Mathematisches Forschungsinstitut Oberwolfach

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Homotopietheorie

March 16th – March 22nd, 2003

The conference was organized by Mike Hopkins (MIT), Karlheinz Knapp (Wuppertal) and Erich Ossa (Wuppertal). 48 participants from Europe, Japan, Singapore, and the United States attended.

All in all, 19 talks were given, covering a wide array of homotopy theory and interconnections to other areas of mathematics, such as representation theory and group cohomology. Some major new developments presented were a proof of the longstanding conjecture that every (quasi)finite loop space is homotopy equivalent to a smooth, compact, parallelizable manifold, and a proof of the Martino-Priddy conjecture on classifying spaces of finite groups. There were several talks on chromatic homotopy theory, some of them opening up ways towards generalizations of elliptic cohomology. Among the highlights were two talks showing how methods and ideas from commutative algebra may now be applied to yield surprising new insights into stable homotopy theory.

Abstracts

Periodic Localization and Goodwillie Calculus

Nicholas Kuhn

Goodwillie's calculus of homotopy functors interacts in beautiful and deep ways with Bousfield localization with respect to periodic homology theories. We have two theorems illustrating this, with input used in proving the theorems coming from consequences of the Nilpotence theorems of Devanitz, Hopkins, and Smith.

Let $K(n)$ denote the n^{th} Morava K–theory, localized at a prime p, and with $n > 0$. Let $T(n)$ denote the mapping telescope of a v_n–self map of a type n finite spectrum. The Bousfield classes compare: $\langle T(n) \rangle \geq \langle K(n) \rangle$, and the telescope conjecture is the statement that equality holds.

Let $\mathcal{A}lg$ denote the category of commutative, augmented S–algebras. If X is an S–module, let $\mathbb{P}X$ denote the free object in $\mathcal{A}lg$ generated by X. This is homotopy equivalent to the wedge over d of $(X^{\wedge d})_{h\Sigma_d}$. Another example of an object in $\mathcal{A}lg$ is $\Sigma^\infty \Omega^\infty X_+$.

$\mathcal{A}lg$ is a fine model category, and the Goodwillie tower of its identity functor, evaluated at an algebra R yields the André–Quillen tower of R. The tower for $\mathbb{P}X$ evidently splits into the product of the homogeneous pieces $(X^{\wedge d})_{h\Sigma_d}$. Our first theorem implies that the André–Quillen tower for $\Sigma^\infty \Omega^\infty X_+$ similarly splits after Bousfield localization with respect to $T(n)$.

Theorem A. *There is a natural map $L_{T(n)}\mathbb{P}X \to L_{T(n)}\Sigma^\infty\Omega^\infty X_+$ of commutative augmented $L_{T(n)}S$–algebras inducing a $T(n)_*$–equivalence of André–Quillen towers.*

This theorem follows from an application of the telescopic functors constructed in the mid 1980's by Bousfield and myself.

Input A The evaluation $\epsilon\colon \Sigma^\infty\Omega^\infty X \to X$ admits a natural section after $T(n)$-localization.

Our second theorem concerns the periodic localization of all functors with domain and range in the stable model category of S–modules.

Theorem B. *All Goodwillie towers $F_1(X) \leftarrow F_2(X) \leftarrow F_3(X) \leftarrow \cdots$ of polynomial functors from S–modules to S–modules split as the product of their homogeneous fibres after localization with respect to $T(n)$, for all $n > 0$.*

This follows from the next theorem, which strengthens to $T(n)$ a result about $K(n)$ proved by Greenlees, Hovey, and Sadofsky.

Input B For all finite groups G, $L_{T(n)}\mathrm{Tate}_G(L_{T(n)}S) \simeq *$.

The deduction of Theorem B from Input B uses an insight about polynomial functors due to McCarthy.

