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Motives and Homotopy Theory of Schemes

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Introduction by the Organisers

This field of motives and homotopy theory of schemes has made rapid advances recently, most visible in the Fields Medal winning proof of the Milnor conjecture by Voevodsky.

This meeting was an attempt to bring together researchers working in the fields related to motives and the homotopy theory of schemes. It was attended by 45 researchers from 12 countries, bringing together people working in algebraic geometry, algebraic number theory, K-theory, quadratic forms and algebraic topology among others, whose work is related to motives and the homotopy theory of schemes. The meeing was organized by Thomas Geisser, Bruno Kahn, and Fabien Morel, and consisted of 18 talks.

The theory of motives goes back to Grothendieck's ideas in the sixties, in which he tried to understand unexplained phenomena related to the cohomology of algebraic varieties. He was able to define pure motives, which — together with some still unproven conjectures — account for the cohomology of smooth projective varieties over a field. He expected the existence of a category of mixed motives $MM(k)$, containing pure motives as its semi-simple part, and such that any k-variety X should have "universal" cohomology groups $h^i(X) \in MM(k)$. Deligne and especially Beilinson in the early eighties suggested that one might more easily construct $MM(k)$ as the heart of a t-structure on a triangulated category $D(k)$ that would be constructed first. Hanamura, Levine and Voevodsky constructed candidates for $D(k)$. In these categories, at least if k is of characteristic 0, any k-scheme X has two associated objects: its motive $M(X)$ and its motive with compact support (or Borel-Moore motive) $M^c(X)$. There is a canonical functor from pure motives $D(k)$ which for X smooth projective carries $h(X)$ onto $M(X)$ or its dual, according to the variance conventions. Taking Hom groups from or to $M(X)$ or $M^c(X)$ to or from various shifted Tate twists defines motivic cohomology, motivic homology, motivic cohomology with proper supports and Borel-Moore motivic homology. These various theories can also be described more concretely: for example, Borel-Moore motivic homology coincides (up to reindexing) with Bloch's higher Chow groups and motivic homology in weight zero is Suslin's homology. Unfortunately, the motivic t-structure on any of these categories is out of reach at the moment.

The homotopy and stable homotopy theory of schemes are much more recent constructions and were developed by Morel and Voevodsky in the nineties. To any field k are associated the homotopy category $\mathcal{H}(k)$ and stable homotopy category $\mathcal{SH}(k)$ of k-schemes. The former is a symmetric monoidal category while the latter is a tensor triangulated category. There are adjoint functors $H : DM(k) \to \mathcal{SH}(k)$ (Eilenberg-Mac Lane functor) and $C : \mathcal{SH}(k) \to DM(k)$ (chain complex functor), where $DM(k)$ is a version of Voevodsky's triangulated category of motives. These functors are analogous to those defined in algebraic topology with $\mathcal{SH}(k)$ replaced by the stable homotopy category and $DM(k)$ replaced by the derived category of abelian groups. Important cohomology theories are representable in $\mathcal{SH}(k)$: for example motivic cohomology and Weibel's homotopy invariant algebraic K-theory. An example of new cohomology theory arising from $\mathcal{SH}(k)$ is algebraic cobordism. The Hom groups with integral coefficients are much richer in $\mathcal{SH}(k)$ than in $DM(k)$: for example, Steenrod operations on mod p motivic cohomology arise from $\mathcal{SH}(k)$, and it is in this way that Voevodsky proved the Milnor conjecture.

The talks covered a wide range of aspects of the subject including some not covered by the above description, like arithmetic and p-adic questions and quadratic forms. We had the impression of a genuine get-together, with participants from very different backgrounds taking the opportunity to discuss with each other and having a very good attendance to all talks, not just those on which they were specialists. This made a very pleasant and, we hope, useful meeting.

Workshop on Motives and Homotopy Theory of Schemes

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Abstracts

The absolute and relative de Rham-Witt complexes

Lars Hesselholt

Let $f\colon X \to S$ be a morphism of noetherian $\mathbb{Z}_{(p)}$-schemes and suppose that p is odd and nilpotent on S. There is a canonical surjective map

$$W_n\Omega^{\cdot}_X \to W_n\Omega^{\cdot}_{X/S}$$

from the absolute de Rham-Witt complex of X considered by the author and Madsen to the relative de Rham-Witt complex of X/S considered by Langer and Zink. In the classical case, where S is a perfect $\mathbb{F}_p$-scheme, the map is an isomorphism and the common complex coincides with the de Rham-Witt complex of Bloch, Deligne, and Illusie. In general, the terms of the complexes are quasi-coherent $W_n(\mathcal{O}_X)$-modules on the small étale site of X, and the differential of the relative de Rham-Witt complex is $f^{-1}W_n(\mathcal{O}_S)$-linear. The kernel $\mathcal{I}$ of the projection is equal to the differential graded ideal generated by the image of the canonical map $f^{-1}W_n\Omega^1_S \to W_n\Omega^1_X$. The graded pieces for the $\mathcal{I}$-adic filtration are differential graded modules over the differential graded ring $W_n\Omega^{\cdot}_{X/S}$, and hence, complexes of quasi-coherent $f^{-1}W_n(\mathcal{O}_S)$-modules on the small étale site of X. We show that the absolute and relative de Rham-Witt complexes are related as follows.

