

Report No. 43/2003

**Arbeitsgemeinschaft mit aktuellem Thema:  
Homotopy of Moduli Spaces**

October 5th – October 11th, 2003

Twenty years ago D. Mumford conjectured that the rational cohomology of the stable moduli spaces of Riemann surfaces is a polynomial algebra generated by certain classes  $\kappa_i$  of dimension  $2i$ . For the purpose of calculating rational cohomology, one may replace the stable moduli space of Riemann surfaces by  $B\Gamma_\infty$ , where  $\Gamma_\infty$  is the group of isotopy classes of automorphisms of a smooth oriented connected surface of “large” genus. Tillman’s insight that the plus construction makes  $B\Gamma_\infty$  into an infinite loop space led to a stable homotopy version of Mumford’s conjecture, stronger than the original. In 2002 Madsen and Weiss proved this integral Mumford’s conjecture, using Harer’s stability theorem, Vassiliev’s theorem concerning spaces of functions with moderate singularities and methods from homotopy theory. The goal of our meeting was to go over this proof. The material was divided into 17 talks.

The Arbeitsgemeinschaft was organized by Michael Weiss (Aberdeen) and Søren Galatius (Aarhus).

# Abstracts

## Mapping class groups and homological stability

JENS HORNBOSTEL

We define the mapping class group  $\Gamma_{g,b} := \pi_0(\text{Diff}(F_{g,b}))$ , where  $F_{g,b}$  is a Riemannian manifold of genus  $g$  with  $b$  boundary circles, and  $\text{Diff}$  denotes orientation-preserving diffeomorphisms that restrict to the identity on the boundary. For  $b = 0$ , this is related (i.e. we have an  $H_*(\cdot, \mathbb{Q})$ -isomorphism) to the moduli space of complex curves. This follows as the zero component  $\text{Diff}_0(F_{g,b})$  is contractible for  $g > 1$  by a Theorem of Earle-Eells-Schatz, and there is a  $\text{Diff}_0$ -principal bundle with total space given by the smooth complex structures on  $F_{g,0}$  and base space the  $6g - 6$ -dimensional Teichmüller space, both being contractible and the Teichmüller space coming with a virtually free action of  $\Gamma_{g,0}$ . We then discuss the stability isomorphism of Harer-Ivanov which implies that  $H_n(\Gamma_{g,b})$  is independent of  $g$  and  $b$  for  $g, b \gg n$ . Moreover  $\Gamma_{g,b}$  is perfect for  $g > 2$ . Next, we recall the theorems about plus construction and group completion. Putting all this together, we deduce a homotopy equivalence  $\Omega B(\bigsqcup_{g \geq 0} B\Gamma_{g,1+1}) \simeq \mathbb{Z} \times B\Gamma_{\infty,1+1}^+$ .

## Tillmann's theorem

JAREK KEDRA

The goal of the talk is to explain the proof of the following fact:

$\Omega B(\bigsqcup_g B\Gamma_{g,2})$  is an infinite loop space.

The argument consists of the following steps:

- (1)  $\Omega B(\bigsqcup_g B\Gamma_{g,2}) \simeq \mathbb{Z} \times B\Gamma_{\infty,2}^+$  [first talk];
- (2)  $\mathbb{Z} \times B\Gamma_{\infty,2}^+ \simeq \Omega B\mathcal{Y}$ , where  $\mathcal{Y}$  is certain category;
- (3)  $\mathcal{Y}$  is a symmetric strict monoidal 2-category;
- (4)  $B\mathcal{Y}$  is its own group completion;
- (5) According to a theorem of May and Segal, there exist an  $\Omega$ -spectrum whose associated infinite loop space is the group completion of  $B\mathcal{Y}$ .

Taking the above steps together we get the statement.

