Mathematisches Forschungsinstitut Oberwolfach

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Profinite Groups and Discrete Subgroups of Lie Groups

May 18th – May 24th, 2003

This workshop was organised by F. Grunewald (Düsseldorf), A. Lubotzky (Jerusalem), and D. Segal (Oxford). There were 46 participants, coming from at least 15 different countries. (In some cases the present email address is not a reliable indicator of a person's origin.)

As reflected by the 35 selected presentations, a whole range of topics were discussed. Among these featured: subgroup growth and variations, self-similar groups, congruence subgroup problems, absolute Galois groups, lattices in Lie groups, and 3-manifold groups.

The following abstracts contain more detailed information about the various contents of the talks.

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Diese Tagung stand unter der Leitung von F. Grunewald (Düsseldorf), A. Lubotzky (Jerusalem) und D. Segal (Oxford). An ihr nahmen 46 Mathematikerinnen und Mathematiker aus mindestens 15 verschiedenen Ländern teil. (In einigen Fällen liefert die momentane Email-Adresse keinen brauchbaren Hinweis auf die Herkunft einer Person.)

Über eine große Anzahl von Themen wurde diskutiert und in den 35 ausgewählten Vorträgen berichtet. Im Mittelpunkt standen unter anderem: Untergruppenwachstum und Variationen, selbstähnliche Gruppen, Kongruenzuntergruppenprobleme, absolute Galoische Gruppen, Gitter in Lieschen Gruppen und Fundamentalgruppen von 3-Mannigfaltigkeiten.

Eine detailliertere Beschreibung der einzelnen Vorträge erfolgt in den nachstehenden Zusammenfassungen.

Abstracts

Fuchsian groups, Riemann surfaces, subgroup growth, random quotients, and random walks

A. Shalev

(joint work with M. Liebeck)

We use character-theoretic and probabilistic methods to study spaces of homomorphisms from a Fuchsian group Γ to symmetric groups $\mathrm{Sym}(n)$. This leads to a variety of results, such as

(1) counting index n coverings of Riemann surfaces with given ramification types,
(2) counting index n subgroups of Fuchsian groups: it follows from our results that $\log a_n(\Gamma)/\log n! \to \mu(\Gamma)$, where $\mu(\Gamma) = -\chi(\Gamma)$ is the measure of Γ,
(3) showing that almost all index n subgroups are maximal,
(4) giving a probabilistic proof of G. Higman's conjecture that Γ surjects onto $\mathrm{Alt}(n)$ for all large n,
(5) solving a similar problem for symmetric quotients $\mathrm{Sym}(n)$,
(6) finding the mixing time of some random walks on $\mathrm{Sym}(n)$ with conjugacy classes as generating sets.

On the structure of analytic groups over pro-p rings other than $\mathbb{Z}_p$

B. Klopsch

(joint work with A. Jaikin-Zapirain)

Some time ago A. Jaikin-Zapirain and I proved: if R is a pro-p ring (i.e. an infinite commutative noetherian complete local ring $(R, \mathfrak{m})$ such that the residue field $R/\mathfrak{m}$ is finite of characteristic p and the graded ring $\mathrm{gr}(R) = \bigoplus \mathfrak{m}^i/\mathfrak{m}^{i+1}$ is an integral domain) and G a finitely generated compact R-analytic group satisfying some group identity, then G is p-adic analytic. If $pR = 0$, e.g. if $R = \mathbb{F}_p[\![t]\!]$, this can be regarded as a special instance of the following conjecture: if R is a pro-p ring of characteristic p and G a finitely generated compact R-analytic group, then G does not map onto any non-discrete p-adic analytic group. In my talk I reported on our present efforts to prove this conjecture under the extra assumption that $R = \mathbb{F}_p[\![t]\!]$.

Uniform bounds for finite groups and closed subgroups of profinite groups

D. Segal

(joint work with N. Nikolov)

The main results on finite groups are the following "uniformity" theorems.

(1) In any finite d-generated group, any element of the derived group is a product of $g(d)$ commutators.
(2) In any finite d-generated group G, for any $q \in \mathbb{N}$, every element of G^q is a product of $f(c, d, q)$ qth powers, where c is maximal such that $\mathrm{Alt}(c)$ is involved in G.

(Here f and g are universal functions, $g(d) = 12d^3 + O(d^2)$. We conjecture that $f(c, d, q)$ is independent of c.)