Theorem. *Let $f\colon X \to S$ be a smooth morphism of noetherian $\mathbb{Z}_{(p)}$-schemes and suppose that p is odd and nilpotent on S. Then there is a canonical isomorphism*

$$f^{-1}W_n\Omega^s_S \otimes^{\mathbb{L}}_{f^{-1}W_n(\mathcal{O}_S)} W_n\Omega^{\cdot-s}_{X/S} \xrightarrow{\sim} \operatorname{gr}^s_{\mathcal{I}} W_n\Omega^{\cdot}_X$$

in the derived category of quasi-coherent $f^{-1}W_n(\mathcal{O}_S)$-modules.

By Langer and Zink there is a canonical isomorphism

$$H^q_{\mathrm{crys}}(X/W_n(S)) \xrightarrow{\sim} H^q(X, W_n\Omega^{\cdot}_{X/S})$$

provided that $S = \operatorname{Spec} R$ is affine. Suppose that the groups $H^q_{\mathrm{crys}}(X/W_n(S))$ are flat $W_n(R)$-modules. This is true, for instance, if X is an abelian S-scheme. Then the isomorphism of the theorem gives rise to an integrable Gauss-Manin connection

$$\nabla\colon H^q_{\mathrm{crys}}(X/W_n(S)) \to W_n\Omega^1_R \otimes_{W_n(R)} H^q_{\mathrm{crys}}(X/W_n(S)).$$

The structure of $W_n\Omega^1_R$ is in general not well-understood, but we have

$$W_n\Omega^1_{\mathbb{Z}_{(p)}} = \prod_{1\le s<n} \mathbb{Z}/p^s\mathbb{Z}\cdot dV^s([1]_{n-s}).$$

The Gauss-Manin connection constructed here is closely related to the monodromy operator of Hyodo and Kato on log-crystalline cohomology.

On the derived category of 1 -motives

Luca Barbieri-Viale

In the first part of the talk (mostly a report of a work in progress jointly with B. Kahn [1]) I made a survey of the theory of 1-motives. The theory is originated by a remark of Grothendieck (see [8]) that the theory of motives h^1 of smooth projective curves is equivalent to the theory of abelian varieties (up to isogeny). Deligne [6] gave a first definition of 1-motives and made some conjectures. I described the *abelian* category $\mathcal{M}_1(k;\mathbb{Z})$ of 1-motives, with torsion, over a field k (introduced in [3] and also studied in [1]) and their Hodge, ℓ-adic, crystalline and De Rham realizations. I mentioned that the subcategory of free objects (= Deligne's 1-motives) is an exact subcategory (in the sense of Quillen) whose derived category is equivalent to $D^b(\mathcal{M}_1(k;\mathbb{Z}))$ (proven in [1]). I mentioned a comparison theorem (see [2]) between the crystalline realisation of a free 1-motive in positive characteristic and the De Rham realisation of a lifting in zero characteristic. Further mentionated constructions and results on Picard and Albanese 1-motives can be found in [5] and [10]. In [4] a Grothendieck-Hodge conjecture for singular varieties is stated. See [3] for a proof of Deligne's conjectures.

In the second part of the talk I considered the derived category of 1-motives up to isogeny, *i.e.*, $D^b(\mathcal{M}_1(k;\mathbb{Q}))$, along with its fully-faithful embedding into Voevodsky's triangulated category of motives

$$\mathrm{Tot} : D^b(\mathcal{M}_1(k;\mathbb{Q})) \to \mathrm{DM}^{\mathrm{eff}}_{\mathrm{gm}}(k;\mathbb{Q})$$

(see [12] and [9]). Regarding Tot as a universal realisation functor we show in [1] that it has a left adjoint

$$\mathrm{LAlb} : \mathrm{DM}^{\mathrm{eff}}_{\mathrm{gm}}(k;\mathbb{Q}) \to D^b(\mathcal{M}_1(k;\mathbb{Q})).$$

Dually, composing with Cartier duality, we obtain RPic.