## Thom-Pontryagin construction and MMM classes

IVAN IZMESTIEV

We construct a map

$$\alpha_\infty : \Omega B(\bigsqcup_g B\Gamma_{g,2}) \cong \mathbb{Z} \times B\Gamma_{\infty,2} \longrightarrow \Omega^{2+\infty} \text{Th}(\mathbb{L}_\infty^\perp)$$

which allows us to formulate the integral Mumford conjecture:  $\alpha_\infty$  is a homotopy equivalence. A proof of this statement will be carried out in subsequent talks. Via calculation of  $H^*(\Omega^{2+\infty} \text{Th}(\mathbb{L}_\infty^\perp); \mathbb{Q})$  this implies the original Mumford conjecture.

In the second part of the talk (that actually took place at the evening catch-up session) we define the Miller-Morita-Mumford classes

$$\kappa_i \in H^{2i}(B\Gamma_g; \mathbb{Z}), \quad i > 0, \quad g \gg i$$

which are viewed as characteristic classes of surface bundles. Then we indicate a relation of MMM classes to the map  $\alpha_\infty$ . Namely, we find elements  $\bar{\kappa}_i \in H^{2i}(\Omega^{2+\infty}\text{Th}(L_\infty^\perp); \mathbb{Z})$  such that  $\kappa_i = \alpha_\infty^*(\bar{\kappa}_i)$ .

## Classifying spaces: what they classify

SIGRID WORTMANN

The first part of the talk was concerned with homotopy theory of sheaves on the category  $\mathcal{X}$  of smooth manifolds (with boundary). After introducing the notion of concordance, the representing space  $|\mathcal{F}|$  of a sheaf  $\mathcal{F}$  was defined and its name justified. In the second part of the talk sheaves with category structure, i.e. taking values in the category of small categories, were discussed. The classifying space  $B|\mathcal{F}|$  of such a sheaf is the geometric realization of the nerve of  $|\mathcal{F}|$ . The answer to the question in the title was given. The main theorem of this talk states that  $B|\mathcal{F}| \simeq |\beta\mathcal{F}|$  (homotopy equivalent). Here  $\beta\mathcal{F}$  is a set-valued sheaf on  $\mathcal{X}$ . Its definition uses a generalization of Steenrod's description of principal  $G$ -bundle in terms of bundle charts. Due to lack of time no details of the proof were given.

## First desingularization procedure

NATHALIE WAHL

Consider the sheaves  $h\mathcal{V}$ ,  $\mathcal{V}$  and  $\mathcal{V}_c$  defined on the category of smooth manifolds as follows: for a manifold  $X$ ,  $h\mathcal{V}(X)$  is the set of pairs

$$(\pi, \hat{f})$$

with  $\pi : E \rightarrow X$  a graphic submersion with 3-dimensional oriented fibres and  $\hat{f}$  a fibrewise non-singular section of the vertical jet bundle  $J_\pi^2(E; \mathbb{R})$  with constant part  $f : E \rightarrow \mathbb{R}$ , such that  $(\pi, f) : E \rightarrow X \times \mathbb{R}$  is proper. The set  $\mathcal{V}(X)$  is the integrable version, that is the set of pairs  $(\pi, f)$  with  $f : E \rightarrow \mathbb{R}$  such that  $(\pi, j_\pi^2(f)) \in h\mathcal{V}$ , and  $\mathcal{V}_c(X)$  is the subset of  $\mathcal{V}(X)$  of  $(\pi, f)$  with connected fibres.

Following Madsen-Weiss, we show that the representing spaces  $|\mathcal{V}_c|$  and  $|h\mathcal{V}|$  are respectively equivalent to  $\coprod_{g \geq 0} B\text{Diff}(F_g)$  and  $\Omega^\infty \mathbb{C}P_{-1}^\infty$ . The first result uses Ehresmann's fibration lemma, whereas for the second we use the Thom-Pontryagin construction and Phillips' submersion theorem. The natural map  $|\mathcal{V}_c| \rightarrow |h\mathcal{V}|$  induces the map studied by Madsen and Tillmann,  $\alpha_\infty : \mathbb{Z} \times B\Gamma_\infty^+ \rightarrow \Omega^\infty \mathbb{C}P_{-1}^\infty$ .