Witt vectors and equivariant stable homotopy

Morten Brun

This talk is guided by the fact that the zeroth homotopy group of the C_n-fixed points of topological Hochschild homology of a commutative ring R is equal to the n-truncation of the ring of big Witt vectors, denoted $W_n(R)$. It has the following generalization: Let A be a commutative ring-spectrum with an action of a finite group G. For a subgroup $H \subseteq G$, let $\pi_0(A^H)$ denote the zeroth homotopy group of the H-fixed points of A. Then there for every $H \subseteq G$ there is a natural ring homomorphism $W_H(\pi_0(A)) \to \pi_0(A^H)$, where $W_H(\pi_0(A)$ is the ring of H-Witt vectors of $\pi_0(A)$ defined by Dress and Siebeneicher. This map is given by a universal property of the function $H \mapsto W_H(\pi_0(A))$. I shall end the talk by discussing what happens on higher homotopy groups.

Cohomological quotients and the telescope conjecture

Henning Krause

A new type of quotients for triangulated categories is introduced. This generalizes the classical quotient modulo a thick subcategory in the sense of Verdier. The concept of a cohomological quotient is motivated by the failure of the telescope conjecture. In fact, if $\mathcal{C}$ is a compactly generated triangulated category, then the cohomological quotients of $\operatorname{comp}(\mathcal{C})$ correspond bijectively to the smashing localizations of $\mathcal{C}$. Here, $\operatorname{comp}(\mathcal{C})$ denotes the subcategory of compact objects in $\mathcal{C}$, and a localization functor $\mathcal{C} \to \mathcal{C}$ is smashing if it preserves coproducts. The cohomological quotients of a triangulated category form a complete and cocomplete lattice. This leads for any exact functor $\mathcal{C} \to \mathcal{D}$ between triangulated categories to a unique factorization $\mathcal{C} \to \mathcal{C}' \to \mathcal{D}$ such that

1) $\mathcal{C} \to \mathcal{C}'$ is a cohomological quotient, and
2) if $\mathcal{C} \to \mathcal{C}'' \to \mathcal{D}$ is another factorization such that $\mathcal{C} \to \mathcal{C}''$ is a cohomological quotient, then $\mathcal{C} \to \mathcal{C}'$ factors through $\mathcal{C} \to \mathcal{C}''$.

As an application, we discuss when a ring homomorphism $R \to S$ induces a cohomological quotient $\operatorname{per}(R) \to \operatorname{per}(S)$ for the categories of perfect complexes. This leads to the question, when every map in $\operatorname{per}(S)$ is (up to an isomorphism) induced from a map in $\operatorname{per}(R)$. The answer generalizes recent work of Neeman and Ranicki.

Profinite spaces, complex cobordism, unstable operations, and cohomology of mapping spaces

François-Xavier Dehon

We present division functors in MU-cohomology related to the cohomology of mapping spaces in the spirit of Lannes' T-functor.

Let $\mathrm{gr}\mathcal{S}\mathrm{et}$ denote the category of graded sets, $\widehat{\mathcal{S}}$ the category of profinite spaces and $\mathrm{h}\widehat{\mathcal{S}}$ its homotopy category (as set up by Morel). For S in $\mathrm{gr}\mathcal{S}\mathrm{et}$ we let $\mathrm{K}(S)$ be the product $\prod_{s\in S} \mathrm{MU}_{|s|}$ where $(\mathrm{MU}_n)_n$ stands for the Ω-spectrum associated to complex cobordism (so that we have $\mathrm{Hom}_{\mathrm{h}\widehat{\mathcal{S}}}(X, \mathrm{K}(S)) \simeq \mathrm{Hom}_{\mathrm{gr}\mathcal{S}\mathrm{et}}(S, \mathrm{MU}^*X))$. We denote G the associated monad $S \mapsto \mathrm{MU}^*\mathrm{K}(S)$ and define the category $\mathcal{K}_{\mathrm{MU}}$ of MU-unstable algebras as the category of G-algebras of $\mathrm{gr}\mathcal{S}\mathrm{et}$. The additive structure of $\mathrm{G}(S)$ is easy to describe from the fact the profinite space $\mathrm{K}(S)$ is torsion free by a result of Wilson and the general case comes from the coequalizer diagram $\mathrm{G}^2(M) \rightrightarrows \mathrm{G}(M) \to M$ for M in $\mathcal{K}_{\mathrm{MU}}$. We obtain an abelian category $\widehat{\mathcal{M}}$ with a tensor product $\hat{\otimes}$ and a decreasing filtration of its objects with the following properties:

– Let $\mathcal{K}$ denote the category of unstable algebras over the Steenrod algebra. For M in $\mathcal{K}_{\mathrm{MU}}$, M/f^1 is naturally in $\mathcal{K}$ and there is a bijection $\mathrm{Hom}_{\mathcal{K}_{\mathrm{MU}}}(M, \mathrm{MU}^*) \simeq \mathrm{Hom}_{\mathcal{K}}(M/\mathrm{f}^1, \mathbb{Z}/p)$.

– For a profinite space X there is a morphism $\mathrm{MU}^*X/\mathrm{f}^1 \to \mathrm{H}^*X$ which is iso if X is torsion free, and a bijection $\pi_0 X \simeq \mathrm{Hom}_{\mathcal{K}_{\mathrm{MU}}}(\mathrm{MU}^*X, \mathrm{MU}^*)$.

– Let W be a torsion free space whose mod p cohomology is degree-wise finite and X a profinite space, then there is an isomorphism $\mathrm{MU}^*W \hat{\otimes} \mathrm{MU}^*X \simeq \mathrm{MU}^*W \times X$.

– The functor $\mathrm{MU}^*W \hat{\otimes} -$ has a left adjoint in $\widehat{\mathcal{M}}$ so it has a left adjoint $(- : \mathrm{MU}^*W)_{\mathcal{K}_{\mathrm{MU}}}$ in $\mathcal{K}_{\mathrm{MU}}$. The morphism $(\mathrm{G}(S)\colon \mathrm{MU}^*W)_{\mathcal{K}_{\mathrm{MU}}} \to \mathrm{MU}^*\mathbf{map}(W, \mathrm{K}(S))$ is an isomorphism.

We now state:

Theorem. *The morphism* $(\mathrm{MU}^*X\colon \mathrm{MU}^*\mathbb{C}\mathrm{P}^\infty)_{\mathcal{K}_{\mathrm{MU}}} \to \mathrm{MU}^*\mathbf{map}(\mathbb{C}\mathrm{P}^\infty, X)$ *is an isomorphism if* X *is torsion free.*

Consequently the map $\mathrm{Hom}_{\mathrm{h}\widehat{\mathcal{S}}}(\mathbb{C}\mathrm{P}^\infty, X) \to \mathrm{Hom}_{\mathcal{K}_{\mathrm{MU}}}(\mathrm{MU}^*X, \mathrm{MU}^*\mathbb{C}\mathrm{P}^\infty)$ is a bijection if X is torsion free. Similar results hold for $W = \mathrm{B}\mathbb{Z}/p^n$.

Configurations, Braids, and Homotopy Groups

Jie Wu

(joint work with Jon Berrick, Fred Cohen, and Yan Loi Wong)

Simplicial and Δ-structures of configuration spaces are investigated. New connections between the homotopy groups of the 2-sphere and the braid groups are given. The higher homotopy groups of the 2- sphere are shown to be the derived groups of the braid groups over the 2-sphere. The higher homotopy groups of the 2-sphere are shown to be isomorphic to the Brunnian braids over the 2-sphere modulo the Brunnian braids over the disk. Moreover the sequence of classical Artin pure braid groups has a canonical Δ-group structure with the property that its Moore homotopy groups are the homotopy groups of S^2.

Subalgebras of group cohomology defined by infinite loop spaces

John R. Hunton

(joint work with David J. Green and Björn Schuster)

