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Complex Geometry: Mirror Symmetry and Related Topics

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Die Tagung fand unter der Leitung von A. Beauville (Nice), F. Catanese (Bayreuth), E. Looijenga (Utrecht) und Ch. Okonek (Zürich) statt.

Wie schon bei früheren Tagungen über komplexe Geometrie in Oberwolfach haben auch dieses Jahr viele bedeutende Mathematiker aus verschiedenen Ländern an der Tagung teilgenommen. So fiel es nicht schwer, ein interessantes Tagungsprogramm zusammenzustellen.

Neben schon bald klassisch zu nennenden Themen, als da zu nennen wären Gromov-Witten Invarianten, abgeleitete Kategorien und Fourier-Mukai Transformationen, wurden neueste Entwicklungen wie offene Gromov-Witten Invarianten erörtert, weitere Beispiele der Spiegelsymmetrie beschrieben und der Versuch gewagt, diese Symmetrie in einen neuen Zusammenhang zu stellen. Als besonders befruchtend erwies sich dabei die Zusammenarbeit mit Physikern. Darüber hinaus wurden aber auch Fragestellungen aus der Eichtheorie und zur Klassifikation komplexer projektiver Mannigfaltigkeiten behandelt.

Abstracts

Open Gromov-Witten invariants

Sheldon Katz

The existence of open Gromov-Witten invariants is predicted by open string theory. I surveyed the relations predicted by physics between closed Gromov-Witten invariants, open Gromov-Witten invariants, and Chern-Simons invariants. Some of these relationships, such as gluing formulae, do not involve Chern-Simons, so are conjectures purely about open and closed Gromov-Witten theory. Several ideas were illustrated concretely in the example of a Calabi-Yau threefold X, the total space of $\mathcal{O}(-1) \oplus \mathcal{O}(-1)$ over $\mathbb{P}^1$, together with a Lagrangian L, which is a particular $\mathbb{R}^2$ bundle over $S^1 \subset \mathbb{P}^1$ obtained as the fixed point locus of an antiholomorphic involution on X.

I then surveyed mathematical progress to understanding these invariants including

- moduli space
- tangent-obstruction theory
- virtual fundamental relative class.

If (X, L) admits an S^1 action, free on L, then C.C. Liu has defined an invariant that cannot yet be computed. Ad hoc definitions using localizations have been proposed by Katz-Liu and Li-Song; these are conjectured to agree with Liu's invariant. Possible connections between open Gromov-Witten invariants and relative Gromov-Witten invariants were discussed in conclusion.

tt^* geometry for singularities

Claus Hertling

tt^* geometry is a generalization of the notion of variation of (polarized mixed) Hodge structures. It turned up around 1990 in work of Cecotti and Vafa on $N = 2$ supersymmetric field theories and independently in the work of Simpson on harmonic bundles; implicitly the semisimple case had turned up already 1979 in the holonomic quantum fields of Jimbo-Miwa-Mori-Sato.

The notion of a single polarized Hodge structure is generalized in terms of a vectorbundle on $\mathbb{C}$ with a flat connection ∇ on $\mathbb{C}^*$, with a pole of order ≤ 2 at 0, with a ∇-flat real subbundle on $\mathbb{C}^*$, and with a pairing, such that a series of conditions are satisfied ("(TERP)-structures"). To formulate them one has to construct an extension of the bundle to $\mathbb{P}^1$ and -if this is the trivial bundle- to define a real structure and a Hermitian pairing on the zero fibre.

A result by Cattani-Kaplan-Schmid ('73 and '86) gives a correspondence between polarized mixed Hodge structures and nilpotent orbits of Hodge structures. This conjecturally generalizes to (TERP)-structures. It units work of Schmid with work of McCoy-Tracy-Wu and Ita-Novokshiniv on Painlevé III (for the semisimple rank 2 case).

In the case of singularities (holomorphic function germs as well as tame functions on affine manifolds), one has variations of (TERP)-structures ($=tt^*$ geometry) on the base space of a semiuniversal unfolding, coming from a Fourier-Laplace transform of the Gauss-Manin system. In the case of a tame function on an affine manifold one may conjecture that the (TERP)-structure is positive definite. Evidence comes from physics and from the known mixed Hodge structures.