## Interpolation theory on manifolds

ALEXANDER SCHMIDT

Let  $M$  be a compact smooth manifold and let  $R = C^\infty(M, \mathbb{R})$  be the ring of smooth real-valued functions on  $M$  together with its natural Frechet-topology. Let  $E$  be a finite-dimensional  $\mathbb{R}$ -vector space. We consider the free  $R$ -module  $Z = C^\infty(M, E)$ . We show that there exist finite dimensional subvector spaces  $P_N$  in  $Z$  such that for every closed  $R$ -submodule  $Y$  in  $Z$  of finite  $\mathbb{R}$ -codimension less or equal to  $N$  the following properties

(i) and (ii) are satisfied:

- (i)  $P_N + Y = Z$  (as  $\mathbb{R}$ -vector spaces)
- (ii)  $Y$  is the closed  $R$ -submodule in  $Z$  generated by  $P_N \cap Y$ .

We use this to show that there exists a natural compact Hausdorff topology on the set  $K(N)$  of closed  $R$ -submodules of codimension  $N$  in  $Z$ .

Finally we discussed openness properties, meaning that if  $V$  is a finite dimensional vector space satisfying property (i) above, then the same is true for every finite dimensional vector space  $V'$  sufficiently near to  $V$ .

## Advanced transversality

KATHARINA LUDWIG

In the first part of my talk the definition of transversality as given in the book [GG] of Golubitsky and Guillemin was recalled. The basic fact about transversality was proven: Given a smooth family of smooth maps which intersects a submanifold of the target transversely, for a dense set of parameters the individual mappings also intersect this submanifold transversely. From that one can deduce the Thom transversality theorem and the multijet Thom transversality theorem.

In the second part the first main theorem of Vassiliev, [V1] or [V2], was (roughly) stated (see also Talk 8) and one theorem feeding into the proof was explained in more detail: Given a  $m$ -dimensional manifold  $M$  with boundary and a set of “forbidden” singularities. The goal is to find an ascending chain of affine finite dimensional subspaces  $D_r$  of  $C^\infty(M, \mathbb{R}^n)$  lying sufficiently generic with respect to the set of functions with forbidden singularities such that the cohomology of  $D_r \setminus \{\text{functions with forbidden singularities}\}$  approximates that of  $C^\infty(M, \mathbb{R}^n) \setminus \{\text{functions with forbidden singularities}\}$ . These spaces, consisting of some sort of polynomials, were defined. That these spaces are generic follows by an application of multijet Thom transversality. The idea is to define submanifolds in a multijet bundle of functions  $M \rightarrow \mathbb{R}^n$  such that transversality implies the generic position of the affine space.

[GG] M. Golubitsky and V. Guillemin, *Stable mappings and their singularities*, Springer-Verlag, Graduate Texts in Math. Series, revised version (1980).

[V1] V. A. Vassiliev, *Topology of spaces of functions without compound singularities*, Funktsional Anal. i Prilozhen 93 no. 4 (1989), p. 24-36; English translation in *Funct. Analysis Appl.* 23 (1989), p.277-286.

[V2] V. A. Vassiliev, *Complements of discriminants of smooth maps: topology and applications*, Transl. of Math. Monographs Vol. 98, revised edition, Amer. Math. Soc. (1994).

## Spaces of functions with moderate singularities

DAN FULEA

The talk used the results and techniques of the last two talks in order to touch the main points in the proof of the following

**Theorem [First Main Theorem, [V1], [V2] as in preceding abstract]:**

Fix  $k, m, n$ . Let  $\mathcal{A}$  be a closed, semialgebraic set inside the jet space  $J^k(\mathbb{R}^m, \mathbb{R}^n)$ , which is invariant under the action of the diffeomorphisms of  $\mathbb{R}^m$ . ( $\mathcal{A}$  models the singularities and its complement the *moderate* ones.)

Let  $M$  be a smooth manifold of dimension  $m$ . Consider  $Z$ , the *space of smooth functions*  $M \rightarrow \mathbb{R}^n$ , and  $W$ , the *space of sections* of the jet bundle  $J^k(M, \mathbb{R}^n)$ .