For a finite group G and a representable cohomology theory $E^*(-)$ we construct a subring, $Ch_E(G)$, closed under the action of the Steenrod algebra, of the group cohomology $H^*(G)$ with coefficients in the field of p elements. For E complex K-theory, we recover Thomas' Chern subring, the subring of $H^*(G)$ generated by Chern classes of unitary representations. We discuss $Ch_E(G)$ for a number of theories E. Theoretically, the ring ought be most accessible when there is both a good model for $E^*(BG)$ and a strong hold on the coalgebraic ring $H_*(\underline{E}_*)$, that is the homology of the spaces in the Ω-spectrum for E. This is so for examples such as $E = K$, π_s^0 and $\widehat{E(n)}$, but it turns out that good results are also obtainable in cases where there is no theory for $E^*(BG)$ such as BP, $k(n)$ and the (uncomplete) Johnson-Wilson spectra $E(n)$. In all cases there are simple descriptions of the varieties associated to the $Ch_E(G)$ in terms of categories of elementary abelian subgroups of G. Corollaries of this work include a 'chromatic' filtration of Quillen's variety of $H^*(G)$, and a new proof and generalisation of Yagita's theorem that the image of $BP^*(BG)$ in mod p-cohomology is F-isomorphic to the whole of $H^*(G)$.

Harrison cohomology and rational homotopy of function spaces

Andrey Lazarev

(joint work with J. Block)

Let X and Y be two rational spaces, both nilpotent and of finite type, $f\colon X \to Y$ be a fixed map and denote by $Map(X,Y)$ the function space from X into Y with f as a basepoint. Denote by $A^*(?)$ the Sullivan-deRham differential graded algebra of ?. Then f induces a map of differential graded algebras $A^*(Y) \to A^*(X)$ making $A^*(X)$ into an $A^*(Y)$-module. We show that the homotopy groups of $\pi_i Map(X,Y)$ for $i > 0$ can be expressed as Harrison cohomology of $A^*(Y)$ with coefficients in $A^*(X)$.

For $i = 0$ the analogous result can be proved under the assumption that $X = Y$ and that X satisfies some additional finiteness conditions (e.g. X could be a finite Postnikov tower). Recall that according to the well-known theorem of Sullivan and Wilkerson, the set of homotopy self-equivalences of a rational space X is a set of $\mathbb{Q}$-rational points of an algebraic group scheme over $\mathbb{Q}$. We show that the zeroth Harrison cohomology of $A^*(Y)$ with coefficients in $A^*(X)$ supplied with the Gerstenhaber bracket is the Lie algebra of the corresponding algebraic scheme.

A proof of the Martino-Priddy conjecture

Bob Oliver

The Martino-Priddy conjecture says that for any prime p and any pair of finite groups G and G', the p-completed classifying spaces $BG^\wedge_p$ and $BG'^\wedge_p$ are homotopy equivalent if and only if there is an isomorphism between the Sylow p-subgroups of G and G' which preserves fusion. In other words, the p-local structure of the group determines the homotopy type of its p-completed classifying space.

A p-subgroup $P \leq G$ is called p-centric in G if $Z(P)$ is a Sylow p-subgroup of $C_G(P)$; equivalently, if $C_G(P) = Z(P) \times C'_G(P)$ for some group $C'_G(P)$ of order prime to p. Let $\mathcal{Z}_G$ be the (contravariant) functor on the p-subgroup orbit category of G, defined by setting $\mathcal{Z}_G(P) = Z(P)$ if P is p-centric, and $\mathcal{Z}_G(P) = 0$ otherwise. The proof of the Martino-Priddy conjecture is first reduced to showing that $\varprojlim^2(\mathcal{Z}_G)$ vanishes for any finite group G. This reduction follows from a homotopy decomposition of Dwyer, who showed that BG is mod p equivalent to the homotopy direct limit of classifying spaces of p-centric subgroups of G. The proof that $\varprojlim^2(\mathcal{Z}_G) = 0$ is then carried out using the classification theorem for finite simple groups.

Finite loop spaces are manifolds

Tilman Bauer

(joint work with Nitu Kitchloo, Dietrich Notbohm, and Erik Pedersen)

We prove the following theorem:

Theorem. *Let X be a loop space such that $H_*(X;\mathbf{Z})$ is finitely generated. Then X is homotopy equivalent to a smooth, compact, parallelizable manifold.*

Many examples different from Lie groups may be constructed by Zabrodsky's method of mixing of homotopy types at different primes.