The motivation for this is the following family of applications to profinite groups.

(3) In any finitely generated profinite group, the (algebraic) derived group is closed.
(4) (Corollary) In any finitely generated profinite group (a) each term of the lower central series is closed, (b) every normal subgroup of odd finite index is open.
(5) In any finitely generated non-universal profinite group, every subgroup of finite index is open. (If the above conjecture is true, we can remove "non-universal" here.)—G is non-universal if there exists a finite group that is not a (closed) section of G.

The hard part of the proof is in the finite group theory. One of the facts used is the following
THEOREM. *There is an absolute constant D such that for every finite quasi-simple group S and any $2D$ automorphisms $\alpha_1, \beta_1, \dots, \alpha_D, \beta_D$ of S, we have $S = \prod_{i=1}^{D} T_{\alpha_i,\beta_i}(S, S)$ where $T_{\alpha,\beta}(x, y) = x^{-1}y^{-1}x^{\alpha}y^{\beta}$.*

Character theory of symmetric groups, subgroup growth of Fuchsian groups, and random walks

J.-C. SCHLAGE-PUCHTA
(joint work with T. Müller)

Let Γ be a group of the form

$$\Gamma = \langle x_1, \dots, x_r, y_1, \dots, y_s, u_1, v_1, \dots, u_t, v_t \mid x_1^{a_1} = \dots = x_r^{a_r} = x_1x_2\dots x_r y_1^{e_1} y_2^{e_2} \dots y_s^{e_s}[u_1, v_1]\dots[u_t, v_t] = 1\rangle.$$

Let $s_n(\Gamma)$ be the number of index n subgroups of Γ. We determine the asymptotic behaviour of $s_n(\Gamma)$ and compute several explicit examples. Our result depends on estimates for the character values and the multiplicities of root number functions, which are essentially optimal. As a by-product, the mixing time for all random walks on $\mathrm{Sym}(n)$ generated by a conjugacy class was determined.

Profinite topologies and free products of groups

L. RIBES
(joint work with P. Zalesskii)

Let $\mathcal{C}$ be a class of finite groups closed under taking subgroups, quotients, and extensions (e.g. the class of all finite soluble groups). We say that a group A, endowed with its pro-$\mathcal{C}$ topology, is 2-product subgroup separable (with respect to this topology) if, whenever H_1 and H_2 are finitely generated closed subgroups of A, then the set H_1H_2 is closed in A.
THEOREM. *The free product of finitely many 2-product subgroup separable groups is 2-product subgroup separable.*

On (non-)linearity of free pro-p groups

E. ZELMANOV

I discussed the proof of the following
THEOREM. *A (non-abelian) free pro-p group is not representable by $n \times n$-matrices for $p \gg n$.*

It means that for all such p, n the group $\mathrm{GL}^1(n, \Lambda)$ satisfies a non-trivial pro-p identity.

Introduction to Grigorchuk's group

C. Leedham-Green

An elementary introduction to the most famous of the Grigorchuk groups was given, in the hope that this would be of use for later speakers. Properties of pro-p groups may correspond to properties of classes of finite p-groups. Thus the study of pro-p groups of finite width relates to the study of classes of finite p-groups of bounded width. The fundamental property of a pro-p group of being just-infinite does not easily translate to a property of finite p-groups; however, the property of having finite obliquity does. Branch groups do not have finite obliquity, so the apparent impossibility of classifying the just-infinite branch groups does not hinder the still impossible aim of classifying all pro-p groups of finite width and finite obliquity.

A fairly new proof that Grigorchuk's group has finite width

S. McKay

Let A be the automorphism group of the infinite rooted 2-ary tree, and $M(\rho)$ be the stabiliser of all vertices at level ρ. Define $A(\rho, i) = (M(\rho) \cap \gamma_i(A))M(\rho + 1)$ so that $A = A(0,1) \subseteq A(1,1) \subseteq A(1,2) \subseteq A(2,1) \subseteq \dots$ is a descending series with quotients of order 2.

