Mathematisches Forschungsinstitut Oberwolfach

Report No. 25/2003

Differentialgeometrie im Großen

June 15th – June 21st, 2003

The conference was organized by Robert Bryant (Durham), Bernhard Leeb (München) and Pierre Pansu (Palaiseau) and attended by forty-four participants from five different countries. Unfortunately, several important international invitations had gotten lost after having been sent out by the institute. Moreover, two of the organizers were not able to attend the meeting.

During the meeting, twenty-three talks were given, covering recent developments in the area of global differential geometry. Joachim Lohkamp presented a local stabilization method for singularities of minimal hypersurfaces, which makes the minimal hypersurface technique available for dimensions greater than 7. Martin Schmidt explained his proof of the Willmore-conjecture and the transformation of the problem to a physically motivated setting. Kenji Fukaya reported on his studies of collapsing complex manifolds related to mirror symmetry. Hartmut Weiß discussed a rigidity result for cone manifolds which has applications to the geometrization of three-dimensional orbifolds. Other talks focused on the geometry of singular spaces, geometric analysis, symplectic geometry, Kähler manifolds and Dirac operators. In an evening session, Hermann Karcher showed differential-geometric animations, many of them in red-green stereo. The abstracts of the talks are listed below in chronological order.

Besides the lectures, there was plenty of time for intensive talks and personal encounter, which contributed a lot to the success of the conference. All participants enjoyed the wonderful and stimulating atmosphere of the institute and its surrounding as well as the friendliness of the staff.

Abstracts

Collapsing and mirror symmetry

Kenji Fukaya

This is a report of a project in progress. We consider a family of complex manifold M_q parametrized by $q \in D^2$ such that M_q will be singular for $q = 0$. We assume M_q is smooth and has a Kähler metric with vanishing Ricci curvature. We scale the metric so that the diameter is one. In case that we assume M_0 is normal crossing and the maximal number of components which meet at one point is k, it is conjectured that the limit (Gromov-Hausdorff limit) X of M_q as $q \to 0$ is of dimension k. (Here the dimension is a real dimension.) In the case when k is equal to n the complex dimension of M_q, then it is expected that the $X - S$ is smooth where S is of (real) dimension at most $n - 2$. Also it is expected that the curvature of M_q is bounded on the complement of a neighbourhood of S. If so it implies that M_q minus a neighbourhood of S is a fibre bundle over $X - S$. If the volume of M_q is of order ϵ^n where ϵ is the Hausdorff distance between M_q and X, then it follows that the fibre is a torus. In this situation the leading term of the metric of M_q is obtained from a solution of the real Monge-Ampere-equation on $X - S$ by tensor calculus.

The main part of the talk is to explain a formula which describes an asymptotic expansion of the complex structure of M_q in terms of pseudo-holomorphic disks of the mirror of M_q. (We hope the metric is also described in a similar way but do not yet know the formula for the metric yet.)

Monopoles over 4-manifolds containing long necks

Kim Froyshov

An important property of the moduli space of monopoles over a closed spin-c Riemannian 4-manifold is that it is always compact. However, if one considers parametrized moduli spaces associated to a 1-parameter family of Riemannian metrics obtained by stretching an initial metric along a hypersurface then the compactness properties depend on the cohomology class of the closed 2-form used to deform the Chern-Simons-Dirac functional over the neck. If this cohomology class is non-zero then the variation of the Chern-Simons-Dirac functional over the neck may tend to infinity uniformly as the length of the neck grows. This is a non-compactness phenomenon that has no analogue in Yang-Mills theory as far as I know. I will also give sufficient conditions for the usual compactness picture to hold and deduce a vanishing result for the (refined) Seiberg-Witten invariant.

Embedding problems and splitting holomorphic curves

Klaus Mohnke

(joint work with Kai Cieliebak)

We demonstrate how to use a basic principle of Symplectic Field Theory introduced by Eliashberg, Givental and Hofer to study embedding problems in symplectic geometry. It shows that holomorphic curves will split as one stretches the neck of an almost complex structure along a hypersurface of contact type. The limit consists of finitely many levels of punctured holomorphic curves in either the convex or concave part completed by half cylinders over the hypersurface, or the symplectization of it.