The proof consists of a surgery and a homotopy-theoretic part. We first show that the existence of a mildly enriched circle fibration with total space X and quasifinite, stably reducible base space makes the surgery and finiteness obstructions for X vanish and thus makes X smoothable. The assumptions on such a circle fibration are such that their existence is a local property: circle fibrations on all p-completions of X, for all primes p, will glue to yield a circle fibration on X.

In the second part, we construct such circle fibrations for the p-completion of a loop space, or, more generally, for p-compact groups. Since p-compact groups have maximal tori, circle subgroups can always be found. The construction utilizes the classification of p-adic pseudo-reflection groups, which occur as Weyl groups of p-compact groups, as well as the classification of all p-compact groups up to rank 2.

E_∞-ring structures for some periodic spectra

Birgit Richter

(joint work with Andrew Baker)

Alan Robinson established an obstruction theory for E_∞ ring structures on commutative ring spectra. The obstruction groups live in Gamma cohomology. This is a cohomology theory for commutative algebras which should be thought of as André-Quillen cohomology for E_∞ algebras.

We prove:

The spectra complex K-theory, its p-localization, the first Johnson-Wilson spectrum $E(1)$ and the I_n-adic completions of all Johnson-Wilson spectra each have a unique E_∞ structure.

The method of proof involves the calculation of the algebra of cooperations and their Gamma cohomology with rational and p-adic coefficients.

The homology of inverse limits of spectra and the chromatic splitting conjecture

Hal Sadofsky

Hopkins's chromatic splitting conjecture predicts that for X finite:

(1) The map $L_{n-1}X \to L_{n-1}L_{K(n)}X$ splits, i.e. $L_{n-1}L_{K(n)}X \simeq L_{n-1}X \vee P$, and
(2) P has a particular form, which we will not discuss here.

We prove 1. for X finite type $n-1$. As a corollary, we deduce that $L_{K(n-1)}L_{K(n)}X \simeq L_{K(n-1)}X \vee P'$ for some P'.

Our methods are of more general interest than our results; given a tower of spectra

$$\cdots \to X_i \to X_{i-1} \to \cdots \to X_1$$

and a multiplicative homology theory E where E_*E is flat over E_*, we produce a spectral sequence which computes (under favourable circumstances) $E_*(\operatorname{holim} L_E X_i)$, with E_2-term given by derived functors of inverse limits of E_*E-comodules.

We then use this to approximate $E(n)_*(L_{K(n)}X)$, and the proof proceeds formally from that point.

Toward higher chromatic analogs of elliptic cohomology

Douglas C. Ravenel

Elliptic cohomology fails to detect v_n-periodic phenomena because the formal group attached to an elliptic curve has height at most 2. In order to go deeper into the chromatic tower one needs a curve of genus > 1 whose Jacobian has a 1-dimensional formal summand of height > 2. I will describe such examples in this talk.

Why do I want to be a pro-homotopy theorist?

Wojciech Chachólski

Many properties of spaces are reflections of more general pro-properties of towers. These Pro properties typically contain much more information than their space level analogues. Pro setting is also very convenient for constructing and studying completions and localization functors. The aim of the talk is to present several motivating examples. I will show that various trivialization conditions can be generalized to the Pro setting and explain what new information is carried by these more general statement. This will be illustrated for contractibility, triviality of fibre bundles, triviality of resolutions.

I will then construct various completion towers and show their relation to the Bousfield-Kan tower and the localization tower.

The essential ideal: a selective survey

David J. Green

Consider the mod-p cohomology ring $H^*(G)$ of a finite group G. By definition, the essential ideal consists of those classes whose restriction to every proper subgroup is zero. Of old, detection results led to important theorems: a good example being one of Quillen's proofs of the Adams Conjecture. More recently, people have started deriving consequences of non-detection, that is of the presence of essential classes. Three good examples are

(1) J. F. Carlson's theorem that the presence of essential classes ensures that Duflot's group-theoretic lower bound for the depth of the cohomology ring is attained.
(2) The cohomological characterisation by Adem and Karagueuzian of those p-groups all of whose order p elements are central.
(3) The result of Pakianathan and Yalçın that if the nilpotence degree of the essential ideal exceeds two, then there is a fixed point theorem for G-CW-complexes in low dimensions akin to a well-known result in the theory of transformation groups which holds for all dimensions if the group is elementary abelian. To date there is only one example of a non-elementary abelian group with this high nilpotency degree property: the Sylow 2-subgroup of $SU_3(4)$.

