Mathematisches Forschungsinstitut Oberwolfach

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Geometrie der Banachräume

April 13th – April 19th, 2003

The conference "Geometrie der Banachräume" was organized by H. König (Kiel), J. Lindenstrauss (Jerusalem) and N. Tomczak-Jaegermann (Edmonton) and attended by 44 people from 14 countries including 10 from Germany. Many of the participants are among the leaders in the area but also a large number of young mathematicians actively participated in the meeting.

Altogether 24 lectures were presented, one third of which were given by young researchers. The lectures outlined important areas of research in the theory of Banach spaces such as structure theory of infinite-dimensional Banach spaces, non-linear Banach space theory, asymptotic theory of normed spaces and connections with convex geometry. The lectures were well attended all week long. We just mention a few contributions. Argyros described some deep constructions closely related to a celebrated example of Gowers and Maurey and the dichotomy theorem of Gowers. Interesting new results concerning approximation properties of Banach spaces were obtained by Lusky and Johnson. Rudelson outlined new random constructions of euclidean subspaces of Banach spaces generated by ± 1 matrices requiring new probabilistic tools and Vershynin reported on important developments concerning restricted invertibility introduced and studied earlier by Bourgain and Tzafriri. Godefroy gave a new and elegant approach to the Lipschitz classification of Banach spaces, Lindenstrauss presented new ideas and new results on the Fréchet differentiability of Lipschitz maps. The participants spent much time on informal discussions resulting in new questions, projects and progress done on existing problems.

Abstracts

Mixing conditional and unconditional structures

Spiros Argyros

By “mixing conditional and unconditional structures” we understand Banach spaces on which coexist such structures. There are some recent results in this direction. The first is a reflexive Banach space X_{ius} which is indecomposable and unconditionally saturated. This is a joint work with A. Manoussakis (to appear in Studia Math.). Moreover this space has few operators, namely every $T \in \mathcal{L}(X_{ius})$ is of the form $T = \lambda I + S$ with S strictly singular. The second, a joint work with A. Tolias, is a dual pair (X_{uh}, X_{uh}^*) with X_{uh} being a reflexive unconditionally saturated Banach space and X_{uh}^* hereditarily indecomposable. The space X_{uh} has the the additional property that every quotient of it has a further quotient which is hereditarily indecomposable. More recently, in a joint work with I. Gasparis and A. Tolias, we have constructed a non reflexive analogue of the last result. The norms of these spaces are obtained by a general method which extends the Gowers Maurey method of constructing H.I. Banach spaces.

Gelfand numbers and metric entropy of convex hulls in Hilbert spaces

Bernd Carl

(joint work with David E. Edmunds)

For a precompact subset K of a Hilbert space we prove the following inequalities:

$$n^{1/2} c_n(\operatorname{cov}(K)) \leq c_K \Big(1 + \sum_{k=1}^{n} k^{-1/2} e_k(K)\Big)\ , \quad n \in \mathbb{N},$$

and

$$k^{1/2} c_{k+n}(\operatorname{cov}(K)) \leq c \left[(\log(n+1)^{1/2} \varepsilon_n(K) + \sum_{j=n+1}^{\infty} \frac{\varepsilon_j(K)}{j(\log(j+1))^{1/2}} \right] ,$$

$k, n \in \mathbb{N}$, where $c_n(\operatorname{cov}(K))$ is the n^{th} Gelfand number of the absolutely convex hull of K and $\varepsilon_k(K)$ and $e_k(K)$ denote the k^{th} entropy and k^{th} dyadic entropy number of K, respectively. The inequalities are, essentially, a reformulation of the corresponding inequalities given in Carl, Kyrezi and Pajor which yield asymptotically optimal estimates of Gelfand numbers $c_n(\operatorname{cov}(K))$ provided that the entropy numbers $\varepsilon_n(K)$ are slowly decreasing. For example, we get optimal estimates in the non-critical case where $\varepsilon_n(K) \preceq (\log(n+1))^{-\alpha},\ \alpha \neq \frac{1}{2}$, $0 < \alpha < \infty$, as well as in the critical case where $\alpha = \frac{1}{2}$. For $\alpha = \frac{1}{2}$ we show the asymptotically optimal estimate $c_n(\operatorname{cov}(K)) \preceq n^{-\frac{1}{2}} \log(n+1)$ which refines the corresponding result of Gao obtained for entropy numbers. Furthermore, we establish inequalities similar to that of Creutzig and Steinwart in the critical as well as non-critical case. Finally, we give an alternative proof of a result by Li and Linde for Gelfand and entropy numbers of the absolutely convex hull of K when K has the shape $K = \{t_1, t_2, \ldots\}$, where $\|t_n\| \leq \sigma_n$, $\sigma_n \downarrow 0$. In particular, for $\sigma_n \leq (\log(n+1))^{-\frac{1}{2}}$, which corresponds to the critical case we get a better asymptotic behaviour of Gelfand numbers, $c_n(\mathrm{cov}(K)) \preceq n^{-\frac{1}{2}}$.

