaneously to functions in $\mathcal{O}(M)$.

A global result can be proved for weakly 2-concave hypersurfaces.

Segre varieties and analytic continuation of holomorphic mappings

Rasul Shafikov

We showed that, given bounded domains $D, D' \subset\subset \mathbb{C}^n$ such that

- ∂D is real-analytic, smooth, and simply connected
- $\partial D'$ is real-algebraic, smooth, and connected

then the following are equivalent:

(1) There exists a proper holomorphic correspondence $F : D \longrightarrow D'$

(2) There exist points $p \in \partial D$ and $p' \in \partial D'$, and small neighborhoods $U \ni p$ and $U' \ni p'$, and a biholomorphic mapping $f : U \longrightarrow U'$ such that $f(p) = p'$ and $f(U \cap \partial D) = U' \cap \partial D'$

Thus equivalence of domains modulo proper holomorphic correspondences can be characterized in terms of local CR equivalence of the boundaries of these domains.

Pluripolar graphs are holomorphic

Nikolay Shcherbina

We proved the following

Theorem: *Let* Ω *be a domain in* $\mathbb{C}^n$ *and* $f : \Omega \longrightarrow \mathbb{C}$ *a continuous function. Then the graph* $\Gamma(f)$ *of* f *is a pluripolar set in* $\mathbb{C}^{n+1}$ *if and only if* f *is holomorphic.*

Semi-classical anlysis of Schrödinger operators and compactness in the $\bar{\partial}$-Neumann problem

Emil J. Straube

(joint work with Siqi Fu)

We study the asymptotic behavior, in a "semi-classical limit" of the first eigenvalues of a class of Schrödinger operators with magnetic fields and the relationship of this behavior with compactness in the $\bar{\partial}$-Neumann problem on Hartogs domains in $\mathbb{C}^2$.

Simultaneous linearization of attracting and semi-attracting fiexd points in two complex variables

Tetsuo Ueda

Let M be a complex manifold of dimension 2 and F a holomorphic automorphism of M with a fixed point p_0. We denote by λ, μ the eigenvalues of the differential $F'(p_0)$ of F at p_0. The basin of (locally uniform) attraction of p_0 is defined by

$$D = \{p \in M \mid F^{\circ n}(p) \longrightarrow p_0, \text{ (locally uniformly) when } n \longrightarrow \infty\}$$

In the cases

- p_0 is attracting $(|\lambda|, |\mu| < 1)$
- p_0 is semi-attracting $(\lambda = 1, |\mu| < 1)$, with an additional condition

it is known that there is a biholomorphic map $\Phi : D \longrightarrow \mathbb{C}^2$.

In this talk we treat a family of holomorphic automorphisms F_t depending holomorphically on a parameter $t \in T \subset \mathbb{C}$. Suppose that, for all $t \in T$ the mapping F_t has a fixed point p_0, such that the eigenvalues $\lambda(t), \mu(t)$ of $F_t'(p_0)$ satisfy $\lambda(t_0) = 1$, and $|\mu(t)| < 1$ (where $t_0 \in T$ is an interior point).

Our result is that there is a family of biholomorphic maps $\Phi_t : D_t \longrightarrow \mathbb{C}^2$, where $t \in \{t \in T \mid |\lambda(t)| < 1\}$, which has a non-tangential limit, as $t \longrightarrow 1$ from inside the unit disc, and such that $F_t|D_t$ is reduced to the form

$$(x, y) \longmapsto (\lambda(t)x + 1, \mu(t)y + h_t(x)\,)$$

where h_t is an entire function.

Prescribing automorphism groups of complex manifolds

Jörg Winkelmann

Generalizing earlier results of Bedford, Dadoc, Saerens, and Zame, we prove

Theorem: *Let G be a real Lie group and G^0 the connected component of e. Assume that*

- $G = G^0$ *(G is connected)*

or

- $G^0 = \{e\}$, *(G is discrete)*

Then there exists a Stein, complete hyperbolic, complex manifold X such that

$$\mathrm{Aut}\,(X) \cong G.$$

We conjuctured that this result holds for arbitrary real Lie groups as well.

Calabi-Yau hypersurfaces, discriminants, and Quillen metrics

Ken-Ichi Yoshikawa

Let X be a Calabi-Yau manifold, i.e. a compact Kähler manifold with trivial canonical line bundle. We set

$$\lambda(\Omega_X) = \bigotimes_{p \geq 0} \lambda(\Omega_X^p)^{(-1)^p\, p} = \bigotimes_{p,q \geq 0} (\det H^q(X, \Omega_X^p)\,)^{(-1)^{p+q}\, p}$$

By modifying the Quillen metric on $\lambda(\Omega_X)$, we introduce a hermitian structure $\| \cdot \|_{\lambda(\Omega_X),Q}$ on $\lambda(\Omega_X)$, which is independent of the choice of a Kähler metric on X. Hence, there exist two hermitian lines

$$(\, H^0(X, \Omega_X^{\mathrm{top}}), \| \cdot \|_{L^2}\,),\ \ (\lambda(\Omega_X), \| \cdot \|_{\lambda(\Omega_X),Q}\ )$$

associated to X. When X is a hypersurface of a Fano manifold V, a member of the anti-canonical system $|K_V^{-1}|$, we can compare these two intrinsic hermitian lines. We prove that the difference of them can be identified with the discriminant of the linear system $|K_V^{-1}|$.

The hermitian structure $\| \cdot \|_{\lambda(\Omega_X),Q}$ is constructed as follows:

$$\| \cdot \|^2_{\lambda(\Omega_X),Q} = A(X, K) \cdot \tau(X, K) \cdot \| \cdot \|^2_{L^2},$$

where

$$\tau(X,K) = \exp\left(-\sum_{p,q\geq 0}(-1)^{p+q}pq\cdot\zeta'_{p,q}(0)\right)$$

where

$$\zeta_{p,q}(s) = \sum_{\lambda\in\sigma(\Box_{p,q})}\lambda^{-s}$$

here $\Box_{p,q}$ denotes the Laplacian acting on (p,q)-forms. Furthermore

$$A(X,K) = \exp\left(\frac{1}{12}\int_X \log\left(\frac{K^n/n!}{\eta\wedge\overline{\eta}}\cdot\frac{\|\eta\|_{L^2}}{\mathrm{Vol}(X,K)}\right)c_n(X,K) + \frac{1}{12}\chi_{\mathrm{top}}(X)\cdot\mathrm{Vol}(X,K)\right)$$

Here, $c_n(X,K)$ is the top Chern form of (TX,K) and $\chi_{\mathrm{top}}(X)$ the Euler number, and $\eta\in H^0(X,\Omega^n_X)\setminus\{0\}$.

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