Mirror symmetry via logarithmic degeneration data

Mark Gross

We study mirror symmetry by studying degenerations $\mathcal{X} \to S$ of Calabi-Yau manifolds over a disk S, with $\mathcal{X}_0$ being singular. The basic idea is that the knowledge of $\mathcal{X}_0$ by itself is not enough to find a mirror family, however in many cases, knowing $\mathcal{X}_0$ with the log structure on $\mathcal{X}_0$ induced by the inclusion $\mathcal{X}_0 \subset \mathcal{X}$ is.

We discuss forthcoming joint results with B. Siebert in this direction. We consider special sorts of degenerations which we will call toric degenerations. These are degenerations such that $\mathcal{X}_0$ is a union of toric varieties meeting along toric strata and such that $\mathcal{X} \to S$ is log smooth away from some well-behaved bad set $Z \subset \mathcal{X}$. Toric log Calabi-Yau spaces are then log spaces which "look like" the central fibre of a toric degeneration. We then develop a mirror symmetry construction for toric log Calabi-Yau spaces, and this construction is an algebro-geometric, or discrete version of SYZ. The next step will be to study smoothability of toric log Calabi-Yau spaces, which will enable us to give a quite general mirror symmetry construction.

A comparison theorem for virtual fundamental classes

Andrei Teleman

(joint work with Ch. Okonek)

We show that Quot spaces over curves have a natural virtual fundamental class in tha algebraic geometry sense (Fulton, Behrend-Fantechi).

Let $Quot^E_{\mathcal{E}_0}$ be the quot space of quotients of $\mathcal{E}_0$ with kernels of fixed topological type E. A special case of the Kobayashi-Hitchin correspondence identifies $Quot^E_{\mathcal{E}_0}$ with a moduli space of generalized vortices. This identification endows $Quot^E_{\mathcal{E}_0}$ with a virtual fundamental class in the differential geometric sense. We show that, if $\operatorname{rank} E = 1$, these two fundamental classes coincide via the cycle class map.

This result can be generalized in the following way (work in progress):

Let $\rho : G \to Gl(V)$ be a representation of a reductive group such that $A(V)^G = \mathbb{C}$. Fix a differentiable principal G-bundle P on a curve Y. The classification problem for pairs (H, φ) consisting of a holomorphic structure H on P and a H-holomorphic section $\varphi \in \Gamma(Y, P \times_\rho V)$ has a stability theory and one can construct with complex geometric methods a moduli space $\mathcal{M}^{sst}$ of semistable objects, which can be identified to a moduli space $\mathcal{M}^{HE}$ of solutions of a generalized Hermite Einstein equation. We state:

1. $\mathcal{M}^{sst}$ has a GIT construction and is a projective variety
2. $\mathcal{M}^{st}$ has a natural algebraic geometric virtual fundamental class $[\mathcal{M}^{st}]^{vir}$
3. The Kobayashi-Hitchin identification maps the differential geometric virtual fundamental class of $\mathcal{M}^{HE}$ onto $[\mathcal{M}^{st}]^{vir}$.

Mirror symmetry for $K3$

Werner Nahm
(joint work with K. Wendland)

For Calabi-Yau manifolds X which are not hyperkähler, the moduli space of conformal quantum field theories in 2 dimension (CFT_2) which can be described by sigma models on X has the form $\mathcal{M}_{\mathrm{cplx}}(X) \times \mathcal{M}_{\mathrm{Kähler}}(X)$, where $\mathcal{M}_{\mathrm{cplx}}(X)$ is the classical moduli space of complex structures on X and $\mathcal{M}_{\mathrm{Kähler}}(X)$ is the moduli space of quantum corrected complexified Kähler structures on X. Mirror symmetry means $\mathcal{M}_{\mathrm{cplx}}(X) = \mathcal{M}_{\mathrm{Kähler}}(\check{X})$, $\mathcal{M}_{\mathrm{Kähler}}(X) = \mathcal{M}_{\mathrm{cplx}}(\check{X})$, with a mirror manifold $\check{X}$. For hyperkähler X the quantum corrections are absent, but the correspondent CFT_2 moduli space has no product structure, so the meaning of mirror symmetry is not clear. On the other hand the SYZ construction of mirror symmetry (dualising the fibres in a torus fibration) is applicable. We investigated these questions for $X = K3$.