Some remarks on purely unrectifiable sets

Marianna Csörnyei

(joint work with G. Alberti and D. Preiss)

A set A is called purely unrectifiable (PU) if it meets every Lipschitz curve in a set of length zero; A is uniformly purely unrectifiable ($PUPU$) if for every $\varepsilon > 0$ there exists an open set G covering A such that G meets every 1-Lipschitz curve in a set of length at most ε. (A set is called a 1-Lipschitz curve if it is a rotated copy of a graph of a Lipschitz function with Lipschitz constant at most 1). A basic open question is whether $PU = PUPU$.

The class of $PUPU$ sets is closely related to the class of sets of width 0: a set A has width 0 if there exists a convex cone C and for every $\varepsilon > 0$ there exists an open set $G \supset A$ such that every Lipschitz curve that goes in the direction of C intersects G in a set of length at most ε.

In the planar case the σ-ideal of the Lebesgue null sets and the σ-ideal generated by sets of width zero coincide. This leads to the following theorem: For any probability measure μ on $\mathbb{R}^2$,

(i) if μ is absolutely continuous with respect to the Lebesgue measure, then every Lipschitz map $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is differentiable almost everywhere with respect to μ;

(ii) if μ is singular then there exists a Lipschitz map $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ that is *non*-differentiable almost everywhere with respect to μ.

In fact for any probability measure μ on $\mathbb{R}^n$ there exists a Lipschitz map $f\colon \mathbb{R}^n \to \mathbb{R}^n$ that is *non*-differentiable almost everywhere with respect to μ, if and only if there are sets $A_1, A_2, \ldots$ of width zero with $\mu(\bigcup A_i) = 1$. However, in dimension greater than 2, it is still not known whether these are precisely the singular measures.

Maximal inequalities and almost sure convergence in L_p-spaces: commutative and non-commutative

Andreas Defant

The by now classical theory of general orthogonal series provides deep theorems on almost everywhere convergence. Probably the most prominent result is the fundamental Menchoff-Rademacher theorem: For an arbitrary orthonormal system (x_n) in $L_2(\mu)$, the Fourier series $\sum \lambda_k x_k$ converges almost everywhere provided $\sum |\lambda_k \log k|^2 < \infty$. It is well known that the latter condition can be weakened if other summation procedures are considered, for example Cesaro, or more generally, Riesz summation. Parts of the theory have been extended to L_p-spaces, $p \neq 2$. For instance, Maurey-Nahoum and independently Bennett proved that for each unconditionally convergent series $\sum x_k$ in L_1, the series $\sum \frac{x_k}{\log(k+1)}$ converges almost everywhere.

The aim of this talk is to present a sort of machinery which allows to transfer a variety of classical commutative maximal theorems on orthogonal series together with their implications on almost everywhere convergence into results which live in non-commutative L_p-spaces $L_p(M, \phi)$, $1 \leq p < \infty$, build over non commutative probability spaces (M, ϕ). We reformulate commutative L_2-results in terms of a maximal inequalities and use in a second step operator theory and the theory of tensor products in Banach spaces to derive appropriate commutative L_p-inequalities, $p \neq 2$. Now recent techniques based on Pisier's ℓ_∞-valued non-commutative L_p-spaces enable us to prove non-commutative L_p-maximal inequalities. In the final step, we use non-commutative Chebycheff type inequalities in order to deduce the anticipated almost everywhere results.