Let G be the Grigorchuk's group. A lower central diagram for G has k in the (ρ, i) position if there exists $g \in \gamma_k(G) \setminus \gamma_{k+1}(G)$ such that $g \in A(\rho, i) \setminus A(\rho, i + 1)$. Then $|\gamma_k(G) : \gamma_{k+1}(G)| = 2^m$ where m is the number of k's in this diagram. It can be shown that the (ρ, i) entry is i for $2^{\rho-1} < i \leq 2^\rho$ and is $i - 2^{\rho-3}$ for $2^{\rho-1} - 2^{\rho-3} < i \leq 2^{\rho-1}$, and empty otherwise. The proof uses the facts that G embeds naturally in $G \wr C_2$ as a subgroup of index 8 and that G acts uniserially on $M(\rho)/M(\rho + 1)$ for all ρ.

It follows easily that G has Hausdorff dimension 5/8 in A, that G has finite width (3 in fact), that G is just-infinite, and that G has finite obliquity.

Zeta functions of groups and enumeration in Bruhat-Tits buildings

C. Voll

We introduce a new method to calculate the local factors of the normal zeta function of nilpotent groups. It is based on an enumeration of vertices in the Bruhat-Tits building for $\mathrm{SL}_n(\mathbb{Q}_p)$. We show how this method can be used to explicitly compute the factors of so-called *non-uniform* zeta functions, encoding the arithmetic of plane curves and—more generally—Pfaffian hypersurfaces. Such groups were first constructed by du Sautoy.

Our translation of subgroup growth into enumeration in Bruhat-Tits buildings is also powerful enough to explain *functional equations* satisfied by the normal zeta functions. These have two sources: We explain how the functional equation of the Hasse-Weil zeta function associated to the Pfaffian hypersurface matches up with a symmetry in the building, first described by Igusa, to give rise to functional equations of the local normal zeta functions. These were conjectured by du Sautoy.

The zeta function of representations of pro-p groups

A. Jaikin-Zapirain

We say that a pro-p group G is perfect if all terms of its derived series are open, or equivalently if $\lambda_i(G) := |\{\lambda \in \mathrm{Irr}(G) | \lambda(1) = p^i\}| < \infty$ for all $i \geq 0$, where $\mathrm{Irr}(G)$ denotes the set of characters of all irreducible smooth complex representations of G. For any perfect pro-p group G consider

$$\zeta_G^{\mathrm{ch}}(s) := \sum_{i=0}^{\infty} \lambda_i(G) p^{-is} = \sum_{\lambda \in \mathrm{Irr}(G)} |\lambda(1)|^{-s}.$$

Main Theorem. *Let G be a perfect p-adic analytic pro-p group. If G is a uniform pro-p group or $p > 2$, then $\zeta_G^{\mathrm{ch}}(s)$ is a rational function in p^s.*

The proof of this theorem is based on a correspondence constructed by R. Howe between the characters of a uniform pro-p group and the orbits of the action of the group on the dual of its Lie algebra. The correspondence is quite explicit and gives an exact formula for the number of characters in some cases. From this we obtain

Theorem. *Let $p \geq 3$ and let N be a uniform pro-p group; if $p = 3$ add the stronger assumption that $[G, G] \leq G^{p^2}$. Then for any $g \in N$,*

$$\sum_{\lambda \in \mathrm{Irr}(G)} \lambda(g) |\lambda(1)|^{-s}$$

is a rational function in p^s.

From this it follows that for any perfect p-adic analytic pro-p group G there exists a constant $c = c(G)$ such that $\lambda_i(G) \leq p^{ci}$ for all i. This conclusion holds also for other known perfect pro-p groups.

Theorem. *Let G be either the Nottingham group or an $\mathbb{F}_p[[t]]$-analytic pro-p group whose associated Lie algebra is perfect. Then here exists c such that $\lambda_i(G) \leq p^{ci}$ for all i.*

Representation growth zeta functions for finitely generated nilpotent groups

B. Martin

(joint work with E. Hrushovski, A. Lubotzky)

Let G be a finitely generated nilpotent group. For each positive integer n, let $a_n(G)$ be the number of subgroups of G of index n. Grunewald, Segal, and Smith proved that for each prime p, the zeta function (formal power series) $\sum_{n=0}^{\infty} a_{p^n}(G) p^{-ns}$ is a rational function of p^{-s}. The proof uses the result that power series arising from definable p-adic integrals are rational.