These functors provide natural complexes of 1-motives (up to isogeny) $\mathrm{LAlb}(X)$, $\mathrm{LAlb}^c(X)$, $\mathrm{LAlb}^*(X)$, $\mathrm{RPic}(X)$, $\mathrm{RPic}^c(X)$ and $\mathrm{RPic}^*(X)$ of an algebraic variety X over k. We can compute the 1-motivic homology $\mathrm{L}_i\mathrm{Alb}(X)$ and cohomology $\mathrm{R}^i\mathrm{Pic}(X)$ for X smooth or a singular curve. We recover in this way Deligne-Lichtenbaum motivic (co)homology of curves (see [6] and [7]). We also recover the previously mentioned Picard and Albanese 1-motives. Note that the counit

$$a_X : M(X) \to \mathrm{TotLAlb}(X)$$

provide a universal map in $\mathrm{DM}^{\mathrm{eff}}_{\mathrm{gm}}(k;\mathbb{Q})$, the motivic Albanese map, which is an isomorphism if $\dim(X) \leq 1$ and it 'contains' the classical Albanese map (and the less classical map in [11]).

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On the étale homotopy type of Morel-Voevodsky spaces

Alexander Schmidt

In the 1960s Artin and Mazur [AM] constructed a functor which associates to each locally noetherian scheme X its *étale homotopy type* X_{et}, an object of pro$-\mathcal{H}$, the pro-category of the homotopy category $\mathcal{H}$ of simplicial sets. For any geometric point x of X, the (pro)groups $\pi_i((X,x)_{et})$, $i \geq 1$, give a natural definition of homotopy groups in algebraic geometry. Artin and Mazur proved that the étale homotopy type X_{et} of a smooth complex variety X is isomorphic in pro$-\mathcal{H}$ to the profinite completion of the topological space $X(\mathbb{C})$. This strongly refined previously known comparison theorems between étale and singular cohomology and led to interesting applications to both algebraic geometry and topology, most prominent, the proofs of the Adam's conjecture given by Friedlander/Quillen [Fr1] and by Sullivan [Su].

In the 1990s Morel and Voevodsky [MV] defined a natural categorical framework for the use of topological methods in algebraic geometry. They embedded the category of smooth schemes of finite type over a field k into a larger category of 'k-spaces', which carries the structure of a closed model category, namely the $\mathbb{A}^1$-model structure. The associated homotopy category is the celebrated $\mathbb{A}^1$-homotopy category of smooth schemes over k.

In this talk we explain (following [Sc]) how both concepts interact. Let k be a field and let $Sm(k)$ be the category of smooth schemes of finite type over k. The first observation is the existence of a connected component functor

$$\Pi : Shv_{et}(Sm(k)) \longrightarrow Sets,$$

which has the property that the connected components of a representable sheaf are the scheme-theoretic ones. This functor naturally extends to a functor Π :

$\Delta^{op} Shv_{et}(Sm(k)) \to \Delta^{op} Sets$. The second observation is that hypercoverings can be replaced by trivial local fibrations ([MV]) in the Artin-Mazur construction. One obtains a natural functor

$$et : \Delta^{op} Shv_{et}(Sm(k)) \longrightarrow \text{pro}-\mathcal{H}$$

such that the composite $Sm(k) \to \Delta^{op} Shv_{et}(Sm(k)) \to \text{pro}-\mathcal{H}$ is the functor 'étale homotopy type' of Artin-Mazur. The key observation is

Theorem 1. *Let M be a simplicial set (also considered as a constant simplicial sheaf) and let $X \in \Delta^{op} Shv_{et}(Sm(k))$. Then there exists a natural isomorphism*

$$Hom_{\mathcal{H}_{s,et}(Sm(k))}(X, M) \cong Hom_{\text{pro}-\mathcal{H}}(X_{et}, M).$$

Using theorem 1 one then can prove:

Theorem 2. *The functor et factors through the simplicial homotopy category, i.e. induces a functor*

$$et : \mathcal{H}_{s,et}(Sm(k)) \longrightarrow \text{pro}-\mathcal{H}.$$

If $char(k) = 0$ and k is of finite virtual cohomological dimension, then et factors through the $\mathbb{A}^1$-homotopy category, i.e. we obtain a functor

$$et : \mathcal{H}_{\mathbb{A}^1,et}(Sm(k)) \longrightarrow \text{pro}-\mathcal{H}.$$

If k has positive characteristic, then the affine line has a highly non-trivial fundamental group [Ra] and the étale homotopy type does not factor through the $\mathbb{A}^1$-homotopy category. However, factorization holds after completion away from the characteristics (this works, more generally, over any base scheme). Via the base change functor from Nisnevich to étale topology, the above results apply also to the (usual) categories which are built using the Nisnevich-topology. In particular, we can attach étale homotopy groups $\pi_i^{et}(X, *)$ to any (geometrically) pointed object in the $\mathbb{A}^1$-homotopy category.