Two examples: (1) For every unconditionally convergent series $\sum x_k$ in $L_1(M, \phi)$ (the predual of M) there is a summation process given by a lower triangle matrix (a_{jk}) such that $\sum a_{jk} x_k$ converges to $\sum x_k$ bilaterally almost surely in the sense of Jajte. This is a natural analogue of a result on orthogonal series due to Zygmund, and even in the commutative case it seems to be unknown. (2) Given an orthonormal series $\sum \lambda_k x_k$ in a non-commutative $L_2(M, \phi)$, the Cesaro means of the partial sums of this series bilaterally almost surely sum $\sum \lambda_k x_k$ provided $\sum |\lambda_k \log \log k|^2 < \infty$. In the commutative situation this is another fundamental result due to Kaczmarz and Menchoff. This talk reports on joint work with Marius Junge and continues our previous work from: Maximal theorems of Menchoff-Rademacher type in non-commutative L_p-spaces; to appear in J. Funct. Anal. 2003.

Non-linear geometry of Banach spaces

Gilles Godefroy

The space of real-valued Lipschitz functions on a Banach space X is a dual space in a canonical way. The predual, which enjoys nice functorial properties, is called the Lipschitz-free space over X. In a joint work with N. J. Kalton, we use this space to prove the following: if a linear quotient map from a Banach space Y onto a separable Banach space X has a Lipschitz right inverse, then it has a linear continuous right inverse. The proof relies on convolution with smooth measures and weak Gateaux differentiability. Separability is essential in this result, since it fails e.g. for every non separable WCG space X. This provides canonical examples of pairs of non separable spaces which are Lipschitz-isomorphic but not linearly isomorphic. Another corollary is the following: if there is an isometric embedding from a separable Banach space X into a Banach space Y, then there is a linear isometric embedding from X into Y. This last statement fails as well for every non separable WCG (e.g. reflexive) space.

Localization method in geometry of convex bodies

OLIVIER GUEDON
(joint work with Matthieu Fradelizi)

In this talk, I have presented a joint work with Matthieu Fradelizi concerning the study of extreme points of the set of s-concave probabilities measures satisfying a linear constraint. This work gives a new approach to a localization theorem due to Kannan, Lovász and Simonovits which happens to be very useful in geometry to obtain inequalities for integrals like concentration and isoperimetric inequalities. Roughly speaking, the study of such inequalities is reduced to these extreme points and the theorem states that these extreme points are some Dirac measures and some s-affine probabilities supported by a segment. This tool appeared in the literature as very powerful to get dimension free inequalities for log-concave measures.

I have also presented a generalization of this result to the study of the extreme points of the set of log-concave probabilities satisfying p linear constraints. They are some Dirac measures, some log-concave probabilities supported by a segment with a density equal to $e^{\max_{i=1,\dots,p} \ell_i}$ where ℓ_i are some affine functions, some log-affine probabilities supported in a p-dimensional subspace and it remains an open question to characterize these extreme points when the support of the measure generates a k-dimensional affine subspace, when $2 \le k \le p-1$.

A quantitative version of Krein's theorem

PETR HAJEK
(joint work with M. Fabian, A. Granero, V. Montesinos, and V. Zizler)

We say that a bounded subset $M \subset X$, where X is a Banach space, is ϵ-weakly relatively compact (ϵ-WRK), if $S = w^* - \mathrm{cl}(M)$ satisfies $S \subset X + \epsilon B_{X^{**}}$. In our work we show the following theorem:

Theorem 1 *Let $M \subset X$ be bounded, ϵ-WRK. Then $co(M)$ is 2ϵ-WRK subset of X.*

The statement of the above theorem when $\epsilon = 0$ coincides with the classical Krein's theorem. In some cases (e.g. WCG spaces) it is possible to preserve the WRK constant when passing to convex hulls, regardless of the the set M and the renorming of X. However, in general this is false and we give examples of sets in l_∞ for which it is necessary to add the constant 2 when passing to the convex hull. We can also demonstrate renormings of the space $l_1([0,1]) \oplus c_0$ under which we preserve the WRK constant for every bounded set M, or different renormings under which passing to the double value for convex hulls is unavoidable for some sets M.

Non-Equivalence of Rearranged Walsh and Trigonometric Systems in L_p

Aicke Hinrichs
(joint work with Jörg Wenzel)

The subject of the talk is the question whether the trigonometric system can be equivalent to some rearrangement of the Walsh system in L_p for some $p \neq 2$. It is shown that this problem is closely related to a combinatorial problem involving the permutation and the structure of the underlying groups. This makes it possible to prove non-equivalence of trigonometric system and rearranged Walsh-system for $p \neq 2$ for a number of rearrangements including the original Walsh order, the Walsh-Kaczmarz order, general dyadically linear and piecewise linear rearrangements and perturbations thereof. Previously, this was only known for the Walsh-Paley order. The method is not limited to the case of the Walsh functions but can be used for systems of characters on compact abelian groups.