Local coordinate transport of  $\mathcal{A}$  from  $\mathbb{R}^m$  to  $M$  introduces the subspaces  $Z_{\mathcal{A}} \subset Z$  and  $W_{\mathcal{A}} \subset W$  of elements with  $\mathcal{A}$ -singularities inside  $Z, W$ .

The jet prolongation map  $Z \rightarrow W$ ,  $f \rightarrow (x \rightarrow (j^k f)_x)$ , realizes  $Z$  as the integrable sections inside  $W$ . The theorem claims that the map

$$Z \setminus Z_{\mathcal{A}} \rightarrow W \setminus W_{\mathcal{A}}$$

is an isomorphism in integral homology in case  $\text{codim } \mathcal{A} \geq m + 2$ .

Using the **excellent guide** [We], I explained the following steps of the proof:

(1) Approximation by finite dimensional subspaces of functions  $D \subset Z$  (and  $D' \subset W$ ),  $d := \dim D < \infty$ , and reduction of the problem to the explicit computation of the homology of  $D \setminus D_{\mathcal{A}}$  for a suitable  $D$  in “general position”.

(2) ALEXANDER DUALITY gives  $H^{\bullet}(D \setminus D_{\mathcal{A}}) \cong H_{d-1-\bullet}^{\text{lf}}(D_{\mathcal{A}})$ .

(3) Let  $RD_{\mathcal{A}}$  be the “resolution” of  $D_{\mathcal{A}}$  given by the following construction of simplicial type:  $RD_{\mathcal{A}}$  is the set of all  $(x, w; f) \in \bigsqcup_{p \geq 1} M^p \times \Delta_{p-1} \times D_{\mathcal{A}}$ , such that  $f \in D_{\mathcal{A}}$  has singularities in  $x = (x_1, \dots, x_p)$  and  $w$  is an element of the simplex  $\Delta_{p-1} \subset \mathbb{R}^p$ . Then the map  $RD_{\mathcal{A}} \rightarrow D_{\mathcal{A}}$ ,  $(x, w; f) \rightarrow f$  is an  $H_{\bullet}$ -isomorphism.

(4) An appropriate truncation  $RD_{\mathcal{A}}^p$  of  $RD_{\mathcal{A}}$  leads to a spectral sequence

$H_{d-1+p+q}^{\text{lf}}(RD_{\mathcal{A}}^p, RD_{\mathcal{A}}^{p-1}) \Rightarrow H_{d-1+p+q}^{\text{lf}}(RD_{\mathcal{A}})$ . Its convergence needs  $\text{codim } \mathcal{A} \geq m + 2$ .

(5) Observe that  $H_{d-1-\bullet}^{\text{lf}}(RD_{\mathcal{A}}^p, RD_{\mathcal{A}}^{p-1}) \cong H_{d-1-\bullet}^{\text{lf}}(RD_{\mathcal{A}}^p \setminus RD_{\mathcal{A}}^{p-1})$

(6) Consider in parallel to (3) the space  $\Delta(\mathcal{A}(M))$  of all  $(x, w) \in \bigsqcup_{p \geq 1} M^p \times \Delta_{p-1}$  and the appropriate truncation  $\Delta(\mathcal{A}(M))^p$ .

(7) There is an  $H_{\bullet}^{\text{lf}}$ -isomorphism  $RD_{\mathcal{A}}^p \setminus RD_{\mathcal{A}}^{p-1} \rightarrow \Delta(\mathcal{A}(M))^p \setminus \Delta(\mathcal{A}(M))^{p-1}$ , given by  $(x, w; f) \rightarrow (x, w)$ .

(8) Collecting all isomorphisms, we can functorially express  $H^{\bullet}(D \setminus D_{\mathcal{A}})$  by a convergent spectral sequence with relevant entries, which are independent of  $D$  approximating  $Z$ . Same independence can be transposed for  $H^{\bullet}(D' \setminus D'_{\mathcal{A}})$  with  $D'$  approximating  $W$ .