Finally, we discussed related results by D. Isaksen [Is] who used a completely different method.

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Postnikov towers in $\mathbb{A}^1$-homotopy theory and the homotopy coniveau tower

Marc Levine

1. Introduction

Recall the Morel-Voevodsky $\mathbb{A}^1$-stable homotopy category $\mathcal{SH}(k)$, being the homotopy category of the category of $\mathbb{P}^1$-spectra over k, $\mathbf{Spt}_{\mathbb{P}^1}(k)$ (see [4] for an introduction to these concepts). Voevodsky [5] has defined the *slice filtration* on the $\mathbb{A}^1$-stable homotopy category $\mathcal{SH}(k)$, as a tower of functors

$$\ldots \to \nu^{\geq n+1} \to \nu^{\geq n} \to \ldots \to \mathrm{id}$$

from $\mathcal{SH}(k)$ to itself. The arrow $\nu^{\geq n}\mathcal{E} \to \mathcal{E}$ is universal for morphisms $\mathcal{F} \to \mathcal{E}$, where $\mathcal{F}$ is in the subcategory $\Sigma^n_{\mathbb{P}^1}\mathcal{SH}(k)^{eff}$. We let $\nu_n\mathcal{E}$ denote the cofiber of $\nu^{\geq n+1}\mathcal{E} \to \nu^{\geq n}\mathcal{E}$.

Let $\mathcal{H}\mathbb{Z}$ be the $\mathbb{P}^1$-spectrum representing motivic cohomology. One important result regarding the functors ν_n is

Theorem 1.1 (Voevodsky [6], Levine [3])**.** *Let k be a perfect field. For $\mathcal{E}$ in $\mathcal{SH}(k)$, the layers $\nu_n\mathcal{E}$ have the natural sturcture of an $\mathcal{H}\mathbb{Z}$-module. For the sphere spectrum §, we have a canonical isomorphism $\nu_0§ \cong \mathcal{H}\mathbb{Z}$.*

We have the "motivic Eilenberg-Maclane" functor $\mathcal{H} : \mathrm{DM}(k) \to \mathcal{SH}(k)$. By an argument of Morel, a $\mathbb{P}^1$-spectrum $\mathcal{E}$ which is given the structure of an $\mathcal{H}\mathbb{Z}$-module is of the form $\mathcal{H}m(\mathcal{E})$ for some motive $m(\mathcal{E})$ in $\mathrm{DM}(k)$; the isomorphism class of $m(\mathcal{E})$ is canonically determined by the $\mathcal{H}\mathbb{Z}$-module structure on $\mathcal{E}$. This justifies the definition:

Definition 1.2. Let $\pi^\mu_n(\mathcal{E})$ be the object of $\mathrm{DM}(k)$ with $\pi^\mu_n(\mathcal{E})[n] = m(\nu_n(\mathcal{E}))$.

Example 1.3. In [3] we compute: Let $\mathcal{K}$ be the $\mathbb{P}^1$-spectrum representing algebraic K-theory. Then

$$\pi^\mu_n(\mathcal{K}) = \mathbb{Z}(n)[n].$$

The slice tower thus yields the spectral sequence of Atiyah-Hirzebruch type

$$E_2^{p,q} := H^p(X_{\mathrm{Nis}}, \pi^\mu_{-q}(\mathcal{E})) \Longrightarrow \mathcal{E}^{p+q,0}(X)$$

for X in $\mathbf{Sm}/k$.

In this abstract, we describe a relation of the functors $\nu^{\geq n}$ with the coniveau filtration.

2. The S^1-stable theory

Let k be a perfect infinite field and let $\mathbf{Spt}(k)$ be the category of functors $E : \mathbf{Sm}/k^{\mathrm{op}} \to \mathbf{Spt}$, where $\mathbf{Spt}$ is the category of spectra. The objects of $\mathbf{Spt}(k)$ are called *S^1-spectra over k*.

For E in $\mathbf{Spt}(k)$, $Y \in \mathbf{Sm}/k$ and $W \subset Y$ a closed subset, set $E^W(Y) := \mathrm{fib}(E(Y) \to E(Y \setminus W))$. Let $\Omega_{\mathbb{P}^1} E : \mathbf{Sm}/k^{\mathrm{op}} \to \mathbf{Spt}$ be the functor

$$(\Omega_{\mathbb{P}^1} E)(X) := E^{X \times 0}(X \times \mathbb{A}^1).$$

2.1. **The tower.** Let $\Delta^n := \mathrm{Spec}\, k[t_0, \ldots, t_n]/\sum_i t_i - 1$. A *face* of Δ^n is a subscheme defined by an ideal of the form $(t_{i_1}, \ldots, t_{i_s})$. For $X \in \mathbf{Sm}/k$, let $\mathcal{S}_X^{(p)}(n)$ be the set of closed subsets $W \subset X \times \Delta^n$ such that

$$\mathrm{codim}_{X \times F} W \cap (X \times F) \geq p$$

for all faces $F \subset \Delta^n$.