Carleson Embeddings for Weighted Bergman Spaces

Hans Jarchow
(joint work with Urs Kollbrunner)

We consider analytic functions on $D = \{z \in \mathbf{C} : |z| < 1\}$. A q-Carleson measure μ for the classically weighted Bergman space A^p_α is a finite, positive Borel measure μ on D such that the operator $I : A^p_\alpha \to L^q(\mu) : f \mapsto f$ is well-defined ('Carleson embedding'). Typical examples come from e.g. pointwise multipliers, composition operators, ... Such measures have been characterized by D.H. Luecking and others in terms of properties of the function $z \mapsto \dfrac{\mu(B(z))^{1/q}}{(1-|z|^2)^{(\alpha+2)/p}}$. Here $B(z)$ is the hyperbolic ball centered at z of radius $1/2$, say. For several operator theoretic properties, the cases $p \leq q$ and $p > q$ are very different. For example, if $p > q$ then I is always compact. But there is no difference for order boundedness and certain related summing properties. Order boundedness of $I : A^p_\alpha \to L^q(\mu)$ is equivalent to μ-integrability of $z \mapsto (1-|z|^2)^{-(\alpha+2)q/p}$. It leads to extensions of I to certain larger function spaces. In many cases it is equivalent to the property that a related Carleson embedding $A^2_{\alpha'} \to L^2(\mu)$ is a Hilbert-Schmidt operator.

Stochastic approximation property

William B. Johnson

(joint work with V. P. Fonf, G. Pisier, and D. Preiss)

We show that a Banach space X has the stochastic approximation property iff it has the stochasic basis property, and these properties are equivalent to the approximation property if X has non trivial type. If for every Radon probability on X, there is an operator from an L_p space into X whose range has probability one, then X is a quotient of an L_p space. This extends a theorem of Sato's which dealt with the case $p = 2$. In any infinite dimensional Banach space X there is a compact set K so that for any Radon probability on X there is an operator range of probability one that does not contain K.

BMO-regularity is a self-dual property

S.V. Kislyakov

A Banach lattice X of measurable functions (ideal space) on the unit circle is said to be BMO-regular if for every nonzero function f in X there exists $g \in X$ with

$$|f| \leq g, \|g\| \leq C\|f\|, \|\log g\|_{\mathrm{BMO}} \leq C.$$

This property has turned out to be intimately related to interpolation of Hardy-type subspaces in lattices of measurable functions.

It is shown in the talk that, if X has the Fatou property and X is BMO-regular, then the order dual X' is also BMO-regular. Surprisingly, the proof involves the Ky Fan–Kakutany fixed point theorem for multivalued maps.

Minimal slabs in the cube

Alexander Koldobsky

(joint work with Franck Barthe)

We study the volume of symmetric slabs in the unit cube. We show that, for $t < 3/4$, the slab parallel to a face has the minimal volume among all symmetric slabs with width t. For large width, we prove the asymptotic extremality of the slab orthogonal to the main diagonal. The proof is based on certain concavity properties of the Laplace transform and on several limit theorems from probability: the central limit theorem and classical principles of moderate and large deviations. Finally, we extend some of the results to more general classes of bodies.

Frechet differentiability of Lipschitz functions between Banach spaces

JORAM LINDENTRAUSS

There are two important types of differentiability for Lipschitz functions between infinite dimensional Banach spaces. For Gâteaux differentiability of such functions there is a satisfactory general existence theorem. However, Gâteaux derivatives are only weak linear approximations of a function. It is much more desirable to have points of Frechet differentiability. It turns out that it is very hard to prove the existence of such points. One source of the difficulty is to find a proper notion of "almost everywhere". In a joint recent work with David Preiss we introduced a new notion of null sets which enables to prove existence results for points of Frechet differentiability in some important special cases (e.g. convex continuous functions from an Asplund space to the reals and any Lipschitz function from a C(K) space with K countable to a space with the RNP).