[We] **M Weiss** *Cohomology of the stable mapping class group*, to appear in *Topology, Geometry and Quantum Field theory*, proceedings of 2002 Oxford conf., Cambridge University Press.

## Second desingularization procedure

DAG OLAV KJELLEMO

We define the sheaves  $\mathcal{W}$  and  $h\mathcal{W}$ , for each manifold  $X$  consisting of pairs  $(\pi, \hat{f})$ , similarly to the sheaves  $\mathcal{V}$  and  $h\mathcal{V}$ , but with  $\hat{f}$  being of Morse type on the fibres of  $\pi : E \rightarrow X$ . The first main result is that the classifying space  $h\mathcal{W}$  is homotopy equivalent to  $\Omega^{\infty} \mathbf{hW}$ , where  $\mathbf{hW}$  is a certain Thom spectrum. This space fits in the fibre sequence of talk 10. The second main result is to show that the jet prolongation map  $j_{\pi}^2 : \mathcal{W} \rightarrow h\mathcal{W}$  induces a homotopy equivalence on classifying spaces. The steps to do this are summarized by the diagram

$$\begin{array}{ccccccc} |\mathcal{W}| & \xrightarrow{\cong} & |\mathcal{W}^0| & \xleftarrow{\cong} & |\beta\mathcal{W}^{\mathcal{A}}| & \simeq & B|\mathcal{W}^{\mathcal{A}}| \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \simeq \\ |h\mathcal{W}| & \xrightarrow{\cong} & |h\mathcal{W}^0| & \xleftarrow{\cong} & |\beta h\mathcal{W}^{\mathcal{A}}| & \simeq & B|h\mathcal{W}^{\mathcal{A}}| \end{array}$$

The sheaves  $\mathcal{W}^0$  and  $h\mathcal{W}^0$  only require  $\hat{f}$  to be fibrewise Morse in a neighbourhood of  $f^{-1}(0)$  (where  $f$  is underlying function of  $\hat{f}$ ); this gives the right codimension conditions required by Vassiliev's theorem applied below.  $\mathcal{W}^{\mathcal{A}}$  and  $h\mathcal{W}^{\mathcal{A}}$  are sheaves of triples  $(\pi, \hat{f}, A)$ , where  $(\pi, \hat{f})$  are in  $\mathcal{W}^0$  or  $h\mathcal{W}^0$ , and  $A \subset \mathbf{R}$  is an “armlet” for  $(\pi, \hat{f})$ .  $f^{-1}(A)$  are then

fibre bundles over  $X$ , and we essentially apply Vassiliev's theorem fibrewise. Technically this is achieved by defining a sheaf  $\mathcal{T}^{\mathcal{A}}$  and maps  $p$  and  $q$  giving a commutative diagram

$$\begin{array}{ccc} \mathcal{W}^{\mathcal{A}} & \xrightarrow{j_{\pi}^2} & h\mathcal{W}^{\mathcal{A}} \\ & \searrow p & \swarrow q \\ & \mathcal{T}^{\mathcal{A}} & \end{array}$$

We define fibre sheaves  $p^{-1}(\tau)$  and  $q^{-1}(\tau)$  over elements  $\tau \in \mathcal{T}^{\mathcal{A}}(*)$ . Their representing spaces are homology equivalent by Vassiliev's theorem, and this in turn induces a homology equivalence  $B|\mathcal{W}^{\mathcal{A}}| \rightarrow B|h\mathcal{W}^{\mathcal{A}}|$ . (These spaces are actually homotopy co-limits of functors from  $\mathcal{T}^{\mathcal{A}}$ .) Lastly, this map must also be a homotopy equivalence since we know that  $|h\mathcal{W}|$  is group complete.