The moduli space of Ricci flat matrices on $K3$ is

$$_{SO(3)\times O(19)}\backslash O^+(3,19)/_{Aut(\Lambda^{3,19})} \times \mathbb{R}^+.$$

Here $\Lambda^{3,19}$ is a even self-dual lattice of signature (3,19). One may write $\mathbb{R}^+ =_{O(1)} \backslash O^+(1,1)$. To implement conventional mirror symmetry one can consider subspaces

$$O^+(2,\rho_1) \times O^+(2,\rho_2) \subset O^+(2,20),\ \ \rho_1 + \rho_2 \leq 20$$

and the $S^2 \times S^2$ bundle $_{SO(2)\times SO(2)}\backslash SO(4)$, which yields

$$\left(_{SO(2)\times O(\rho_1)}\backslash O^+(2,\rho_1)/A_1\right) \times \left(_{SO(2)\times O(\rho_2)}\backslash O^+(2,\rho_2)/A_2\right)$$

where $A_i = O^+(2,\rho_i) \cap Aut(\Lambda^{4,20})$. For Kummer orbifolds $(T^2 \times T^2)/_{\mathbb{Z}_2}$ one finds a stratum with $\rho_1 = \rho_2 = 1$.

More generally, classical data which yield equivalent CFT_2, can be joined by a path in moduli space, which yields an element of $Aut(\Lambda^{4,20})$. We found this element for the torus fibrations (with singular fibres) $(T^2 \times T^2)/\mathbb{Z}_N$, $N = 2,3,4,6$.
(1) On the subgroup of $H^{\mathrm{even}}(X,\mathbb{Z}) \cong \Lambda^{4,20}$ which comes from $H^{\mathrm{even}}(T^2 \times T^2, \mathbb{Z})$ the action of the mirror symmetry is known by SYZ. The action on the exceptional divisor can be determined (up to obvious symmetries) by unique extension of this action to $\Lambda^{4,20}$.
(2) In CFT_2, the corresponding data are the transformations of the twist fields. For CFT_2 on T^4 with period lattice Λ, one has (for B-field $B = \mathcal{O}$) vertex operators for the elements of $\Lambda \oplus \Lambda^4$. Let T_t be a twist field for $\theta \in \mathbb{Z}_N$, $V(p,z)$ a vertex operator with $p \in \Lambda \oplus \Lambda^4$, $z \in \mathbb{C}$, $T_t(w)$ a twist field. Then $V(p,z) \longmapsto V(\theta p, z)$ for a path around w. One finds

$$V(p,z)T_t(w) \sim \sum_{t'} W_{tt'} T_{t'}(w),$$

where the matrices $W(p)$ form a Weyl algebra W. For maximal commutative subalgebras of W, the the representation splits into one-dimensional irreducible representations generated by T_x, where t corresponds to classical fixed points of θ. Mirror symmetry selects a different commutative subalgebra of W, which leads to a mirror symmetry action on the T_t.
(3) Comparing the transformation on the exceptional divisors as in (1) and the transformation on the twist field as in (2), one gets a transformation between twist fields and exceptional divisors which just is the McKay correspondence between representations of $Sl_2 \supset \mathbb{Z}_N$ ($\sim \mathbb{Z}_N$-covariant flat line bundles on the torus) and the exceptional divisors. The approach should generalize to other hyperkähler manifolds. Moreover, if allows to calculate the CFT's for several strata in the moduli space of the sigma models on $K3$ which eventually should lead to a complete understanding of the corresponding family of CFT's.

Conclusion: mirror symmetry for $K3$ are elements of $Aut(\Lambda^{4,20})$ and many $\gamma \in Aut(\Lambda^{4,20})$ of finite even order (not only order 2) have geometric interpretations as mirror symmetries in the SYZ sense.