Now define $b_n(G)$ to be the number of irreducible n-dimensional complex characters of G, modulo a natural equivalence relation. We have proved that for each p, the series $\sum_{n=0}^{\infty} b_{p^n}(G) p^{-ns}$ is a rational function of p^{-s}. Here the method of definable p-adic integrals no longer works. In my talk I described a model-theoretic result of Hrushovski and explained how we used it to prove rationality. We obtain a new tool for proving that zeta functions are rational.

A characterisation of finite soluble groups

F. Grunewald

(joint work with T. Bandman, B. Kunyavskii, E. Plotkin, G.-M. Greuel, G. Pfister)

Theorem. *For elements x, y of a group G define $e_1(x,y) := x^{-2}y^{-1}x$ and for $n > 1$ recursively $e_{n+1}(x,y) := [xe_n(x,y)x^{-1}, ye_n(x,y)y^{-1}]$, where commutators are defined as $[a,b] := aba^{-1}b^{-1}$. Now let G be a finite group. The following two statements are equivalent.*

(1) *G is soluble.*
(2) *There is an n such that $e_n(x,y) = 1$ for all $x, y \in G$.*

Exchange lemmas, growth, and pro-p groups

J. Wilson

(joint work with N. Romanovskii)

The exchange lemmas discussed can be reformulated as the following assertion for the categories of abstract and pro-p groups. Let G have finite presentation $\langle X \mid R\rangle$ where $|X| > |R|$ and let S be any generating set for G. Then S has a subset of cardinality $|X| - |R|$ which freely generates a free group. They lead to the solution of a problem of M. Gromov on growth of such groups G.

The main ingredient in the proof is a Freiheitssatz for pro-p groups due to N. Romanovskii. The strategy of a generalisation of this Freiheitssatz was described in the lecture.

Classification and statistics of finite index subgroups in free products (the Poincaré-Klein Problem)

T. Müller

(joint work with J.-C. Schlage-Puchta)

I presented a solution of the Poincaré-Klein Problem for the class of groups $\Gamma = C_{p_1}^{*e_1} * \ldots * C_{p_t}^{*e_t} * F_r$, where $p_1, \ldots, p_t$ are different primes. More specifically, solutions of the realisation, asymptotics, and distribution problems were given.

Realisation. Which isomorphism types allowed by the Kurosh Subgroup Theorem are realised as finite index subgroups of Γ?

Asymptotics. Let τ_i be a sequence of isomorphism types. What can one say concerning the asymptotics of the sequence $s_{\tau_i}(\Gamma)$ counting finite index subgroups of type τ_i in Γ for each i?

Distribution. How to describe limit distributions for finite index subgroups as the index tends to infinity (with respect to various weight functions)?

Automorphisms of free pro-p groups

W. Herfort

(joint work with P. Zalesskii)

A whole number of results were discussed, culminating in

Theorem. Let G be a finitely generated pro-p group. Then the following are equivalent.

(1) G is a virtually free pro-p group.
(2) G acts on a pro-p tree with finite (vertex) stabilisers.
(3) G is the pro-p fundamental group of a finite graph of finite groups.
(4) G contains a dense abstract group G_0 which is finitely generated, virtually free, with $F_0 \trianglelefteq G_0$ such that G_0/F_0 is a p-group, and such that $G_0^{\wedge p} \cong G$.

Ihara zeta function for infinite groups and integrable maps

A. Zuk

(joint work with R. Grigorchuk)

We define the Ihara zeta function for Cayley graphs of infinite finitely generated groups and infinite graphs which are limits of covering sequences of finite graphs. We present several computations of the zeta functions and associated spectral measures for groups generated by finite automata. These computations are based on the integrability of some 2-dimensional mappings and are related to the invariant measures for quadratic maps.

Subgroup growth of lattices in semisimple Lie groups

A. Lubotzky

(joint work with D. Goldfeld, N. Nikolov, L. Pyber)

Theorem. *Let Γ be a non-uniform lattice in a simple Lie group G of R-rank at least two. Define $s_n(\Gamma) := \#\{H \leq \Gamma \mid |\Gamma : H| \leq n\}$. Then $\lim \log s_n(\Gamma)/((\log n)^2/\log\log n)$ exists and is independent of Γ. In fact, it is equal to $\rho(G)$ which is explicitly given.*

We conjecture that the same holds for all lattices.

Counting congruence subgroups in SL_2

L. Pyber

(joint work with D. Goldfeld, A. Lubotzky)

The number of congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$ is essentially $n^{((3-2\sqrt{2})/4)(\log n/\log\log n)}$.