Definition 2.2. For $X \in \mathbf{Sm}/k$. let

$$E^{(p)}(X, n) := \operatorname*{hocolim}_{W \in \mathcal{S}_X^{(p)}(n)} E^W(X \times \Delta^n).$$

This gives the simplicial spectrum $n \mapsto E^{(p)}(X, n)$ with total spectrum denoted $E^{(p)}(X)$. Varying p, we have the tower

$$\ldots \to E^{(p+1)}(X) \to E^{(p)}(X) \to \ldots \to E^{(0)}(X).$$

Let $E^{(p/p+1)}(X)$ denote the cofiber of $E^{(p+1)}(X) \to E^{(p)}(X)$. Note that $X \mapsto E^{(p)}(X)$ is functorial with respect to smooth morphisms.

2.3. **Some axioms.** We impose some axioms:

- A1. E is homotopy invariant: $p_1^* : E(X) \to E(X \times \mathbb{A}^1)$ is a weak equivalence for all $X \in \mathbf{Sm}_k$.
- A2. E satisfies Nisnevic excision: If $f : X' \to X$ is an étale map in $\mathbf{Sm}/k$, and $W \subset X$ is a closed subset such that $f : f^{-1}(W) \to W$ is an isomorphism, then $f^* : E^W(X) \to E^{W'}(X')$ is a weak equivalence.
- A3. $E = \Omega^4_{\mathbb{P}^1} E'$ for some E' satisfying A1 and A2.

From now on, we assume our E's satisfy A1-A3.

Proposition 2.4. (1) *$X \mapsto E^{(p)}(X)$ extends (up to canonical weak equivalence) to a functor $E^{(p)} : \mathbf{Sm}/k^{\mathrm{op}} \to \mathbf{Spt}$*

(2) *$E^{(p)}$ is homotopy invariant and satisfies Nisnevic excision.*

(3) *Let $i : Z \to X$ be a closed codimension d embedding in $\mathbf{Sm}/k$ with trivial normal bundle. Then there is a natural weak equivalence*

$$(\Omega^d_{\mathbb{P}^1} E)^{(p-d)}(Z) \to (E^{(p)})^Z(X).$$

Applying the above to $X \times 0 \to X \times \mathbb{A}^1$ yields

Corollary 2.5. *There is a natural weak equivalence*

$$(\Omega_{\mathbb{P}^1} E)^{(p)} \xrightarrow{\theta_p} \Omega_{\mathbb{P}^1}(E^{(p+1)}).$$

Also, we have

Corollary 2.6. *(1) Let $W \subset Y$ have codimension $> p$, with $Y \in \mathbf{Sm}/k$. Then $(E^{(p/p+1)})^W(Y) \sim *$.*

(2) Let $j : U \to X$ be open and dense, with $X \in \mathbf{Sm}/k$. Then $j^* : E^{(0/1)}(X) \to E^{(0/1)}(U)$ is a weak equivalence.

2.7. **The layers.** The main result on the layers follows from

Proposition 2.8. *$(E^{(p)})^{(q)}$ is weakly equivalent to $E^{(\max(p,q))}$. In particular*

$$(E^{(p/p+1)})^{(p)} \sim E^{(p/p+1)}.$$

Corollary 2.9. *For $X \in \mathbf{Sm}/k$, $E^{(p/p+1)}(X)$ is weakly equivalent to a simplicial spectrum $E^{(p)}_{sl}(X,-)$ with n-simplices*

$$E^{(p)}_{sl}(X,n) \sim \bigvee_{W \in \mathcal{S}^{(p)}_X(n),\ irreducible} (\Omega^p_{\mathbb{P}^1} E)^{(0/1)}(k(W)).$$

Applying this to $E = K$-theory and noting that $\Omega_{\mathbb{P}^1} K = K$, $K^{(0/1)}(k(W)) = EM(\mathbb{Z}, 0))$, gives the isomorphism

$$\pi_n(K^{(p/p+1)}(X)) \cong H^{2p-n}(X, \mathbb{Z}(p)).$$

This gives a new proof of the main result needed for the construction of the Bloch-Lichtenbaum/Friedlander-Suslin spectral sequence from motivic cohomology to K-theory [1], [2].