### Localisation

CHRISTIAN SERPÉ

In this talk we introduce the sheaves  $\mathcal{W}_{loc}$  and  $h\mathcal{W}_{loc}$  and have a look at the diagram

$$\begin{array}{ccccc} |\mathcal{V}| & \longrightarrow & |\mathcal{W}| & \longrightarrow & |\mathcal{W}_{loc}| \\ \downarrow jet & & \downarrow jet & & \downarrow jet \\ |h\mathcal{V}| & \longrightarrow & |h\mathcal{W}| & \longrightarrow & |h\mathcal{W}_{loc}|. \end{array}$$

The horizontal maps are just inclusions of the sheaves and the vertical maps come from the jet prolongation. We show that the bottom row is a homotopy fibre sequence and that the map in the right column is a homotopy equivalence. The main ingredients for the proofs are the Thom–Pontryagin construction, obstruction theory, immersion theory and submersion theory.

### Homotopy Co-limits

MARKUS SZYMIK

The homotopy co-limit of a diagram of spaces is much like a co-limit. However, it is invariant under objectwise equivalences. In the talk, homotopy co-limits were defined as actual co-limits of suitable resolutions. To show existence, transport categories were used. These lead to good functoriality properties, which were also discussed in some detail. At the end, an application to sheaves was presented.

**Homotopy co-limit decomposition  
and  
return to surface bundles**

MARIA CASTILLO, BALASZ VISY, SEBASTIAN GRENSING,  
CARL-FRIEDRICH BÖDIGHEIMER, HANNES EBERT

The three talks 12, 13 and 14 were concerned with the following diagram of sheaves:

$$\begin{array}{ccc}
 \mathcal{W} & \longrightarrow & \mathcal{W}_{\text{loc}} \\
 \uparrow \simeq & & \downarrow \simeq \\
 \mathcal{L} & \longrightarrow & \mathcal{L}_{\text{loc}} \\
 \uparrow \simeq & & \uparrow \simeq \\
 \text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{L}_T & \longrightarrow & \text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{L}_{\text{loc},T} \\
 \downarrow \simeq & & \downarrow \simeq \\
 \text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{W}_T & \longrightarrow & \text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{W}_{\text{loc},T}
 \end{array}$$

All sheaves in this diagram, apart from  $\mathcal{W}$  and  $\mathcal{W}_{\text{loc}}$ , and all maps with exception of the top horizontal one were defined. Then all vertical maps were shown to be weak equivalences. The final result is therefore: One can replace the top map

$$\mathcal{W} \longrightarrow \mathcal{W}_{\text{loc}}$$

by the bottom map

$$\text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{W}_T \longrightarrow \text{hocolim}_{T \text{ in } \mathfrak{K}} \mathcal{W}_{\text{loc},T} .$$

For fixed  $T$ , the fibre of the map

$$\mathcal{W}_T \longrightarrow \mathcal{W}_{\text{loc},T}$$

is known to be a sheaf of surface bundles.

**Surgery, I**

PAUL MITCHENER

We have seen in the earlier talks that we have a commutative diagram of sheaves

$$\begin{array}{ccccc}
 \mathcal{W} & \rightarrow & \mathcal{W}_{\text{loc}} & & \\
 & & \downarrow & & \\
 h\mathcal{V} & \rightarrow & h\mathcal{W} & \rightarrow & h\mathcal{W}_{\text{loc}}
 \end{array}$$

where the lower row is a homotopy fibre sequence, and the vertical arrows are weak equivalences. In order to prove the Mumford conjecture, we need to identify the sheaf  $\mathcal{V}$  with the homotopy-fibre of the canonical map  $\mathcal{W} \rightarrow \mathcal{W}_{\text{loc}}$ .

In this talk, we indicate the first steps of this process. The obstacle is that the space  $|\mathcal{W}|$  is a classifying space for certain surface bundles; we need to do some sort of surgery to replace  $|\mathcal{W}|$  by the corresponding classifying space for *connected* surface bundles.

## Surgery, II

THOMAS SCHICK

The goal of this talk is a proof of theorem 6.1.4. This is the basic theorem which explains how to pass from not necessarily connected surfaces to connected surfaces.

This is done by a well organized surgery (i.e. connected sum) process. To be able to do this, we have to enlarge our sheaves by adding the corresponding surgery data. One half of the task is to show that this does not change the homotopy type of the classifying space.