Counting congruence subgroups—the general case

N. Nikolov

(joint work with A. Lubotzky)

My talk complemented the one of A. Lubotzky on the subgroup growth of lattices in simple Lie groups. I explained that, if Γ is a lattice in a simple Lie group G, then

$$\limsup_{n\to\infty} \frac{s_n(\Gamma)}{(\log n)^2/\log\log n} \geq \rho(G),$$

where $\rho(G)$ is defined only in terms of the Dynkin diagram of the Lie type of G. The main new ingredient is the following.

Let $X(\mathbb{F}_q)$ be a finite simple group of Lie type X over the finite field $\mathbb{F}_q$ of characteristic $p > 3$. For any subgroup H of $X(\mathbb{F}_q)$ define

$$f(H) := \frac{\log[X(\mathbb{F}_q) : H]}{\log|H^{\mathrm{ab}}_{p'}|},$$

where $H^{\mathrm{ab}}_{p'}$ is the maximal abelian quotient of H whose order is coprime to p; set $f(H) := \infty$ if $|H^{\mathrm{ab}}_{p'}| = 1$.

Theorem. *Given the Lie type X (twisted or untwisted) then*

$$\liminf_{q\to\infty, H<X(\mathbb{F}_q)} f(H) \geq R(X),$$

where $R(X)$ is the ratio of the number of positive roots to the Lie rank of the root system of untwisted Lie type corresponding to X.

On subgroups of certain branch groups

R. Grigorchuk

(joint work with J. Wilson)

A group G is called a branch group if it has a faithful action on a spherically homogeneous tree $T = T_{\bar{m}}$, where $\bar{m} = (m_n)$, $m_n \geq 2$, denotes the branch index, such that for any $n \geq 1$ the rigid stabiliser $\mathrm{rist}_G(n)$ has finite index in G. Branch groups constitute one of three classes into which the class of just-infinite groups naturally splits and they posses many interesting properties.

One of the strategies in the study of branch groups is to obtain a good understanding of basic examples, like the 2-group J known as Grigorchuk's group or Gupta-Sidki p-groups. Recently, E. Pervova proved that every maximal subgroup in J has finite index.

Recall that two groups are abstractly commensurable if they have isomorphic subgroups of finite index. We have proved

Theorem 1. *If H is abstractly commensurable to J, then all maximal subgroups of H have finite index in H.*

Theorem 2. *Every infinite finitely generated subgroup of J is abstractly commensurable to J.*

Recall that a group G is called subgroup separable if each finitely generated subgroup is an intersection of subgroups of finite index in G.

Theorem 3. *The group J is subgroup separable and consequently the generalised word problem for J is solvable.*

Contracting self-similar groups

V. Nekrashevych

Let X be a finite alphabet, and let X^* be the set of all finite words over X. A faithful action of a group G on X^* is self-similar if for all $g \in G$ and $x \in X$ there exist $h \in G$ and $y \in X$ such that $g(xw) = yh(w)$ for all $w \in X^*$. If an action is self-similar, then for every $g \in G$ and $v \in X^*$ there exists a uniquely defined $h = g|_v \in G$ such that $g(vw) = g(v)h|_v(w)$ for all $w \in X^*$. A self-similar group is contracting if there exists a finite set $N \subseteq G$ such that for every $g \in G$ one can find $n \in \mathbb{N}$ such that $g|_v \in N$ for all v of length at least n.

Contracting groups (i.e. groups with a contracting action) have many interesting properties and applications.

(1) Their word problem is solvable in polynomial time.
(2) They have easy to construct finitely presented HNN-extensions.
(3) A naturally defined limit space can be associated to every contracting action.
(4) They appear naturally in holomorphic dynamics as iterated monodromy groups of expanding coverings.

Dimension and randomness in groups acting on rooted trees

M. Abért

(joint work with B. Virág)

We explore the structure of the p-adic automorphism group of the infinite rooted regular tree. We describe the relationship between dimension and other properties of groups such as solubility, existence of dense free subgroups, and the normal subgroup structure. We show that three random elements generate with probability one a full-dimensional closed subgroup and that there exist finitely generated closed subgroups of arbitrary dimension. Our results solve a problem of A. Shalev and answer a question of S. Sidki.