3. The $\mathbb{P}^1$-stable theory

3.1. **$\mathbb{P}^1$-spectra.**

Definition 3.2. A $\mathbb{P}^1$-spectrum over k, $\mathcal{E}$, is a sequence $(E_0, E_1, \ldots)$ of S^1-spectra over k, together with maps $\epsilon_n : E_n \to \Omega_{\mathbb{P}^1} E_{n+1}$. Denote this category by $\mathbf{Spt}_{\mathbb{P}^1}(k)$.

There is a model structure on $\mathbf{Spt}_{\mathbb{P}^1}(k)$ for which the associated honmotopy category is $\mathcal{SH}(k)$. If $\mathcal{E} = (E_0, E_1, \ldots)$ is a $\mathbb{P}^1$-spectrum over k and each E_n satisfies A1 and A2, and in addition each ϵ_n is a point-wise weak equivalence, then each E_n satisfies A3. We call such an $\mathcal{E}$ *weakly fibrant*; for $\mathcal{E}$ weakly fibrant, the map to a fibrant model $\mathcal{E} \to \mathcal{E}_{fib}$ is a point-wise weak equivalence. Henceforth, all $\mathcal{E}$ we use are assumed to be weakly fibrant.

Let $\mathcal{E} = (E_0, E_1, \ldots)$. Let $\mathcal{E}^{(p)}$ be the $\mathbb{P}^1$-spectrum $(E_0^{(p)}, E_1^{(p+1)}, \ldots, E_n^{(p+n)}, \ldots)$, where the connecting map $\epsilon_n^{(p)} : E_n^{(p+n)} \to E_{n+1}^{(p+n+1)}$ is the composition

$$E_n^{(p+n)} \xrightarrow{(\epsilon_n)^{(p+n)}} (\Omega_{\mathbb{P}^1} E_{n+1})^{(p+n)} \xrightarrow{\theta_{p+n}} \Omega_{\mathbb{P}^1}(E_{n+1}^{(p+n+1)}).$$

The main result is:

Theorem 3.3. *For $\mathcal{E}$ in $\mathbf{Spt}_{\mathbb{P}^1}(k)$, there is a natural isomorphism in $\mathcal{SH}(k)$*

$$\nu^{\geq p}\mathcal{E} \cong \mathcal{E}^{(p)}.$$

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“Consanguinity” of symmetric forms in triangulated categories

Paul Balmer

We introduce the concept of the “consanguinity” of a collection $\alpha_1, \dots, \alpha_n$ of symmetric forms in a triangulated category as an obstruction for their product $\alpha_1 \otimes \cdots \otimes \alpha_n$ to be non-degenerate. The interaction of the product with the symmetric cone (considered as a differential on symmetric forms) simply follows a Leibniz rule, at least when the formula makes sense. We give examples where the forms α_i are ”diagonal”, that is, are given by global sections of line bundles. Multiplying such diagonal forms with differentials of other diagonal forms, we produce interesting geometric examples, like the class generating the Witt group of the projective n-space, over any regular basis (a theorem proved by Charles Walter, using descriptions of the derived categories). These types of classes obtained via non-consangunity methods are locally trivial. That is, this method complements the classical method of detecting information in the Witt group of the quotient field.

The slice filtration from the point of view of Hodge theory

Anette Huber

Let k be a perfect field, A a commutative ring. Consider the commutative diagram of categories

$$\begin{array}{ccc} D\mathcal{M}^{\mathrm{eff}}_{\mathrm{gm}}(k,A) & \longrightarrow & D\mathcal{M}_{\mathrm{gm}}(k,A) \\ \downarrow & & \downarrow \\ D\mathcal{M}^{\mathrm{eff}}_{-}(k,A) & \xrightarrow{\iota} & D\mathcal{M}_{-}(k,A) \end{array}$$

where we follow the notation of [V]. The category $D\mathcal{M}_-$ is defined from $D\mathcal{M}^{\mathrm{eff}}_-$ by formally inverting the object $A(1)$. In particular $D\mathcal{M}_- = \bigcup_{n\in\mathbb{Z}} D\mathcal{M}^{\mathrm{eff}}_-(n)$.

Lemma 1 (H., Kahn). The inclusion $\iota : D\mathcal{M}^{\mathrm{eff}}_-(n) \xrightarrow{\iota} D\mathcal{M}_-$ has a right adjoint $\nu^{\geq n}$.

The functors $\nu^{\geq n}$ induce a filtration on objects in $D\mathcal{M}_-$. It is called *slice filtration.* It is finite on geometric objects.

The graded pieces of the filtration, the *slices*, take values in the category of birational motives studied by Kahn and Sujatha. A similar construction for the homotopy category of schemes is considered by Voevodsky.

We study the slice filtration by using the Hodge realization functor

$$\underline{H}_{\mathcal{H}} : D\mathcal{M}_{\mathrm{gm}} \to \mathrm{MHS}$$

where MHS is the category of mixed $\mathbb{Q}$-Hodge structures. We restrict to the case $k = \mathbb{C}$, $A = \mathbb{Q}$.