This is applied to situations where the hypersurface is the boundary of a neighbourhood of a Lagrangian submanifold which is symplectomorphic to the unit disk bundle with respect to a fixed Riemannian metric on the Lagrangian.

In the case of Lagrangian tori in $\mathbb{C}\mathrm{P}^n$ we use a flat metric on the torus and certain (simple) Gromov-Witten-invariants of $\mathbb{C}\mathrm{P}^n$ to show the following

Theorem. *For any embedded Lagrangian torus $L \subset \mathbb{C}\mathrm{P}^n$ there is a map $u : D^2 \to \mathbb{C}\mathrm{P}^n$, $u(\partial D) \subset L$, $u^*\omega \geq 0$ such that*

$$0 < \int_D u^*\omega < \frac{\pi}{n+1}$$

Local rigidity of 3-dimensional cone-manifolds

Hartmut Weiss

Let C be a 3-dimensional cone-manifold of curvature $\kappa \in \{-1, 0, 1\}$ and cone-angles $\leq \pi$. Then the singular locus $\Sigma \subset C$ is a trivalent graph. The smooth part $M = C \setminus \Sigma$ carries a smooth Riemannian metric of constant sectional curvature κ. Let $\mathcal{E} \to M$ be the flat vector-bundle of infinitesimal isometrics. In the flat case $\mathcal{E}_{trans} \subset \mathcal{E}$ is a parallel subbundle.

I discuss the following results:

Theorem 1. *Let C be a cone-manifold of curvature $\kappa \in \{-1, 0, 1\}$ with cone-angles $\leq \pi$. Then the following holds:*

i. $H^1_{L^2}(M, \mathcal{E}) = 0$ *if* $\kappa = \pm 1$
ii. $H^1_{L^2}(M, \mathcal{E}_{trans}) \cong \{\omega \in \Omega^1(M, \mathcal{E}) | \nabla\omega = 0\}$ *if* $\kappa = 0$

The proof uses a Bochner formula for $\mathcal{E}$-valued 1-forms and a Hodge theorem for the de Rham complex on M with values in $\mathcal{E}$. It essentially consists of the study of the selfadjoint extensions of the corresponding Hodge-Laplace operator.

¿From Theorem 1 I deduce local rigidity in the hyperbolic and the spherical case via an analysis of the variety of representations of $\pi_1 M$ into $\mathrm{SL}(2, \mathbb{C})$, resp. $\mathrm{SU}(2) \times \mathrm{SU}(2)$:

Theorem 2. *Let C be a cone-manifold of curvature $\kappa = \pm 1$ with cone-angles $\leq \pi$. Then the family of cone-angles provides a local parametrization of the space of cone-manifold structures of curvature κ near the given structure.*

Cone-manifolds with large cone-angles

Juan Souto

We show that every closed oriented 3-manifold admits a cone-manifold structure with singular locus a link and cone-angles arbitrarily close to 2π. We use existence results, due to Bonahon and Otal for convex-cocompact hyperbolic metrics with given bending lamination on open hyperbolic 3-manifolds and the local rigidity theorem of Hodgson and Kerckhoff for cone-manifold structures with cone-angles less than 2π.

The Yamabe problem and a Yamabe-type problem for Dirac operators

Bernd Ammann

Let M be a compact manifold of dimension ≥ 3 with a fixed conformal class. According to theorems by Trudinger, Aubin and Schoen, there is a metric in this conformal class with constant scalar curvature. The solution of this problem is known as the Yamabe problem.

We begin with a historical overview over how to solve the Yamabe problem. In particular, we sketch Aubin's solution for $\dim M \geq 6$, M not locally conformally flat. The remaining cases are reduced to showing that if one develops asymptotically the Green's function of the conformal Laplacian in conformal normal coordinates, then the zero-order coefficient in this development is positive. This statement was finally proven by Schoen and Yau. Witten realized that the positivity of this coefficient can be derived with the help of spinors. Using analysis on asymptotically flat spaces, he constructed a harmonic, asymptotically parallel spinor on the stereographic blowup of M. I will present a simpler version of this proof (joint work with E. Humbert). We show that such a harmonic, asymptotically parallel spinor can be easily obtained from the Green's function of the Dirac operator on M. Our proof is short and only uses standard analytical tools of Dirac operators on compact manifolds.