On Banach spaces with the commuting bounded approximation property

WOLGANG LUSKY

We consider separable Banach spaces X with a sequence of bounded linear finite rank operators $R_n : X \to X$ which satisfy $\lim_{n\to\infty} R_n x = x$ for all $x \in X$ and $R_n R_m = R_{\min(n,m)}$ whenever $n \neq m$. Such a sequence will be called a commuting approximating sequence (c.a.s.). We study under which additional conditions on R_n the space X has a basis or even an unconditional basis. For example, if the operators $R_n - R_{n-1}$ factor uniformly through $l_p^{m_n}$-spaces then X has a basis. If in addition there is $\lambda > 0$ such that, for any sequence of indices k_n with $k_n \neq k_{n'}$ whenever $n \neq n'$ and any linear $U_n : X \to X$ with $||U_n|| \leq 1$, we have $|| \sum_n (R_{k_n} - R_{k_n-1}) U_n (R_n - R_{n-1}) || \leq \lambda$ then X has an unconditional basis. If X is a subspace of a $\mathcal{L}_p$-space Z such that $Z = \overline{X+Y}$ for some subspace $Y \subset Z$, $(\hat{R}_n)$ is a c.a.s. of Z with $\hat{R}_n|_X = R_n$ and $\hat{R}_n|_Y$ are the projections of a basis of Y then $X \oplus l_p$ has a basis. We apply these results to certain subspaces of $C(\mathbb{T})$ and $L_1(\mathbb{T})$ of the form $\overline{\text{span}}\{ \ z^k \ : \ k \in \Lambda \ \}$, where $\mathbb{T} = \{ \ z \in \mathbb{C} \ : \ |z| = 1 \ \}$ and $\Lambda \subset \mathbb{Z}$ is a given subset.

Reflexivity and approximate fixed points

EVA MATOUSKOVA

A Banach space X is reflexive if and only if every bounded sequence $\{x_n\}$ in X contains a norm attaining subsequence. This means that it contains a subsequence $\{x_{n_k}\}$ for which $\sup_{f \in S_{X^*}} \limsup_{k\to\infty} f(x_{n_k})$ is attained at some f in the dual unit sphere S_{X^*}. A Banach space X is *not* reflexive if and only if it contains a normalized sequence $\{x_n\}$ with the property that for every $f \in S_{X^*}$, there exists $g \in S_{X^*}$ such that $\limsup_{n\to\infty} f(x_n) < \liminf_{n\to\infty} g(x_n)$.

Combining this with a result of Shafrir, we conclude that every infinite-dimensional Banach space contains an unbounded closed convex set which has the approximate fixed point property for nonexpansive mappings.

$L \log L$ spaces and convergence of Fourier series associated with 1D periodic Dirac operators

Boris Mityagin

We study Dirac operators on the interval $[0, 1]$ with periodic, antiperiodic or Dirichlet boundary conditions. For any compact set of potentials in L^2 we present a domain in $\mathbb{C}$ (a rectangle and a sequence of small discs centered at $k\pi$, k integers), which contains spectra of Dirac operators. Spectral decompositions converge in $L^p, 1 < p < \infty$ if a potential lies in L^2. Under more strict conditions on the potential we guarantee unconditional L^2-convergence and uniform convergence of spectral decompositions of Dirac operator.

Conditioned Brownian Motion and Multipliers into SL^∞

Paul F. X . Müller

(joint work with Peter W. Jones)

We show how to *construct* bounded harmonic functions which obey *pointwise constraints* and also satisfy *uniform estimates* for their Littlewood - Paley square function: Given a measurable set E, contained in the boundary of the unit disk $\mathbb{T}$, we constructively determine a multiplier $m : \mathbb{T} \to [0, 1]$ with the following property:

$$h = m \cdot 1_E$$

is non trivial, and its mean value satisfies the following lower bound,

$$\int h(e^{i\alpha})d\alpha \geq e^{-1} \int u(e^{i\alpha})d\alpha,$$

h is also smooth enough so that its Littlewood-Paley square function is uniformly bounded, that is

$$\sup_\alpha \left(\frac{1}{\pi} \int_{\mathbb{D}} |\nabla h(z)|^2 \frac{1 - |z|^2}{|e^{i\alpha} - z|^2} \log \frac{1}{|z|} dA(z) \right)^{1/2} \leq A_0$$

where A_0 is a universal constant. A large proportion of the presentation was devoted to finding an explicite formula for the multiplier $m : \mathbb{T} \to [0, 1]$.

On some operator property related to power-boundedness

Krysztof Oleszkiewicz

(joint work with N. Kalton, S. Montgomery-Smith and Y. Tomilov)

Let $T : F \to F$ be a bounded operator on a Banach space F. Let

$$L = \limsup_{n \to \infty} n \|T^{n+1} - T^n\|.$$

We prove that $L < 1/e$ implies that T is power bounded and that the constant $1/e$ cannot be improved. We also prove an Esterle type theorem: Let $C = \liminf_{n\to\infty} n\|T^{n+1} - T^n\|$. If both $C < 1/e$ and spectrum of T is equal to $\{1\}$ then $T = Id$. The constant $1/e$ again is optimal (Esterle and Berkani proved it for $C < 1/96$ and $C < 1/12$ respectively).