Symplectic resolutions for symplectic singularities

Baohua Fu

A complex variety W, smooth in codimension 1, is said to have symplectic singularities, if there exists a holomorphic symplectic 2-form ω on W_{reg}, such that for any resolution of singularities $\pi : X \to W$, $\pi^*\omega$ can be extended to be a holomorphic symplectic 2-form on the whole of X. If furthermore $\pi^*\omega$ can be extended to be a holomorphic *symplectic* 2-form, then π is said to be symplectic. In this talk, we will discuss the following aspects (and corresponding results) for symplectic resolutions:

(I) Existence:

Theorem 1. *Let $\mathcal{O}$ be a nilpotent orbit in a semisimple complex Lie algebra $\mathfrak{g}$. Suppose that $\pi : Z \to \bar{\mathcal{O}}$ is a symplectic resolution. Then Z is isomorphic to $T^*(G/P)$ for some parabolic subgroup P of G and π is the collapsing of the zero section.*

(II) Uniqueness:

Theorem 2. *Let S be a symplectic smooth surface and $S^{(n)}$ its n-th symmetric product. Then any symplectic resolution of $S^{(n)}$ is isomorphic to the classical one: $S^{[n]} \to S^{(n)}$.*

(III) Finiteness:

Conjecture 3. *Any symplectic singularity admits at most finitely many non-isomorphic symplectic resolutions.*

This conjecture is true in the following cases:

- Nilpotent orbit closures
- dimension $= 4$

(IV) Deformations

Conjecture 4. *Suppose we have two symplectic resolutions π and π' of a symplectic singularity. Then there exists deformations φ_s and φ'_s of π and π' respectively, such that φ_s and φ' are isomorphic for $s \neq 0$.*

This conjecture is true in the following cases:

- if the singularity is projective
- if the singularity is a nilpotent orbit closure in $sl(n+1, \mathbb{C})$ for some n.

Remark 5. *For a nilpotent orbit closure $\bar{\mathcal{O}}$ in a simple Lie algebra of classical type, X is deformation equivalent to X^+, if $X \to \bar{\mathcal{O}}$ and $X^+ \to \bar{\mathcal{O}}$ are two symplectic resolutions for $\bar{\mathcal{O}}$.*

Twisted Fourier-Mukai transforms for homomorphic symplectic manifolds

Sawon

Studying fibrations on holomorphic symplectic manifolds seems to be one of the most promising approaches to understanding them. Matsushita has proved that for a (irreducible, projective) holomorphic symplectic manifold X^{2n} and a proper morphism $X \to B$, with $0 < \dim B < \dim X$, the generic fibre must be an n-dimensional abelian variety and the base B must be fano with the same Hodge numbers as $\mathbb{P}^n$. In particular, for $n = 2$ B must be $\mathbb{P}^2$. Similar results for $n = 2$ were obtained by Markushevich, who also showed that if a holomorphic symplectic fourfold X is fibred by Jaconians of genus 2 curves then it is birational to the Hilbert scheme $Hilb^2 S$ of two points on a K^3 surface (if it admits a section).

in my talk I discussed deformations of this fibration by Jacobians, ny using the techniques of twisted Fourier-Mukai transforms. A Fourier-Mukai transform is an equivalence between the derived category of some space X and the derived category of some fine moduli space M of sheaves on X. Examples include the case when X is an elliptic curve E and M is its dual $Pic^0 E$ (more generally abelian varieties). In the relative case, i.e. an elliptic fibration, the universal sheaf need not extend globally to a universal sheaf on $X \times J$, where X is the elliptic fibration and J its relative Jacobian. However, the failure of the existence of a global universal sheaf is "controlled" by a gerbe $\beta \in H^2(J, \mathbb{O}^*)$. This gerbe can be incorporated into the construction to give a twisted Fourier-Mukai transform: an equivalence between the derived category of X and the derived category of β-twisted sheaves on J. These ideas were described in Andrei Căldăraru's thesis. My talk described an example extending the technique to fibrations by abelian surfaces, namely the fibration by Jacobians over $\mathbb{P}^2$ which comes from a deformation of $Hilb^2 S$. It should be possible to extend the ideas further, to abelian fibrations on deformations of $Hilb^n S$ for larger n.