After this presentation of the Yamabe problem, we study a similar problem for Dirac operators. We sketch how to obtain existence of solutions if a certain "spectral condition" is satisfied. In the locally flat case of sufficiently high dimension we obtain a local test spinor. The spectral condition follows in this case. At the end we show that solutions in dimension 2 provide existence results for periodic conformal branched surfaces in $\mathbb{R}^3$ with constant mean curvature.

Publications are available under http://www.berndammann.de/publications.

Positive Scalar Curvature

Joachim Lohkamp

For Spin-manifolds one has a Dirac-Operator and from Lichnerowicz-formula and the Atiyah-Singer-Index Theorem obstructions for positive scalar curvature. This instrument is not available for Non-Spin-manifolds and the currently only available tool is the inductive minimal surface technique.

This has the essential drawback that minimal hypersurfaces may have codimension 7 singularities which can behave very unpleasantly. In our talk we sketch a local stabilization method using Green's function as a tool to deform the metric while keeping the scalar curvature positive. This makes the minimal hypersurface technique available in arbitrary dimensions whereas its range of application was previously restricted to dimension ≤ 7.

Positive scalar curvature and rho-invariants

THOMAS SCHICK

(joint work with Nigel Higson and Paolo Piazza)

Eta-invariants and higher eta-invariants show up as correction terms in "(higher) Atiyah-Patodi-Singer index theorems". Rho-invariants, the differences of such eta-invariants, are locally computable and have geometric meaning. To us, in particular two higher variants are of importance: the L^2-index theorem where one is looking at a normalized index (using von Neumann dimensions) of the Dirac operator lifted to the universal covering, and non-commutative higher indices in non-commutative de Rham cohomology of suitable smooth subalgebras of the C^*-algebra of the fundamental group.

We use these rho-invariant to distinguish metrics with positive scalar curvature.

Theorem. *Assume that M is a closed spin-manifold with a metric with* $\mathrm{scal} > 0$ *of dimension* $m \equiv 3 \pmod 4$*,* $m > 4$*. Assume that the fundamental group* Γ *contains a non-trivial element of finite order. Then there are infinitely many non-bordant metrics of positive scalar curvature on* M*, in particular the space of metrics with positive scalar curvature has infinitely many components. These are distinguished by the* L^2*-rho-invariant.*

The main ingredients of the proof are: Lichnerowicz type considerations, the (higher) Atiyah-Patodi-Singer index theorem, additionally this is based on an example studied by Botvinnik-Rosenberg (for finite fundamental groups), together with induction considerations.

On the contrary, we conjecture, that these rho-invariants vanish for torsion free fundamental groups. In this direction, we prove:

Theorem. *Assume that* M *is a closed spin-manifold with a metric with positive scalar curvature. Assume that the fundamental group of* M *is torsion-free, and that the max-Baum-Connes map*

$$\mu_{max}\colon KO_*(B\pi_1(M)) \to KO(C^*_{max}\pi_1(M))$$

is an isomorphism. Then all rho-invariants and higher rho-invariants associated to M *vanish.*

The max-Baum-Connes map is known to be an isomorphism e.g. for amenable group, in particular solvable groups, and lattices in $SO(n,1)$ as well as $SU(n,1)$. The map is known *not* to be surjective for groups with property T, like lattices in Lie groups of higher rank.

Apart from the above input, this is based on a geometric description of KO-theory.