Achievements and failures of operator theory in Banach spaces

Albrecht Pietsch

This historical survey deals with the following topics:

- finite-dimensional pattern from linear algebra:
  diagonal form, Jordan form, triangular form, Cramer's rule
- infinite-dimensional phenomena:
  non-complemented subspaces, spaces without basis,
  quasi-nilpotent operators
- Riesz–Schauder theory of compact operators
- Fredholm operators
- completeness of root vectors
- invariant subspaces
- spectral operators, decomposable operators
- Fredholm determinants
- eigenvalue distributions
- the relationship between spaces and operators

Random ± 1 matrices generate Kashin's subspaces

Mark Rudelson
(joint work with A. Litvak, A. Pajor and N. Tomczak-Jaegermann)

Let $\delta > 0$ and let m, n be integer numbers such that $m < (1-\delta)n$. Kashin's theorem states that if $S : \mathbb{R}^n \to \mathbb{R}^n$ is a random orthogonal matrix, then with high probability the L_1 and L_2 norms are equivalent on the subspace $E = S\mathbb{R}^m$. There was a question whether a random orthogonal matrix can be replaced by a matrix of a simpler structure, in particular by one having a small number of different entries. We prove that if S is a matrix whose entries are i.i.d. random variables taking values 1 and -1 with probability $1/2$, then the subspace $S\mathbb{R}^m$ satisfies Kashin's theorem with probability close to 1.

Strictly convex renorming

Stanmir Troyanski

(joint work with A. Moltó and J. Orihuela)

For a set A by $D(A)$ we denote the diagonal of A^2, i.e. $D(A) = \{(x, x) : \ x \in A\}$.

Definition. Let X be a linear topological space and let M be a symmetric subset of X^2, i.e. if $(x, y) \in M$ then $(y, x) \in M$. We say that M is

a) *quasidiagonal* if $x = y$ whenever (x, x), $(y, y) \in \overline{\text{conv } M}$.

b) *strictly quasidiagonal* if $x = y$ whenever either $(x, x) \in \overline{\text{conv } M}$ or $(y, y) \in \overline{\text{conv } M}$.

Theorem. *Let X be a normed space and F a subspace of X^* which is 1–norming for X. Let us consider the following conditions*

(i) the norm of X is strictly convex;

(ii) S_X^2 is a countable union of strictly quasidiagonal sets with respect to $(X, \sigma(X, F))$;

(iii) S_X^2 is a countable union of quasidiagonal sets with respect to $(X, \sigma(X, F))$;

(iv) X^2 is a countable union of quasidiagonal sets with respect to $(X, \sigma(X, F))$;

(v) X admits an equivalent $\sigma(X, F)$ strictly convex norm.

Then (i)⇒(ii)⇒ (iii)⇔(iv)⇔(v).

Using the above theorem we obtain some necessary and some sufficient condition for the existence of dual strictly convex norm in $C(K)^*$ in terms of the compact K.

Restricted Invertibility of Operators on Random Subspaces

Roman Vershynin

The principle of the restricted invertibility due to Bourgain and Tzafriri finds a large set of isomorphism of a given linear operator T on l_2^n, that is a coordinate subspace on which T is a nice isomorphism. A natural question is - how many such sets (subspaces) are there? In particular, can this subspace be chosen at random? I will present a sharp positive result. It is dimension free and more general than Bourgain-Tzafriri's principle.

Narrow operators on Banach spaces

Dirk Werner

(joint work with V. Kadets, N. Kalton and R. Shvidkoy)

An operator $T : X \to Y$ between Banach spaces is called narrow if for every $\varepsilon > 0$, for every slice S of the unit ball of X, every x in the unit sphere of X and every $y \in S$ there exists some $v \in S$ such that $\|x - v\| \geq 2 - \varepsilon$ and $\|Ty - Tv\| \leq \varepsilon$. A Banach space X has the Daugavet property if $\|\mathrm{Id} - T\| = 1 + \|T\|$ for every rank-1 operator $T : X \to X$. It can easily be shown that X has the Daugavet property if and only if there exists a narrow operator on X and that on a space with the Daugavet property, every narrow operator satisfies $\|\mathrm{Id} - T\| = 1 + \|T\|$.