The second half of the task is to show that the connected sum procedure gives rise to a homotopy inverse to the map induced by the inclusion of the set of connected surfaces into the set of all surfaces.

### Reduction to Harer's theorem and conclusion

THILO KUESSNER

We show that there is a homotopy fibration

$$\mathbb{Z} \times B\Gamma_{\infty,2}^+ \rightarrow |\mathcal{W}| \rightarrow |\mathcal{W}_{loc}|.$$

This finishes the proof of Mumford's conjecture.

The proof relies on the homology stability of surface mapping class groups.

Other activities during the meeting:

- evening sessions after the dinner with the purpose to catch up on the untold and clarify the told;
- walk to a restaurant 10 kilometres away from MFO in order to have some cakes;
- discussion of the subject for the next Arbeitsgemeinschaft;
- piano and flute concert performed by Paolo Salvatore and Michael Weiss.

*Edited by Ivan Izmestiev*



## Participants

**Prof. Dr. Joseph Ayoub**

jayoub@clipper.ens.fr  
61, rue Gabriel Peri  
F-75005 Paris VII

**Prof. Dr. Carl-Friedrich Bödigheimer**

boedigheimer@math.uni-bonn.de  
Mathematisches Institut  
Universität Bonn  
Beringstr. 1  
D-53115 Bonn

**Dr. Oliver Bültel**

bueltel@mathi.uni-heidelberg.de  
Mathematisches Institut  
Universität Heidelberg  
Im Neuenheimer Feld 288  
D-69120 Heidelberg

**Prof. Dr. Ulrich Bunke**

bunke@uni-math.gwdg.de  
Mathematisches Institut  
Georg-August-Universität  
Bunsenstr. 3-5  
D-37073 Göttingen

**Maria Castillo**

castillo@math.uni-bonn.de  
Mathematisches Institut  
Universität Bonn  
Beringstr. 1  
D-53115 Bonn

**Prof. Dr. Christopher Deninger**

deninger@math.uni-muenster.de  
Mathematisches Institut  
Universität Münster  
Einsteinstr. 62  
D-48149 Münster

**Johannes Ebert**

jebert@uni-bonn.de  
Mathematisches Institut  
Universität Bonn  
Beringstr. 1  
D-53115 Bonn

**Dr. Konstantin Feldman**

feldman@maths.ed.ac.uk  
School of Mathematics  
University of Edinburgh  
James Clerk Maxwell Bldg.  
King's Building, Mayfield Road  
GB-Edinburgh, EH9 3JZ

**Dr. Dan Fulea**

fulea@euklid.math.uni-mannheim.de  
dan@mathi.uni-heidelberg.de  
Mathematisches Institut  
Universität Heidelberg  
Im Neuenheimer Feld 288  
D-69120 Heidelberg

**Dr. Swiatoslaw R. Gal**

sgal@math.uni.wroc.pl  
Institute of Mathematics  
Wroclaw University  
pl. Grunwaldzki 2/4  
50-384 Wroclaw – Poland

**Prof. Dr. Soren Galatius**

galatius@imf.au.dk  
Department of Mathematical Sciences  
University of Aarhus  
Building 530  
Ny Munkegade  
DK-8000 Aarhus C

**Sebastian Grensing**

grensing@math.uni-bonn.de  
Mathematisches Institut  
Universität Bonn  
Beringstr. 1  
D-53115 Bonn

**Prof. Dr. Frank Herrlich**

Frank.Herrlich@math.uni-karlsruhe.de  
Mathematisches Institut II  
Universität Karlsruhe  
D-76128 Karlsruhe

**Dr. Thilo Kuessner**

kuessner@mathematik.uni-muenchen.de  
Mathematisches Institut  
Universität München  
Theresienstr. 39  
D-80333 München

**Dr. Jens Hornbostel**

jens.hornbostel@mathematik.uni-regensburg.de  
Naturwissenschaftliche Fakultät I  
Mathematik  
Universität Regensburg  
D-93040 Regensburg

**Katharina Ludwig**

ludwig@math.uni-hannover.de  
Institut für Mathematik  
Universität Hannover  
Welfengarten 1  
D-30167 Hannover