Grothendieck's problem concerning profinite completions—a solution

M. Bridson

(joint work with F. Grunewald)

In 1970 A. Grothendieck posed the following question: Let Γ_1, Γ_2 be finitely presented, residually-finite groups and suppose that $u : \Gamma_1 \to \Gamma_2$ induces an isomorphism of profinite completions $\widehat{u} : \widehat{\Gamma_1} \to \widehat{\Gamma_2}$. Does it follow that u is an isomorphism?

We settle this question by exhibiting pairs of groups $u : P \hookrightarrow \Gamma$ such that Γ is a residually-finite product of hyperbolic groups and P is a finitely presented subgroup of infinite index, with $\widehat{u} : \widehat{P} \to \widehat{\Gamma}$ an isomorphism.

The proof relies on finiteness criteria for fibre products, D. Wise's variation on the Rips construction, and the construction of just-infinite groups with aspherical balanced presentations, as well as the Platonov-Tavgen approach to constructing counterexamples in the finitely generated case.

3-manifolds, property (τ), and pro-p groups

M. Lackenby

A subtitle for this talk was "The geometry of finite quotient groups". Let G be a group with a finite generating set S. Let $\{G_i\}$ be its finite index normal subgroups. Then the family of Cayley graphs $X(G/G_i, S)$ of G/G_i with respect to S can be viewed as geometric objects. It is possible to relate their geometric properties to algebraic properties of G_i. For example, I explained a theorem giving necessary and sufficient conditions for some G_i to have infinite abelianisation in terms of the Cheeger constants of $X(G/G_i, S)$. I also gave a theorem relating the Cheeger constants of $X(G/G_i, S)$, the rank of G_i and its possible decomposition into an amalgamated free product or HNN-extension. This result suggests a possible approach to the virtually Haken conjecture in 3-manifold theory. I explained how a question of Lubotzky about pro-p groups would, if answered in the affirmative, provide a positive solution to this conjecture.

Branch algebras

L. Bartholdi

The Grigorchuk group G is generated by elements a, b, c, d such that $\phi : G \to G \wr C_2$, with $a\phi = (1,1)\epsilon$, $b\phi = (a,c)$, $c\phi = (a,d)$, $d\phi = (1,b)$, is an injection. I consider the algebra P generated by the action of the Grigorchuk group G on $\{0,1\}^{\mathbb{N}}$, induced from the embedding $\phi^* : G \hookrightarrow \ldots \wr C_2 \wr C_2$, over the field with two elements. Then in analogy with G, the algebra P is branch in a suitable sense, has a nice presentation, and has linear width. One can go on to define a related Lie algebra, which again is branch in a suitable sense.

Profinite groups of finite virtual cohomological dimension

P. Zalesskii

A profinite group is called projective if any epimorphism onto it splits. A profinite group is said to be virtually projective if it contains an open projective subgroup. Note that in the category of profinite groups projective groups are exactly groups of cohomological dimension 1.

Theorem. *Let G be a virtually projective group. Then $G/\langle \mathrm{tor}(G)\rangle$ is projective.*

In the case $\langle \mathrm{tor}(G)\rangle = 1$, i.e. when G is torsion-free, the result is due to J.-P. Serre.

Zeta functions of abelian groups

G. Bhowmik

We associate a zeta function to the number of subgroups of a finite abelian group. To obtain an effective counting function, we use recurrences on types of abelian groups. This helps us give a constructive proof of the rationality of the zeta function. We also mention analytic properties of this function.

Relatively projective groups as absolute Galois groups

M. Jarden

(joint work with D. Haran, F. Pop)

One of the central problems in Galois theory is the classification of absolute Galois groups among all profinite groups. For a field K let K_s be the separable closure of K and $\mathrm{Gal}(K) = \mathrm{Gal}(K_s/K)$ the absolute Galois group of K. Then we have the basic theorem: (a) if K is PAC, then $\mathrm{Gal}(K)$ is projective [J. Ax, D. Haran], (b) if G is projective, then there exists a PAC field K with $\mathrm{Gal}(K) \cong G$ [A. Lubotzky-L. v.d.Dries].

Extending this basic theorem beyond PAC fields and projective groups requires a local-global principle.