After presenting these results I discussed in detail the proof of the theorems of Carlson and Duflot. The key techniques are comodule structure and systems of parameters with nice properties. One further corollary is that the essential ideal is free and finitely generated as a module over a certain polynomial algebra on d generators, where d is the p-rank of the centre of the group. Curiously, for this one has to assume that the group does not have the cyclic group of order p as a direct factor.

Self-maps of loop spaces

Fred Cohen

Let X denote a path-connected CW complex and $P^n(p^r) = S^{n-1} \cup_{p^r} e^n$ the mod p^r Moore space with top cell in dimension n.

Theorem. *If $n \geq 4$ and the order of the identity for $P^n(p^r)$ is p^q, then the p^{q+1}-power map on $\Omega^2(X \wedge P^n(p^r))$ is null-homotopic.*

Corollary. *If $X = \Sigma^3 A$ and the p^q-th co-H-power map on X is null, then $p^{q+1}\pi_* X - 0$.*

Methods: (I) The first step involves combinatorial group theory to give the "universal self-maps" in the group $[\Omega\Sigma^2 X, \Omega\Sigma^2 X]$. This group is an inverse limit, $\varprojlim_n H_n$, with

$$\begin{array}{ccccc} H_n & \longleftarrow & \mathrm{Lie}(n) & \longleftarrow & 1 \\ \downarrow & & & & \\ H_{n-1} & & & & \\ \downarrow & & & & \\ 1 & & & & \end{array}$$

where $\mathrm{Lie}(n) = \mathrm{center}(H_n)$.
(II) The second step is a homotopical version of the Eilenberg-Moore spectral sequence based on work with Dai Tamaki. An analysis of "attaching maps" for $\Omega^2\Sigma^2(P^n(p^r) \wedge X)$ based on "cells" $(P^n(p^r) \wedge X)^{nq}$ gives the requisite null-homotopies using part (I).

Galois theory of commutative $\mathbb{S}$-algebras

John Rognes

Let G be a finite group, E a spectrum. A map $A \to B$ of commutative $\mathbb{S}$-algebras is an *E-local G-Galois extension* if G acts on B through A-algebra maps so that the canonical maps $i\colon A \to B^{hG}$ and $h\colon B \wedge_A B \to \prod_G B$ are E-equivalences. A homomorphism $R \to T$ is a G-Galois extension of commutative rings if and only if the map $HR \to HT$ is a G-Galois extension of commutative $\mathbb{S}$-algebras, where H is the Eilenberg-Mac Lane functor. $c\colon KO \to KU$ is a (quadratic) C_2-Galois extension, while $J_p \to KU_p$ is a $K(1)$-local $\mathbb{Z}_p^*$-pro-Galois extension, and more generally $L_{K(n)}\mathbb{S} \to E_n$ is a $K(n)$-local $S_n \rtimes C_n$-pro-Galois extension, where S_n is the nth Morava stabilizer group. We show that B is *faithful* and *dualizable* as an A-module. G-Galois extensions are preserved by arbitrary base change, and detected by faithful and dualizable base change. It follows that for subgroups $H \subset G$ the extension $B^{hH} \to B$ is H-Galois, and if H is normal, $A \to B^{hH}$ is G/H-Galois. Applications are envisaged to the geometry of *Spec*$(\mathbb{S})$, descent for algebraic K-theory of commutative $\mathbb{S}$-algebras, and $K(n)$-compact groups.

Arithmetic equivariant elliptic cohomology

NEIL P. STRICKLAND

Recall that an *elliptic spectrum* is a triple $\mathbf{E} = (E, C, t)$, where

(a) E is a homotopy-commutative ring spectrum whose homotopy is two-periodic and concentrated in even degrees, giving a formal group P over the scheme $S := \mathrm{spec}(\pi_0 E)$;
(b) C is an elliptic curve over S;
(c) t is an isomorphism $P \simeq \widehat{C}$ of formal groups over S, where $\widehat{C}$ is the formal neighbourhood of the zero section in C.