Vertex algebras and mirror symmetry on tori

Christian van Enckevort

We discuss two aspects of mirror symmetry, the mirror map between the moduli spaces and the homological mirror conjectures. For 2-tori both aspects are well understood, but for higher dimensional tori there are problems. Computing Hodge numbers shows that the number of deformations of the Kähler structure is not equal to the number of complex structure deformations. So the mirror map cannot simply interchange complex structure and Kähler deformations. Following Kapustin and Orlov we show that for special tori of dimension 4 the lattice of charges of holomorphic D-branes can have rank 6, whereas for Lagrangien D-branes the rank is at most 5. This shows that it is necessary to extend the definition of Lagrangian D-branes to include coisotropic D-branes. We then propose solutions for these problems based on input from physics. Using CFT we motivate the introduction of two independent complex structures j_1 and j_2,one for the left-moving sector and one for the right-moving sector. Both have to be compatible with the metric G. Encoding the data B, G, j_1, j_2 in a different way, we discuss a nice description od how isomorphisms and mirror morphisms act on the moduli. Before we only discussed the geometric part of the moduli space defined by $j_1 = \bar{j_2}$. The problem is that this condition is not preserved by mirror morphisms. For the mirror morphism corresponding to the SYZ-fibration, the restriction of the Kähler form and the B-field to the fibre of the SYZ-fibration has to vanish for the mirror image to be in the geometrical part again. Returning to homological mirror symmetry we discuss the CFT description of D-branes using a gluing map. This allows for a nice description of the transformation of D-branes under mirror symmetry. However, the gluing matrix has to satisfy a list of conditions. To check the compatibility of these conditions with mirror symmetry, we replace the gluing map with a lattice motivated by boundary states. Finally, we discuss BPS-branes and point out that mirror symmetry interchanges the conditions defining Lagrangian and holomorphic D-branes.

Branes in Poisson manifolds

Giovanni Felder

There are two places in mathematics where the idea from string theory of integrating over the space of maps from a disk (or a 2-manifold with boundary) to a manifold is relevant. One is the open Gromov-Witten invariants or Fukaya category, where one considers maps to symplectic manifolds with boundary conditions on Lagrangian submanifolds. The other is Kontsevich's deformation quantization of Poisson manifolds, where one has a perturbative evaluation of a path integral over maps close to the trivial map to a point.

One should expect these to be two special cases of a more general theory of coisotropic D-branes in a Poisson manifold. Such submanifolds give allowed boundary conditions for the poisson σ-model. Gauge invariant observables are functions on a coisotropic submanifold which are constant along its characteristic foliation. In the case where the coisotropic submanifold is the manifold itself, we get the Kontsevich formula. In hte general case we get a generalization of this formula which gives (under certain assumptions) a quantization of the algebra of functions on the leaf space. For a pair of coisotropic submanifolds, we obtain a quantization of the functions on the intersection constant along the intersection of the foliations as abimodule over the algebras associated to the two submanifolds.

Derived equivalence of algebraic varieties

Yujiro Kawamata

I considered finitely log version of derived equivalence conjecture in the following sense: let (X, B) and (Y, C) be pairs of normal projective varieties with $\mathbb{Q}$-divisors having standard coefficients. Assume that they have smooth Deligne-Mumford stack covering $\mathcal{X}$ and $\mathcal{Y}$ and a common birational resolution satisfying $f^*(K_X + B) \leq g^*(K_Y + C)$. Then there exists a fully faithful exact functor $D(\mathcal{X}) \to D(\mathcal{Y})$. In particular, if they are log K-equivalent, then they have equivalent derived categories. There is a small list of evidences.

For the opposite direction, if there exists such a functor, then there exists an object on the product stack which represents the given functor. In this way, the existence of the functor has geometric implications.

The Brauer group of analytic $K3$ surfaces

Stefan Schröer
(joint work with D. Huybrechts)

We show that for a complex analytic $K3$ surface any torsion class in $H^2(X, \mathbb{O}_X^*)$ comes from an Azumaya algebra. In other words the Brauer group equals the cohomological Brauer group. For algebraic surfaces, such results go back to Grothendieck.

In our situation, we use twistor spaces to deform a given $K3$ surface to a suitable projective $K3$ surface, and then stable bundles and hyperholomorphy conditions to pass back and forth between the members of the twistor families.

Genus one Gromov-Witten invariants of quintic manifolds

Jun li
(joint work with A. Zinger)

We derived a formula for genus 1 Gromov-Witten invariants of quintic manifolds. Based on this formula one can use localization method to compute the genus one Gromov-Witten invariants and the number of primitive elliptic curves in a quintic manifold.