Theorem. (a) *Let* $\mathbf{K} = (K, X, K_x, V_x)_{x\in X}$ *be a proper Henselian field valuation structure. Suppose that* $\mathbf{K}$ *has the block approximation property. Then* $\mathrm{Gal}(\mathbf{K}) = (\mathrm{Gal}(K), X, \mathrm{GAL}(K_s))$ *is a proper projective group structure.*

(b) *Let* $\mathbf{G} = (G, X, G_x)_{x\in X}$ *be a proper projective group structure. Suppose* $\mathbf{G}$ *has a Galois approximation. Then there exists a Henselian field valuation structure* $\mathbf{K} = (K, X, K_x, V_x)_{x\in X}$ *having the block approximation property and there exists an isomorphism* $\phi : \mathbf{G} \to \mathrm{Gal}(\mathbf{K})$ *of group structures.*

A follow-up result is the classical theorem: Let $\mathcal{F}$ be a finite set of classical local fields (i.e. each $F \in \mathcal{F}$ is either $\mathbb{R}$, or $\mathbb{C}$, or a finite extension of $\mathbb{Q}_p$ for some p). Then G is the absolute Galois group of a P$\mathcal{F}$C field if and only if G is $\mathcal{F}$-projective and $\mathrm{Subgr}(G, \mathrm{Gal}(F))$ is strictly closed in $\mathrm{Subgr}(G)$ for each $F \in \mathcal{F}$.

Galois pro-p groups unramified at p

N. Boston

Unlike the Galois groups of pro-p extensions of number fields ramified at p, which are well-understood and have found wide application in the theory of p-adic representations, those unramified at p are poorly understood. For instance, not one explicitly presented infinite such group is known. On the other hand the important Fontaine-Mazur conjecture states that all such groups should have no infinite analytic quotients.

I indicate how the Fontaine-Mazur conjecture can be reduced to two purely group-theoretical conjectures, one characterising those pro-p groups satisfying certain properties, the other showing that all groups in such a family have no infinite, analytic quotients. I give one such explicit infinite family, which turns out to have surprising connections with quantum field theory.

Probabilistic generation of pro-(finite soluble) groups

T. Weigel

W. Gaschütz introduced for any finite soluble group G a function $P(G,s)$ whose value at any natural number $s=k$ coincides with the probability that a random k-tuple generates the group G. Using some elementary properties of first degree cohomology groups, one obtains the following formula for the minimal number of generators of a pro-(finite soluble) group G:

$$d_G = \sup\left\{ \left\lfloor \frac{\dim \mathrm{H}^1(G,M)}{\dim M} \right\rfloor + \vartheta_M \,\middle|\, M \in S(G) \right\}$$

where $S(G)$ denotes the class of isomorphism types of simple discrete G-modules and ϑ_M equals 0 or 1 depending on whether M is trivial or not. Using an explicit construction one can parametrise the irreducible modules for the free pro-(finite metabelian) group $F^{\wedge}_{\mathrm{met}}(d)$ on d generators. This is relevant for the computation of $P(F^{\wedge}_{\mathrm{met}}(d),s)$:

$$P(F^{\wedge}_{\mathrm{met}}(d),s) = \prod_{k=0}^{d} \zeta(s-k) \cdot \prod_{i=1}^{d} E^{(d)}(s-i)$$

for an explicitly given, but mysterious formal Dirichlet series $E^{(d)}(s)$.

Random generation in pro-p groups

B. Szegedy

Almost all d-tuples in the Nottingham group generate a free subgroup. This proves a conjecture of A. Shalev.

New combinatorial properties of linear groups and the conjecture of Formanek-Zelmanov

V. Platonov

Let $\mathrm{Aut}(F_n)$ be the automorphism group of a free group F_n of rank $n \geq 2$. E. Formanek and E. Zelmanov conjectured that for any representation $\rho : \mathrm{Aut}(F_n) \to \mathrm{GL}_m(K)$ over an arbitrary field K, $\rho(F_n)$ is virtually soluble.

Theorem 1. (with A. Potapchik) *There exists $s > 0$ such that $\rho(x_1)^s$, $\rho(x_2)^s$, ..., $\rho(x_n)^s$ are unipotent for any $\rho : \mathrm{Aut}(F_n) \to \mathrm{GL}_m(K)$.*

Theorem 2. *If $\rho(x_n)$ has at most $n+3$ Jordan blocks and $\rho(x_n)$ is unipotent, then $\rho(F_n)$ is unipotent.*

Let $G = \langle g_1, \ldots, g_l \rangle \subseteq \mathrm{GL}_m(K)$ such that all primitive words are unipotent. Then we conjecture that G is unipotent. Compare this with E. Zelmanov's conjecture that, if $H = \langle h_1, \ldots, h_l \rangle \subseteq \mathrm{GL}_m(K)$ and all primitive words have finite exponent, then H is virtually soluble.