Rigidity of asymptotically symmetric spaces

MARIO LISTING

A well known result is that an asymptotically flat Riemannian spin manifold of non-negative scalar curvature must be the Euclidean space. Using the non-compact Bochner technique analogous results can be shown for hyperbolic manifolds $\mathbb{K}\mathrm{H}^m$ with $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$. For instance M. Min–Oo proved that a strongly asymptotically hyperbolic spin manifold of scalar curvature $\mathrm{scal} \geq -n(n-1)$ is isometric to the real hyperbolic space. Nevertheless, scalar curvature rigidity of the complex and quaternionic hyperbolic space needs a holonomy assumption. M. Herzlich and H. Boualem proved that a Kähler spin manifold which is strongly asymptotic to $\mathbb{C}\mathrm{H}^m$ and of scalar curvature $\mathrm{scal} \geq \mathrm{scal}_0$ must be the complex hyperbolic space. Rigidity of the quaternionic hyperbolic space was independently shown by O. Biquard and M. Listing. A rigidity result of real hyperbolic quotients is due to

M. Listing. Let (M, g) be complete, orientable and strongly asymptotic to a hyperbolic quotient $\mathbb{R}\mathrm{H}^n/\Gamma$. If the sectional curvature satisfies $K \geq -1$, the isometry group of (M, g) is greater than or equal to the isometry group of $\mathbb{R}\mathrm{H}^n/\Gamma$.

CR geometry and Einstein asymptotically complex hyperbolic 4-manifolds

Marc Herzlich
(joint work with Olivier Biquard)

It is well known that any pseudo-convex domain in a complex vector space is endowed with a distinguished complete Kähler-Einstein metric: the Cheng-Yau metric. Following an idea of Fefferman, this metric was the key element in the study of local invariants of CR manifolds (a class of manifolds sharing common features with the real hypersurfaces in a complex vector space). However, the method only catches invariants defined locally from a finite jet of the CR structure. In this work, we show that one can also catch a different (and new) invariant of CR manifolds of dimension 3 when one replaces the Cheng-Yau metric by an Einstein asymptotically complex hyperbolic metric (such as that previously defined by Biquard) which induces the CR structure at infinity. This involves the study of characteristic polynomials in the curvature of the Einstein metric.

A duality theorem for singular Riemannian foliations in nonnegative curvature.

Burkhard Wilking

A singular Riemannian foliation is a subdivision of a Riemannian manifold into complete submanifolds without boundary which are locally equidistant.

A path is then called horizontal if it intersects the leafs perpendicularly. Given such a singular Riemannian foliation one can define the dual singular foliation as follows. Define the leaf of a point p as the set of all points that can be joined with p by a horizontal path.

In general the dual foliation is not any thing reasonable. However we prove that if the ambient manifold has nonnegative curvature and the dual foliation consists of complete submanifolds then it is again a Riemannian singular foliation. Furthermore we show that in nonnegative curvature the dual foliation has automatically complete leafs if a) the original foliation is not singular and b) if the original foliation is given by the orbits of an isometric group action.

We give several applications to open manifolds of nonnegative curvature. Among them is the statement that the Sharafutdinov retraction is of class C^∞. The latter result is based on a theorem of Perelman who showed that the Sharafutdinov retraction is a Riemannian submersion of class C^1. A very different and independent proof of this corollary was recently announced by Cao and Shaw.

Homogeneous Einstein metrics and simplicial complexes

Christoph Böhm

On compact homogeneous spaces we investigate the Hilbert action restricted to the space of homogeneous metrics of volume 1. Based on a detailed understanding of the asymptotic behaviour of this action we assign to a compact homogeneous space G/H a simplicial complex $\Delta_{G/H}$, defined by certain subgroups of G containing H.

Theorem. *Let G/H be a compact homogeneous space with both G and H connected. If the simplicial complex of G/H is not contractible, then G/H admits a G-invariant Einstein metric.*

This theorem is proved by variational methods based on scalar curvature estimates, certain purely Lie-theoretical properties of compact Lie groups and topological properties of real semi-algebraic sets.

We also introduce the notion of *prime homogeneous spaces*. Arbitrary simply connected homogeneous spaces are either products of prime homogeneous spaces or total spaces of a principal torus bundle over such a product. In both cases, the factors of this product are called the *prime factors* of the compact homogeneous space.