We *assume* some "standard" conjectures: existence of a motivic t-structure on $D\mathcal{M}_{\mathrm{gm}}$, existence of a weight filtration, generalized Hodge conjecture for pure Grothendieck motives.

Recall that a Hodge structure is called *effective* if its Hodge type is concentrated in $p \geq 0, q \geq 0$.

Corollary 2. Under the assumption of the above conjectures, an object M of $D\mathcal{M}_{\mathrm{gm}}$ is effective if and only if $\underline{H}_{\mathcal{H}}(M)$ is effective.

Let H^0 be the cohomological functor defined by the motivic t-structure on $D\mathcal{M}_{\mathrm{gm}}$ and hence on $D\mathcal{M}_-$.

We deduce the following from the conjectures:

Proposition 3. The functors $H^0\nu^{\geq n}$ are left exact. On pure motives, they agree with Grothendieck's coniveau fitlration.

Finally, we consider the following example: let X be a generic quintic in $\mathbb{P}^4$, $M = H^{-3}(X)^\vee(2)$. By [PS] Corollary 18 its Hodge realization is simple. By the Hodge conjecture this implies that M itself is simple. Moreover, M is not effective. Hence $H^0\nu^{\geq 0}M = 0$. Now we study $\mathrm{Ext}^1(\mathbb{Q}(0), M)$. The image of this group under the Abel-Jacobi map is an infinite dimensional subspace of $\mathrm{Ext}^1(\mathbb{Q}(-2), \underline{H}^3_{\mathcal{H}}(X))$ ([C] Theorem 6). On the other hand, we can study for each element E the induced long exact sequence of motives

$$0 \to H^0\nu^{\geq 0}E \to \mathbb{Q}(0) \to H^1\nu^{\leq 0}M \to \cdots$$

If $H^1\nu^{\leq 0}M$ vanishes or is pure of the same weight as M, then the map $H^0\nu^{\geq 0}E \to \mathbb{Q}(0)$ trivializes the extension class of E. This shows:

Proposition 4.
1. $H^0\nu^{\geq 0}$ is not exact.
2. The functors $\nu^{\geq n}$ do not commute with the weight filtration.
3. The functors $\nu^{\geq n}$ do not respect geometrical motives.

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The boundary motive

Jörg Wildeshaus

Let X be a scheme over a field. One knows how to associate to X its *motive* $M_{gm}(X)$, its *motive with compact support* $M^c_{gm}(X)$, and the morphism $m_X : M_{gm}(X) \to M^c_{gm}(X)$. In fact, $M_{gm}(X)$, $M^c_{gm}(X)$, and m_X are associated (in a functorial way — see [VSF] for details) to a monomorphism

$$l_X : L(X) \hookrightarrow L^c(X)$$

of Nisnevich sheaves.

Definition 3.1. The *boundary motive* $\partial M_{gm}(X)$ of X is the one associated to $\mathrm{Coker}(l_X)$.

By definition, there is an exact triangle

$$M_{gm}(X) \longrightarrow M^c_{gm}(X) \longrightarrow \partial M_{gm}(X) \longrightarrow M_{gm}(X)[1] .$$

Note that this definition does not require a compactification of X. However, if X^* is one such, then one can show that $\partial M_{gm}(X)$ is represented by the diagram of motives associated to

$$\begin{array}{ccc} \emptyset & \longrightarrow & X \\ \downarrow & & \downarrow \\ \partial X^* & \longrightarrow & X^* \end{array}$$

($\partial X^* := X^* - X$), if the field admits resolution of singularities.

In order to efficiently *compute* $\partial M_{gm}(X)$, we study more generally diagrams of motives associated to

$$\begin{array}{ccc} Y' - Y & \longrightarrow & W - Y \\ \downarrow & & \downarrow \\ Y' & \longrightarrow & W \end{array}$$

for closed immersions $Y \hookrightarrow Y' \hookrightarrow W$ (with W not necessarily proper). One should think of this as $M_{gm}(Y, i^! j_! \mathbb{Z})$, the "motive of Y with coefficients in $i^! j_! \mathbb{Z}$" ($i : Y \hookrightarrow W$, $j : W - Y' \hookrightarrow W$).

We described the two main tools which we have at our disposal, corresponding to (A) the *Čech complex* associated to a closed covering of Y, (B) the invariance of $i^! j_!$ under *deformation to the normal cone* along Y; this necessitates resolution of singularities.

We then discussed the case of a (pure) *Shimura variety.* Tool (A) gives the motivic version of the spectral sequence for the natural stratification of the boundary of the *Baily–Borel compactification.* For each individual stratum, tool (B) allows to prove the motivic version of *Pink's theorem on degeneration* of étale sheaves on Shimura varieties [P].