Enumeration of subgroups in the fundamental groups of surfaces and circle bundles over surfaces

V. Liskovets

We concentrate mainly on the enumeration of subgroups *up to conjugacy*. Typically for natural finitely generated groups (at least, topologically motivated) this problem turns out to be reducible in a sense to the enumeration of subgroups themselves. Here is the first result of this kind (VL'71):

$$N_{F_r}(n) = \frac{1}{n} \sum_{m|n} M_{F_r}(m) \sum_{d|\frac{n}{m}} \mu\Big(\frac{n}{md}\Big) d^{(r-1)m+1},$$

where $M_{F_r}(n)$ and $N_{F_r}(n)$ are the number of n-index subgroups and conjugacy classes of n-index subgroups, respectively, in the free group F_r of rank r (in turn, $M_{F_r}(n)$ is expressed by a famous formula of M. Hall). Similar but much more difficult formulas were obtained by A. Mednykh for the fundamental groups of closed surfaces. I discussed in detail some recent results, including joint work with J.H. Kwak and A. Mednykh on the "Enumeration of branched coverings of non-orientable surfaces with cyclic branch points", and a specific complicated technique (based on the well-known combinatorial orbit-counting lemma applied to regular permutations) of manipulations and calculations in $\mathbb{Z}_\ell \wr \mathbf{S}_m$.

The congruence kernel of an arithmetic lattice in a rank one algebraic group over a local field

A. Mason

The problem of determining whether or not a subgroup of finite index in an arithmetic group is a *congruence subgroup* (the so-called *Congruence Subgroup Problem*) has attracted much attention since the 19th century. It is known that the set of non-congruence subgroups is generally much more extensive when the rank of the ambient algebraic group is low.

Let k be a local field and let G be a connected, semisimple algebraic group over k of k-rank 1. Let Γ be an arithmetic lattice in (the locally compact group) $G(k)$ and let C_Γ be its *congruence kernel*.

When Γ is *cocompact*, it is proved that C_Γ is isomorphic to $\hat{F}_\omega$, the free profinite group on countably many generators.

When Γ is *non-uniform*, P. Zalesskii has proved that C_Γ is a free profinite product of groups, the torsion-free part of which is a projective profinite group P, all of whose open, proper subgroups are isomorphic to $\hat{F}_\omega$. In support of a more general conjecture, it is proved that $P = \hat{F}_\omega$, for example, when $G = \mathrm{SL}_2$.

Non-arithmetic super-rigid groups

H. Bass

(joint work with A. Lubotzky)

Let Γ be a finitely generated, residually finite group. Call Γ rigid if there exist only finitely many classes of irreducible representations $\rho : \Gamma \to \mathrm{GL}_N(\mathbb{C})$ for all N. Call Γ super-rigid if $A(\Gamma)$, the pro-algebraic completion, is finite dimensional. Call Γ linear if there exists an injection of Γ into $\mathrm{GL}_N(\mathbb{C})$ for some N.

V. Platonov conjectured that, if Γ is rigid and linear, then Γ is of arithmetic type. We construct a counterexample as follows. Let $G = F_4^{-20}$, a rank 1 form of F_4. Let $\Gamma \leq G$ be a uniform lattice. There exist a finite index subgroup $\Gamma_1 \leq \Gamma$ and $P \leq \Gamma_1 \times \Gamma_1$, containing the diagonal of infinite index, such that P and $\Gamma_1 \times \Gamma_1$ have the same representation theory. Thus, $P \leq G \times G$, like $\Gamma_1 \times \Gamma_1 \leq G \times G$, satisfies Margulis super-rigidity. In particular, P is super-rigid, yet the infinite index $|\Gamma_1 \times \Gamma_1 : P|$ prevents P from being of arithmetic type.

The proof uses results of various other mathematicians: K. Corlette, M. Gromov-R. Schoen, A. Grothendieck, V. Platonov-O. Tavgen, S. Kumaresan, D. Vogan-G. Zuckerman, A. Ol'shanskii-E. Rips, and M. Gromov.

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