**Dr. Ivan Izmestiev**

izmestie@math.fu-berlin.de  
Institut für Informatik  
Freie Universität Berlin  
Takustr. 9  
D-14195 Berlin

**Dr. Gregor Masbaum**

masbaum@math.jussieu.fr  
U. F. R. de Mathématiques  
Case 7012  
Universite de Paris VII  
2, Place Jussieu  
F-75251 Paris Cedex 05

**Prof. Dr. Sadok Kallel**

Dadok.Kallel@agat.univ-lille1.fr  
UFR de Mathématiques  
Cité Scientifique  
Bâtiment M2  
F-59655 Villeneuve d'Ascq Cedex

**Dr. Paul Mitchener**

mitch@uni-math.gwdg.de  
Mathematisches Institut  
Georg-August-Universität  
Bunsenstr. 3-5  
D-37073 Göttingen

**Dr. Jarek Kedra**

kedra@univ.szczecin.pl  
Mathematisches Institut  
Universität München  
Theresienstr. 39  
D-80333 München

**Niko Naumann**

naumannn@uni-muenster.de  
Fachbereich Mathematik  
Universität Münster  
Einsteinstr. 62  
D-48149 Münster

**Dr. Dag Olav Kjellemo**

kjellemo@math.ntnu.no  
NTNU Trondheim  
N-7491 Trondheim

**Gereon Quick**

gquick@math.uni-muenster.de  
Fachbereich Mathematik  
Universität Münster  
Einsteinstr. 62  
D-48149 Münster

**Dr. Stefan Kühnlein**

stefan.kuehnlein@mathematik.uni-karlsruhe.de  
Mathematisches Institut II  
Universität Karlsruhe  
D-76128 Karlsruhe

**Prof. Dr. Paolo Salvatore**  
salvator@axp.mat.uniroma2.it  
salvator@mat.uniroma2.it  
Dipartimento di Matematica  
Universita di Roma "Tor Vergata"  
V.della Ricerca Scientifica, 1  
I-00133 Roma

**Thomas Schick**  
schick@uni-math.gwdg.de  
Mathematisches Institut  
Georg-August-Universität  
Bunsenstr. 3-5  
D-37073 Göttingen

**Dr. Alexander Schmidt**  
schmidt@mathi.uni-heidelberg.de  
Mathematisches Institut  
Universität Heidelberg  
Im Neuenheimer Feld 288  
D-69120 Heidelberg

**Dr. Christian Serpe**  
serpe@uni-muenster.de  
SFB 478  
Geom. Strukturen in der Mathematik  
Universität Münster  
Hittorfstr. 27  
D-48149 Münster

**Markus Szymik**  
szymik@mathematik.uni-bielefeld.de  
Fakultät für Mathematik  
Universität Bielefeld  
Universitätsstr. 25  
D-33615 Bielefeld

**Balazs Visy**  
visy@math.uni-bonn.de  
Mathematisches Institut  
Universität Bonn  
Beringstr. 1  
D-53115 Bonn

**Prof. Dr. Rainer Vogt**  
rainer.vogt@mathematik.uni-osnabrueck.de  
Fachbereich Mathematik/Informatik  
Universität Osnabrück  
D-49069 Osnabrück

**Dr. Nathalie Wahl**  
wahl@imf.au.dk  
Department of Mathematical Sciences  
University of Aarhus  
Building 530  
Ny Munkegade  
DK-8000 Aarhus C

**Julia Weber**  
julia.weber@math.uni-muenster.de  
Mathematisches Institut  
Universität Münster  
Einsteinstr. 62  
D-48149 Münster

**Dr. Michael Weiss**  
m.weiss@maths.abdn.ac.uk  
Department of Mathematics  
University of Aberdeen  
GB-Aberdeen AB24 3UE

**Dr. Sigrid Wortmann**  
wortmann@mathi.uni-heidelberg.de  
Mathematisches Institut  
Universität Heidelberg  
Im Neuenheimer Feld 288  
D-69120 Heidelberg