Theorem. *Let G/H be a compact simply connected homogeneous space with G connected, simply connected and semisimple. Suppose that there exists a field $\mathbb{F}$ such that the reduced homology with coefficients in $\mathbb{F}$ of the simplicial complexes of all prime factors of G/H does not vanish. Then G/H admits a G-invariant Einstein metric.*

On a proof of the Willmore conjecture

Martin Schmidt

The global Weierstraß representation is used to transform the variational problem of the Willmore functional on tori immersed in $\mathbb{R}^3$ into a variational problem with constraints on the space of Fermi curves of two-dimensional Dirac operators with periodic potentials. These Dirac operators are the Lax operators of an integrable system. The Fermi curves are the integrals of motion. The Willmore functional is the corresponding first integral. In turns out that the moduli space of these Fermi curves is not complete. After adding the Fermi curves of finite rank perturbations of such Dirac operators the subsets of the moduli space, on which the first integral is bounded, becomes compact. We show that the Fermi curves of relative minimizers of this constrained variational problem have dividing real parts. This allows to classify all relative minimizers, whose first integral is not larger than 8π. The total minimum is realized by the Fermi curve of the Clifford torus.

Topology of Symplectomorphisms

Jarek Kedra

I prove that in many cases the rational cohomology ring of groups of symplectomorphisms is infinitely generated (as a ring!). The first part of the talk is concerned with algebraic topology methods (rational homotopy, Gottlieb groups etc). The main result appears as an (not so direct) application. Details can be found in my recent paper "Evaluation fibrations and topology of symplectomorphisms" (math.AT/0305325).

Levi-flat hypersurfaces and a new Liouville type theorem for differential forms

Jianguo Cao

(joint work with Mei-Chi Shaw and Li-he Wang)

In this talk, we discuss the relations between curvature and existence of real Levi-flat hypersurfaces in compact Kähler manifolds.

We show that there is no $C^{2,\alpha}$-smooth real Levi-flat hypersurface in $\mathbb{C}\mathrm{P}^n$. In addition, we show that there is no non-trivial L^2 holomorphic (p,q)-form on any pseudo-concave domain of $\mathbb{C}\mathrm{P}^n$ with distinct integers p and q.

The earlier results in this directions were obtained by Iordan, Lins Neto and Siu, see the recent two papers of Siu appeared in Annals of Math.

Because any compact Kähler manifold with positive curvature is bi-holomorphic to $\mathbb{C}\mathrm{P}^n$, the results above hold for corresponding domains in any positively curved Kähler manifold.

In order to prove the result above, we derive a series of new boundary regularity results for the $\bar{\partial}$-equation on Kähler domains with positive sectional curvature, which are of independent interest.

l_p-cohomology, some geometric applications

Marc Bourdon

The talk is divided into two parts. In the first part I recall some generalities about the l_p cohomology of finitely generated groups. The second part is devoted to word hyperbolic groups. I present a joint work with Herve Pajot which identifies the first l_p-cohomology group with Besov spaces on the boundary. Some applications to the quasi-conformal geometry of the boundary are given.

Buildings and spherical joins

Alexander Lytchak

We prove the following

Theorem. *Let X be a finite dimensional geodesically complete* CAT(1) *space. If it contains a proper subset A containing all the antipodes of all of its points, then X is a building or a spherical join.*

The result can be extended to boundaries X of locally compact geodesically complete Hadamard spaces. Our theorem is related to a deep result of Leeb about rigidity of symmetric spaces and affine buildings. Moreover it generalizes a theorem of Eberlein to singular spaces, that can be used to simplify the proof of higher rank rigidity by Ballmann and Burns-Spatzier. As a corollary of the theorem we get the following rigidity result:

Corollary. *Let G be a building or a spherical join, $f : G \to X$ a 1-Lipschitz surjective map onto a space X as above. Then X is itself a spherical join or a building.*

Resolutions of Riemannian Metrics

Daniel Grieser
(joint work with Richard Melrose)

We consider the following problem: Given an algebraic (or analytic) variety V embedded in a smooth space M, can one refine the standard resolution of singularities of V (whose existence was proved by Hironaka) so that any metric on V_{reg} induced by a smooth Riemannian metric on M is also "resolved"?