We write $k = \mathcal{O}_S = \pi_0 E$. Given S and C, there is often a canonical (or even unique) choice of E as above.

Now fix an elliptic spectrum $\mathbf{E} = (E, C, t)$. Given a compact Lie group G, it is natural to hope for a G-equivariant version of E, represented by an equivariant ring spectrum E_G. This would almost surely be automatic, if we had a sufficiently geometric construction of elliptic cohomology. Conversely, by building E_G in a more *ad hoc* manner, we hope to gain intuition and experimental data to point the way to a geometric construction.

In the case of a finite abelian group A, the theory of equivariant formal groups gives a detailed picture of what E_A should look like, assuming that it exists. We will describe a method that works backwards from this picture to give a construction of E_A. The main part of the construction assumes that E is I_n-torsion and v_n-periodic for some $n \in \{0, 1, 2\}$; it uses geometric fixed points, the Borel construction, and an algebraic extension. The $K(n)$-local case then follows by passage to inverse limits, and the general case can be addressed using chromatic fracture squares, although there are obstructions to the final steps of this approach.

p odd versus $p = 2$ for p-compact groups

JESPER M. MØLLER

The classification theorem for p-compact groups at odd primes has been completed recently. It says that two p-compact groups are isomorphic if and only if their maximal torus normalizers are isomorphic. Focus is therefore now on the case $p = 2$.

The main purpose of the talk was to clarify the differences between 2-compact groups and p-compact groups for $p > 2$ when it comes to classification. The major differences are listed in this table over the first few cohomology groups $H^1(W; \check{T})$ where W is the Weyl group and $\check{T}$ the maximal torus of a connected p-compact group:

$H^i(W;\check{T})$	$p > 2$	$p = 2$
$i = 0$	$\check{Z}(X)$	$\subset \check{Z}(X)$
$i = 1$	0	$(\mathbf{Z}/2)^e$
$i = 2$	0	$(\mathbf{Z}/2)^f$

This means that when $p = 2$,

- the invariants for the Weyl group action are no longer necessarily equal to the center,
- there are automorphisms of the maximal torus normalizers that do not extend to automorphisms of the 2-compact group,
- the maximal torus normalizer does not necessarily split.

I outlined a classification program for 2-compact groups that take these differences into account. The program essentially reduces the classification conjecture to the case of the connected, simple, centerfree 2-compact groups. The program does go through for the 2-compact groups in the infinite A-family and leads to the following concrete result

Theorem. *The* 2*-compact group* $\mathrm{PU}(\mathrm{n}+1)$ *is* N*-determined.*

A 2-compact group is N-determined if it is determined up to isomorphism by its maximal torus normalizer. i.e. if it satisfies the classification conjecture.

New twists to duality in topology and algebra

John Greenlees

(joint work with W. G. Dwyer and S. Iyengar)

The talk discussed a number of examples of strictly commutative ring spectra R with a 'residue field' k.

CA: A commutative local Noetherian ring R with residue field k.
CX: The cochains on a space $R = C^*(X; k)$.
GH: The chromatic example with $R = L_n X^0$ (the L_n-local sphere) and $k = K(n)$ (nth Morava K theory).

It then discussed what it means for R to be regular or Gorenstein, and in the latter case, what it means to be orientable. In Case CA, both regular and Gorenstein mean the usual thing, and all Gorenstein rings are orientable. In Case CX, if k is of characteristic p and X is connected and p-complete then R is regular if and only if X is the classifying space of a p-compact group. It is Gorenstein if X is a manifold and it is orientably Gorenstein if X is orientable over k. It is also Gorenstein if X is the classifying space of any compact Lie group, and orientable if the adjoint bundle is orientable over k. In case GH Gross-Hopkins duality states that the example is Gorenstein, and precisely identifies the non-orientability.

It was also discussed how the notion of proxy-regularity lets one deduce duality statements when R is Gorenstein. This gives the local cohomology theorem for group cohomology rings and the conventional form of Gross-Hopkins duality.

Edited by Björn Schuster

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