Let $Q \subset \mathbb{P}^4$ be a quintic manifold. We consider the moduli space $\mathcal{M}_{1,0}(Q,d) \subset \mathcal{M}_{1,0}(\mathbb{P}^4,d)$. Let $\mathcal{X} \xrightarrow{f} \mathbb{P}^4$ be the universal family over $\mathcal{M}_{1,0}(\mathbb{P}^4,d)$. Then

$$\mathcal{M}_{1,0}(Q,d) = f^* s^{-1}(0) \subset \mathcal{M}_{1,0}(\mathbb{P}^4,d),$$

where $s^{-1}(0) = Q$. Let $\mathcal{M}_{1,0}(\mathbb{P}^4,d)_*$ be the irreducible component of $\mathcal{M}_{1,0}(\mathbb{P}^4,d)$ consisting of maps whose genus 1 domain components are not ghost components. Then

$$GW_1(d) = \int_{[\mathcal{M}_{1,0}(\mathbb{P}^4,d)_*]} e(\pi_* f^* \mathcal{O}(5)) + \frac{1}{12} GW_0(d),$$

where $\pi : \mathcal{X} \to \mathcal{M}_{1,0}(\mathbb{P}^4,d)$ is the projection and $e(\pi_* f^* \mathcal{O}(5))$ is defined to be the Euler class of a desingularisation of $\pi_* f^* \mathcal{O}(5)$ after pull back to some blowing up of $\mathcal{M}_{1,0}(\mathbb{P}^4,d)_*$.

All the constructions can be done equivariantly, thus given a $\mathbb{C}^*$ action on $\mathbb{P}^4$ we can use localization to evaluate the above integral.

Large absolute and relative Gromov-Witten invariants

Andreas Gathmann

Let Y be a smooth hypersurface in a smooth projective variety X. Our current research is focused on the question how to compute the Gromov-Witten invariants (in any genus) of Y in terms of those of X. The most important case is $X = \mathbb{P}^n$: as the Gromov-Witten invariants of projective spaces are all known, we can then hope to access the higher genus Gromov-Witten invariants of hypersurfaces in projective spaces, most prominently of the quintic threefold.

Our strategy is to blow up $Y \times \{0\}$ in $X \times \mathbb{P}^1$, which realizes X as a deformation of the normal crossing scheme $X \cup_Y N$, where N denotes the projective closure of the normal bundle of Y in X. Using Jun Li's theory of relative Gromov-Witten invariants we can then express the (absolute) invariants of X in terms of the relative invariants of X and N relative Y. The relative invariants of the bundle N can in turn be related to the invariants of the base Y using localization techniques. Combining these ideas we get relations between the Gromov-Witten invariants of X and Y.

The most important results that we have obtained so far are:

(1) For curves of genus 0 the invariants of Y can always be reconstructed from those of X. In the case when $-K_Y$ is nef we reprove the well-known "mirror theorem".
(2) In genus 1 our strategy computes the elliptic Gromov-Witten invariants of the quintic threefold and verifies the numbers conjectured by physicists.

On Seifert matrices

Duco van Straten
(joint work with Ch. Meyer)

Unimodular integral bilinear forms $S : H \times H \to \mathbb{Z}$ first arose in the classical work of Seifert. In the theory of vanishing cycles and exceptional collections on Fano manifolds such forms also arise and have the additional properties of upper triangularity and quasi-unipotence of the monodromy. According to Cecotti-Vafa this matrices up to braid group equivalence classify $N = 2$ supersymmetric theories. A list of such 4×4 matrices was presented.

Hamiltonian Gromov-Witten invariants for toric varieties

Mihai Halic

The purpose of the talk was that of presenting the algebraic construction of the Hamiltonian GW-invariants of a smooth and projective toric variety. The presentation was divided into two parts:

(i) the description of the compactification of the space of morphisms from a curve into a toric variety and of its virtual fundamental class, and
(ii) the construction of the natural classes which live on this compactification.

The invariants are obtained by intersecting the natural cycles with the virtual fundamental class.