We propose a notion of "resolved metric" and conjecture the existence of such a resolution; this notion is similar to the one proposed by B. Youssin (conference "Geometric Analysis and Singular Spaces", OW 1998). We discuss which centres of blow-up are admissible in the proposed resolution scenario. This problem, while trivial in the classical setting, is non-trivial here.

New complete quaternionic-Kähler metrics and distribution of 4-planes in 7 dimensions

Olivier Biquard

Quaternionic Contact Structures (QCS) on $(4m-1)$-dimensional manifold are defined as $(4m-4)$-distributions such that at each point, the induced nilpotent Lie algebra is isomorphic to quaternionic Heisenberg algebra.

The first result is that on $(4m-1)$-spheres, QCS close to the standard one are boundaries at infinity of complete quaternionic-Kähler metrics on the $(4m)$-ball, and this is a 1:1 correspondence (in which the standard QCS on the sphere is the boundary of quaternionic hyperbolic space). There is one restriction on this result: dimension has to be at least 12.

The case of dimension 8 has been solved by my student David Duchemin. He founds an integrability condition on 7-dimensional QCS which is equivalent to being the boundary at infinity of a quaternionic-Kähler metric.

For a 4-plane field in dimension 7, being a QCS is no more a differential system, but an open condition. On the other hand, in dimension 8, there exists a notion of quaternionic-symplectic structure (a quaternionic structure for which the fundamental 4-form is only closed; in higher dimension this implies that it is parallel); only recently some examples have been constructed. It is an interesting open question whether 7-dimensional QCS (without the integrability condition) are boundaries of 8-dimensional quaternionic-symplectic structures.

Proper affine hyperspheres which fibre over projective special Kähler manifolds

Vicente Cortés
(joint work with Oliver Baues)

We show that the natural S^1-bundle over a projective special Kähler manifold carries the geometry of a proper affine hypersphere endowed with a Sasakian structure. The construction generalizes the geometry of the Hopf-fibration $S^{2n+1} \longrightarrow \mathbb{C}\mathrm{P}^n$ in the context of projective special Kähler manifolds. As an application we have that a natural circle bundle over the Kuranishi moduli space of a Calabi-Yau threefold is a Lorentzian proper affine hypersphere.

Moduli Spaces of Affine Locally Homogeneous Spaces

Gregor Weingart

In the talk we discussed an explicit construction of the moduli space of germs of affine homogeneous spaces as a cone over a real projective variety with cone tip giving by flat space. Interestingly this construction centers around the definition of the Singer invariant for affine spaces. In particular the algebraic variety is formally defined by infinitely many equations and an upper bound for the number of equations really needed is equivalent to an upper bound for the Singer invariant for every homogeneous space.

We use this construction to associate a complex to each point in the moduli space whose cohomology in degree 1 describes the formal tangent space parametrizing the infinitesimal deformations of the point in question modulo isometrics. In a very precise sense this complex is an inverted Spencer complex not describing the higher prolongations but the higher order integrability conditions for the affine Killing equations. For a family of examples of homogeneous spaces with high Singer invariant constructed by C. Meusers we can use this complex to show that this family is isolated in the moduli space.

Volume and total curvature for hypersurfaces in $\mathbb{R}^{n+1}$

Alexandru Oancea

The subject of the talk belongs to the broader theme of finding intrinsic *quantitative* constraints on the extrinsic geometry of submanifolds. My starting point was a result of Burago and Zalgaller (Geometric Inequalities, Springer, 1988) stating the following: let $\varphi : M^2 \longrightarrow \mathbb{R}^3$ be an isometric C^2-immersion of a closed surface satisfying $\varphi(M) \subset B(0,R)$. Then $Area(M) \leq R^2 T(M)$, where $T(M) = \int_M |K| dVol$ is the total curvature of the immersion and K is the Gauss curvature.