The talk at the meeting discussed examples of narrow operators (e.g., weakly compact operators, Radon-Nikodym operators, ℓ_1-singular operators and their sums), characterisations of such operators on $C(K)$- and L_1-spaces, applications to unconditional expansions (leading to another argument that L_1 does not sign-embed into a space with an unconditional basis), and counterexamples, based on a construction by Bourgain and Rosenthal, that show that a completely continuous operator need not be narrow.

Geometry of weakly Lindelöf determined spaces

Vaclav Zizler

(joint work with M. Fabian, G. Godefroy, P. Hájek and V. Montesinos)

A Banach space X is weakly Lindelöf determined if its dual ball in its weak star topology is homeomorphic to a set S in some $[-1, 1]^\Gamma$ so that every element of S is countably supported on Γ (Corson compacts). In this class of Banach spaces we give a characterization of spaces that are subspaces of weakly compactly generated spaces in two ways.

First, by using a countable splitting biorthogonal systems in such spaces and second, by using a certain uniformity in the directions of Gâteaux derivatives of norms on such spaces.

This is used in giving new short elementary proofs to several results on Eberlein compacts and on uniform Eberlein compacts (sets homeomorphic to weak compact sets in Banach spaces, resp. Hilbert spaces). This applies to the result on continuous images and to the result on containment of dense metrizable subsets in such compacts.

The methods can be used for other spaces like Vašák spaces, Hilbert generated spaces and their subspaces, etc.

Edited by Carsten Schütt

Participants

Dr. Razvan Anisca
anisca@math.ualberta.ca
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1
Canada

Prof. Dr. Spiros A. Argyros
sargyros@math.ntua.gr
Department of Mathematics
National Technical University
of Athens
Zografou Campus
15780 Athens - Greece

Dr. Shiri Artstein
artst@post.tau.ac.il
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040
Tel Aviv 69978 – Israel

Prof. Dr. Ehrhard Behrends
behrends@math.fu-berlin.de
Institut für Mathematik II (WE2)
Freie Universität Berlin
Arnimallee 2-6
D–14195 Berlin

Prof. Dr. Klaus Dieter Bierstedt
klausd@uni-paderborn.de
Institut für Mathematik
Universität Paderborn
Warburger Str. 100
D–33098 Paderborn

Prof. Dr. Bernd Carl
carl@minet.uni-jena.de
Fakultät für Mathematik und
Informatik
Friedrich-Schiller-Universität
Ernst-Abbe-Platz 1-4
D–07743 Jena

Dr. Marianna Csörnyei
mari@math.ucl.ac.uk
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT

Prof. Dr. Andreas Defant
Defant@math.uni-oldenburg.de
Fachbereich 6 Mathematik
Carl von Ossietzky
Universität Oldenburg
D–26111 Oldenburg

Prof. Dr. Tadeusz Figiel
t.figiel@impan.gda.pl
Instytut Matematyczny
Polskiej Akademii Nauk
ul. Abrahama 18
81-825 Sopot – Poland

Prof. Dr. Vladimir Fonf
fonf@black.bgu.ac.il
fonf@math.bgu.ac.il
Dept. of Mathematics
Ben-Gurion University of the Negev
Beer Sheva 84 105 - Israel

Prof. Dr. Gilles Godefroy
gig@ccr.jussieu.fr
Equipe d'Analyse; Boite 186
Université Pierre et Marie Curie
(Université Paris VI)
4, Place Jussieu
F-75252 Paris Cedex 05

Prof. Dr. Yehoram Gordon
gordon@tx.technion.ac.il
Department of Mathematics
Technion
Israel Institute of Technology
Haifa 32000 – Israel

Dr. Olivier Guedon
guedon@ccr.jussieu.fr
Institut de Mathématiques
Projet Analyse Fonctionnelle
Boite 186, Université Paris 6
4, place Jussieu
F-75005 Paris

Prof. Dr. Petr Hajek
hajek@math.cas.cz
Institute of Mathematics of the
AV CR
Zitna 25
115 67 Praha 1 - Czech Republic

Dr. Aicke Hinrichs
hinrichs@minet.uni-jena.de
Mathematisches Institut
Universität Jena
Ernst-Abbe-Platz 1-4
D–07743 Jena

Prof. Dr. Hans Jarchow
jarchow@math.unizh.ch
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Prof. Dr. William B. Johnson
johnson@math.tamu.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368 – USA