Fourier-Mukai partners of $K3$ surfaces and applications

K. Oguiso

I reported my joint work with S. Hosono, B. Lian and S.T. Tau about Fourier-Mukai partners of $K3$ surfaces. In this talk, after recalling a fundamental result due to Mukai and Orlov, which characterizes Fourier-Mukai partners of $K3$ (reps. abelian surfaces) from three viewpoints:

(1) categorical: $Y \in FM(X)$
(2) arithmetic: $(T(Y), \mathbb{C}\omega_Y) \cong (T(X), \mathbb{C}\omega_X)$ Hodge isometric
(2') $(\tilde{H}(Y, \mathbb{Z}), \mathbb{C}\omega_Y) \cong (\tilde{H}(X, \mathbb{Z})), \mathbb{C}\omega_X)$
(3) geometrical: $Y \cong M_H((r, H, s))$ 2-dimensional compact fine moduli space of stable sheaves on X with respect to $\exists$ ample H

I derived the following counting formula:

$$|FM(X)| = \sum_{i=1}^{m} |_{(}S_i) \backslash O(A_{NS(X)}) / _{O_{Hodge}(T(X), \mathbb{C}\omega_X)}|,$$

where X is a $K3$ surface and $\{S_1 = NS(X), S_2, \ldots, S_m\}$ is the genus of $NS(X)$.

From this formula, we have $|FM(X)| = 2^{T(n)-1}$ when X is a $K3$ of $\rho(X) = 1$ and and $NS(X) = \mathbb{Z}H$, $H^2 = 2n$ (Here $T(n)$ is the number of prime factors of n). Moreover,

$$FM(X) = \{M_H(r, H, s) \mid rs = n, r \geq s > 0, (r, s) = 1\}.$$

I also discussed the meaning of the number $2^{T(n)-1}$ from the view of Homological mirror symmetries.

Movable curves

Thomas Peternell

(joint work with S. Boucksom, J.P. Demailly, and M. Paun)

A central conjecture in classification theory of algebraic varieties states: A projective manifold X has Kodaira dimension $\kappa(X) = -\infty$ if and only if X is uniruled, i.e. covered by rational curves. This is known for threefolds by the work of Mori, Miyaoka, and others, using the existence of minimal models.

In general the conjecture can be split into two parts:

A. K_X is not pseudo-effective $\implies$ X is uniruled.

B. K_X is pseudo-effective $\implies$ $\kappa(X) \geq 0$.

The statement A. follows from the more general

Theorem 6. *L is a pseudo-effective line bundle, i.e. $c_1(L)$ is in the closure of the cone of classes of effective divisors, iff $c_1(L) \cdot C_t \geq 0$ for all covering families of curves in X.*

This in turn is the consequence of a duality theorem of "the cone of pseudo-effective divisors" and "the cone of movable curves".

Finally I mention results concerning (B.) in dimension 4.

Self-dual manifolds and Mirror Symmetry for the quintic threefold

Michele Grassi

We develop the theory of self-dual manifolds, which are a natural generalization of almost-Kahler manifolds. We then construct a two-dimensional family of them, which interpolates between the anticanonical family in $\mathbb{P}^n$ and its mirror dual.

Sectional curvatures of Kähler moduli

Pelham M.H. Wilson

We investigate a new property for compact Kähler manifolds. Let X be such a manifold of complex dimension n and $H^{1,1}$ denote the $(1,1)$ part of its real second cohomology. On this space, we have a degree n form given by cup product. Let $\mathcal{K}$ denote the open cone of Kähler classes in $H^{1,1}$, and $\mathcal{K}_1$ the level set consisting of classes in $\mathcal{K}$ on which the form takes value one. This is a Riemannian manifold, with tangent space at a given point being the primitive classes of type $(1,1)$, and the metric defined via the Hodge Index Theorem. In the Calabi–Yau case, we conjecture that $\mathcal{K}_1$ has non-positive sectional curvatures. This would place new restrictions on the possible location of the Kähler cone in cohomology, giving potentially useful information as to which smooth manifolds may support Calabi–Yau structures.

The conjecture is motivated by a Mirror Symmetry argument; this argument suggests that one should develop a mirror version of the Weil–Petersson theory of complex moduli.

The outline of such a theory will be described, and the Conjecture verified under certain extra assumptions. Finally, we investigate in more detail the case when X is a Kähler threefold with $h^{1,1} = 3$, where we only have the one sectional curvature on $\mathcal{K}_1$ to consider. We prove a formula (5.1) relating this curvature to the classical invariants of the ternary cubic form given by cup product, and we discuss various implications of this formula.

Edited by Markus Dürr

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