I explore the case of isometric C^2-immersions $\varphi : M^n \longrightarrow \mathbb{R}^{n+1}$, $n \geq 3$ with $\varphi(M) \subset B(0,R)$. I show there can be no general analogue of the above inequality of the form $Vol(M) \leq C_n R^n T(M)$, with C_n a constant depending only on the dimension and I give a sufficient condition involving the Ricci curvature in dimension $n = 3$. I prove various inequalities of isoperimetric type involving the total curvature. One example is the following:

Theorem. *Under the above assumptions, let $K \subset U \subset M$ such that K is compact, U is open and $K \neq 0$ on U. Supposing $n \geq 1$, one has*

$$Vol(K) \leq C_n \frac{R^n}{d_{\mathbb{S}^n}(K, \partial U)^{n-1}} T(U) + \frac{R}{n} Vol(\partial K) \ .$$

Here $d_{\mathbb{S}^n}$ denotes a suitably defined distance, measured on the sphere $\mathbb{S}^n$ via the Gauss map.

By letting the curvature decrease to zero, this inequality converges to the weak isoperimetric inequality $Vol(K) \leq \frac{R}{n} Vol(\partial K)$ valid for compact sets $K \subset B^n(0,R)$.

On the spectrum of Dirac operators

WERNER BALLMANN
(joint work with Jochen Brüning and Gilles Carron)

We investigate spectral properties of Dirac operators on Dirac bundles (in the sense of Gromov and Lawson) over non-compact, complete Riemannian manifolds. We are interested in the essential spectrum, finiteness of the dimension of L^2-kernels, and index theorems (in the case when the operator is Fredholm). In my talk I concentrate on the finiteness of the dimension of L^2-kernels. I explain part of the proof of one of our results, namely that the L^2-kernel is in fact finite dimensional if the underlying manifold has pinched negative sectional curvature and finite volume and the Dirac bundle has uniformly bounded curvature.

Edited by Jonathan Alze

Participants

Prof. Dr. Uwe Abresch
abresch@math.ruhr-uni-bochum.de
Fakultät für Mathematik
Ruhr-Universität Bochum
Universitätsstr. 150
D–44801 Bochum

Jonathan Alze
alze@mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
80333 München

Dr. Bernd Ammann
ammann@math.uni-hamburg.de
Fachbereich Mathematik
Universität Hamburg
Bundesstr. 55
D–20146 Hamburg

Prof. Dr. Christian Bär
baer@math.uni-hamburg.de
Fachbereich Mathematik
Universität Hamburg
Bundesstr. 55
D–20146 Hamburg

Prof. Dr. Werner Ballmann
ballmann@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Prof. Olivier Biquard
biquard@math.u-strasbg.fr
I.R.M.A.
Université Louis Pasteur
7, rue René Descartes
F-67084 Strasbourg -Cedex

Dr. Christoph Böhm
boehm@math.uni-kiel.de
Mathematisches Seminar
Christian-Albrechts-Universität Kiel
Ludewig-Meyn-Str. 4
D–24118 Kiel

Prof. Dr. Marc Bourdon
bourdon@agat.univ-lille1.fr
Mathématiques
UMR 8524 CNRS
Université de Lille 1
F-59655 Villeneuve d'Asq

Prof. Dr. Jianguo Cao
jianguo.cao.7@nd.edu
Department of Mathematics
University of Notre Dame
Notre Dame, IN 46556-5683 – USA

Prof. Dr. Vicente Cortes
cortes@iecn.u-nancy.fr
Institut Elie Cartan
-Mathématiques-
Université Henri Poincaré, Nancy I
Boite Postale 239
F-54506 Vandoeuvre les Nancy Cedex

Dr. Anand Dessai
dessai@math.uni-augsburg.de
Institut für Mathematik
Universität Augsburg
D–86135 Augsburg

Sven Engel
engel@math.ruhr-uni-bochum.de
svengel@math.uni-muenster.de
Mathematisches Institut
Universität Münster
Einsteinstr. 62
D–48149 Münster

Dr. Kim A. Froyshov
froyshov@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D–33501 Bielefeld