Prof. Dr. Sergei V. Kisliakov
skis@pdmi.ras.ru
St. Petersburg Branch of
Mathematical Institute of
Russian Academy of Science
Fontanka 27
191011 St. Petersburg – Russia

Prof. Dr. Alexander Koldobsky
koldobsk@math.missouri.edu
Dept. of Mathematics
University of Missouri-Columbia
Columbia, MO 65211 - USA

Prof. Dr. Hermann König
hkoenig@math.uni-kiel.de
Mathematisches Seminar
Universität Kiel
Ludewig-Meyn-Str. 4
D–24118 Kiel

Rafal Latala
rlatala@mimuw.edu.pl
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2
02-097 Warszawa - Poland

Prof. Dr. Joram Lindenstrauss
joram@math.huji.ac.il
Mathematics Department
Hebrew University
Givat Ram
Jerusalem 91904 - Israel

Prof. Dr. Alexander Litvak
alexandr@math.ualberta.ca
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1- Canada

Prof. Dr. Wolfgang Lusky
lusky@uni-paderborn.de
Institut für Mathematik
Universität Paderborn
Warburger Str. 100
D–33098 Paderborn

Prof. Dr. Piotr Mankiewicz
piotr@impan.gov.pl
Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 137
ul. Sniadeckich 8
00-950 Warszawa - Poland

Prof. Dr. Eva Matouskova
matouse@matsrv.math.cas.cz
Institute of Mathematics of the
AV CR
Zitna 25
115 67 Praha 1 – Czech Republic

Prof. Dr. Boris Mityagin
borismit@math.ohio-state.edu
Department of Mathematics
Ohio State University
231 West 18th Avenue
Columbus, OH 43210-1174 - USA

Dr. Paul F.X. Müller
pfxm@bayov.uni-linz.ac.at
Institut für Mathematik
Universität Linz
Altenberger Str. 69
A-4040 Linz

Prof. Dr. Assaf Naor
anaor@microsoft.com
Microsoft Research
One Microsoft Way
Redmond, WA 98052-6399 - USA

Prof. Dr. Niels Jorgen Nielsen
njn@imada.sdu.dk
Department of Mathematics and
Computer Science
University of Southern Denmark
Campusvej 55
DK-5230 Odense M

Prof. Dr. Edward Odell
odell@math.utexas.edu
odell@mail.ma.utexas.edu
Dept. of Mathematics
University of Texas at Austin
RLM 8.100
Austin, TX 78712-1082 - USA

Prof. Dr. Krzysztof Oleszkiewicz
koles@mimuw.edu.pl
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2
02-097 Warszawa – Poland

Prof. Dr. Aleksander Pelczynski
olek@impan.impan.gov.pl
Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 137
ul. Sniadeckich 8
00-950 Warszawa - Poland

Prof. Dr. Albrecht Pietsch
pietsch@minet.uni-jena.de
Mathematisches Institut
Universität Jena
Ernst-Abbe-Platz 1-4
D–07743 Jena

Prof. Dr. David Preiss
dp@math.ucl.ac.uk
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT

Prof. Dr. Mark Rudelson
rudelson@math.missouri.edu
Dept. of Mathematics
University of Missouri-Columbia
Columbia, MO 65211 - USA

Prof. Dr. Thomas Schlumprecht
schlump@math.tamm.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368 - USA

Dr. Carsten Schütt
schuett@math.uni-kiel.de
Mathematisches Seminar
Universität Kiel
Ludewig-Meyn-Str. 4
D–24118 Kiel

Prof. Dr. Nicole Tomczak-Jaegermann
nicole@ellpspace.math.ualberta.ca
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1 - Canada

Prof. Dr. Stanimir Troyanski
stroya@um.es
Departamento de Matematicas
Facultad de Matematicas
Universidad de Murcia
Campus de Espinardo
E-30100 Espinaro

Prof. Dr. Roman Vershynin
rvershynin@math.ualberta.ca
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1 – Canada

Prof. Dr. Dirk Werner
werner@math.fu-berlin.de
Institut für Mathematik II (WE2)
Freie Universität Berlin
Arnimallee 2-6
D–14195 Berlin

Prof. Dr. Przemyslaw Wojtaszczyk
wojtaszczyk@mimuw.edu.pl
Institute of Applied Mathematics
University of Warsaw
ul.Banacha 2
02-097 Warszawa - Poland

Prof. Dr. Vaclav Zizler
vzizler@math.ualberta.ca
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1 - Canada