Prof. Dr. Kenji Fukaya
fukaya@math.kyoto-u.ac.jp
Dept. of Mathematics
Faculty of Science
Kyoto University
Kitashirakawa, Sakyo-ku
Kyoto 606-8502 – JAPAN

Prof. Dr. Sebastian Goette
sebastian.goette@mathematik.uni-regensburg.de
Naturwissenschaftliche Fakultät I
Mathematik
Universität Regensburg
D–93040 Regensburg

Dr. Daniel Grieser
grieser@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Prof. Dr. Ursula Hamenstädt
ursula@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Dr. Marc Herzlich
herzlich@math.univ-montp2.fr
Mathématiques, case 51
Université des Sciences et
Techniques du Languedoc
Place Eugène Bataillon
F-34095 Montpellier Cedex 5

Prof. Dr. Hermann Karcher
unm416@uni-bonn.de
Mathematisches Institut
der Universität Bonn
Wegelerstr. 10
D–53115 Bonn

Jarek Kedra
kedra@mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
D–80333 München

Prof. Dr. Bernhard Leeb
leeb@mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
D–80333 München

Prof. Dr. Enrico Leuzinger
enrico.leuzinger@math.uni-karlsruhe.de
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
D–76131 Karlsruhe

Mario Listing
listing@mathematik.uni-leipzig.de
Mathematisches Institut
Universität Leipzig
Augustusplatz 10/11
D–04109 Leipzig

Joachim Lohkamp
lohkamp@math.uni-augsburg.de
Institut für Mathematik der
Universität Augsburg
Universitätsstr. 14
D–86159 Augsburg

Dr. Alexander Lytchak
lytchak@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Prof. Dr. Vladimir S. Matveev
matveev@email.mathematik.uni-freiburg.de
Mathematisches Institut
Universität Freiburg
Eckerstr.1
D–79104 Freiburg

Jan Metzger
jan@everest.mathematik.uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
D–72076 Tübingen

Klaus Mohnke
mohnke@mathematik.hu-berlin.de
Institut für Mathematik
Humboldt-Universität Berlin
Unter den Linden 6
D–10099 Berlin

Alexandru Oancea
oancea@math.polytechnique.fr
Unité de Mathématiques Pures et Appliquées
Ecole Normale Supérieure de Lyon
46 Allée d'Italie
F-69364 Lyon Cedex 07

Dr. Anna Pratoussevitch
anna@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Dr. Thomas Püttmann
puttmann@math.ruhr-uni-Bochum.de
Fakultät für Mathematik
Ruhr-Universität Bochum
D–44780 Bochum

Thomas Schick
schick@uni-math.gwdg.de
Mathematisches Institut
Georg-August-Universität
Bunsenstr. 3-5
D–37073 Göttingen

Prof. Dr. Jean Marc Schlenker
schlenker@picard.ups.tlse.fr
Laboratoire de Mathématiques
E. Picard
Université Paul Sabatier
118 route de Narbonne
F-31062 Toulouse Cedex 4

Dr. Martin Schmidt
mschmidt@aei-potsdam.mpg.de
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
D–14476 Golm

Prof. Dr. Viktor Schroeder
vschroed@math.unizh.ch
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Uwe Semmelmann
semmelma@rz.mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
80333 München

Juan Souto
souto@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Dr. Wilderich Tuschmann
wtusch@math.uni-muenster.de
SFB 478
Geom. Strukturen in der Mathematik
Hittorfstr. 27
D–48149 Münster

Gregor Weingart
gw@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Hartmut Weiss
hartmut.weiss@mathematik.uni-muenchen.de
Mathematisches Institut
Universität München
Theresienstr. 39
D–80333 München

Anna Wienhard
wienhard@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
D–53115 Bonn

Prof. Dr. Burkhard Wilking
wilking@math.uni-muenster.de
Mathematisches Institut
Universität Münster
Einsteinstr. 62
D–48149 Münster

Prof. Dr. Wolfgang Ziller
wziller@math.upenn.edu
Department of Mathematics
University of Pennsylvania
Philadelphia, PA 19104-6395 – USA