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After the talk, F. Morel pointed out that in analogy to topology, the boundary motive should rather be defined as the shift by $[-1]$ of the above construction — the result would be an exact triangle

$$\partial M_{gm}(X) \longrightarrow M_{gm}(X) \longrightarrow M^c_{gm}(X) \longrightarrow \partial M_{gm}(X)[1] \, .$$

Since this modification also leads to a slightly more esthetic formula in the motivic analogue of Pink's theorem, it will very probably be used from now on.

I also noted immediately after the talk that tool (B) does unfortunately not work in the generality announced in my talk; basically it does work in smooth situations, where it amounts to *purity*, which is already known [VSF]. What is true, and what still allows to deduce the motivic analogue of Pink's theorem, is the following:

Theorem 3.2. Analytical invariance. Assume given closed immersions $Y \hookrightarrow Y_1' \hookrightarrow W_1$ and $Y \hookrightarrow Y_2' \hookrightarrow W_2$, and an isomorphism

$$f : (W_1)_Y \xrightarrow{\ \sim\ } (W_2)_Y$$

of formal completions along Y inducing an isomorphism $(Y_1')_Y \cong (Y_2')_Y$, and compatible with the identification of $Y \subset W_1$ with $Y \subset W_2$. Then f induces an isomorphism

$$M_{gm}(Y, i_1^! j_{1\,!}\mathbb{Z}) \xrightarrow{\ \sim\ } M_{gm}(Y, i_2^! j_{2\,!}\mathbb{Z}) \, .$$

Cycle modules and triangulated mixed motives

Frédéric Déglise

Introduction. The purpose of this talk was to relate the notion of a cycle module defined by M. Rost in [Ros96] and the one of a homotopy invariant sheaf with transfers introduced by V. Voevodsky in [Voe00]. The first one is rather algebraic in nature and inspired by the theory of Mackey functors as the other one is more geometric in flavour and, seen as certain motivic complexe, can be thought as an analog of the spectra in algebraic topology.

The result obtained in the author's thesis (cf [Deg02]) is that the category of Voevodsky's sheaves is a full subcategory of the category of Rost's cycle modules. Moreover, one can describe exactly the category of cycle modules in terms of certain modules in the category of homotopy invariant sheaves with transfers.

Notations. In all this report, we fix a perfect field k.

We use the notations introduced in [Voe00] :

1. $\mathscr{S}m_k$ denotes the category of smooth separated k-scheme of finite type,
2. $\mathscr{S}mCor_k$ denotes the category with the same objects but with finite correspondances as morphisms,
3. $\mathrm{HI}(k)$ denotes the category of homotopy invariant sheaves with transfers. It is abelian and monoidal with an internal hom functor.

1. Homoriented modules

1.1. **Stabilizing the Tate sphere.** Recall the definition of the Tate sphere S^1_t, seen as an object of $\mathrm{HI}(k)$. It is defined by the short exact sequence in $\mathrm{HI}(k)$

$$h_0\mathrm{L}[\{1\}] \to h_0\mathrm{L}[\mathbb{G}_m] \to S^1_t \to 0,$$

i.e. it is the reduced sheaf of the pointed scheme $(\mathbb{G}_m, \{1\})$.

Definition 1. A homoriented module is a couple (F_*, ϵ) such that :

1. F_* is a graded sheaf in $\mathrm{HI}(k)$.
2. $\epsilon : S^1_t \otimes^{Htr} F_* \to F_*$ is a graded morphism of degree 1.
3. The adjoint morphism to ϵ
$$F_* \to \underline{\mathrm{Hom}}_{\mathrm{HI}(k)}\left(S^1_t, F_*\right)$$
is an isomorphism.

We denote by $\mathrm{H}\mathscr{M}^{\mathrm{tr}}_k$ the corresponding category.

1.2. **Basic properties.** From the properties of the category $\mathrm{HI}(k)$, we obtain easily the following :

Proposition 1. The category $\mathrm{H}\mathscr{M}^{\mathrm{tr}}_k$ is abelian. Moreover, the canonical functor $\mathrm{H}\mathscr{M}^{\mathrm{tr}}_k \to \mathbb{Z} - \mathrm{HI}(k)$ is exact.

Recall now that from the cancellation theorem of V. Voevodsky, we obtain : for all $F, G \in \mathrm{HI}(k)$,

$$\mathrm{Hom}_{\mathrm{HI}(k)}\left(S^1_t \otimes^{Htr} F, S^1_t \otimes^{Htr} G\right) \simeq \mathrm{Hom}_{\mathrm{HI}(k)}(F, G)\,. \tag{1.1}$$

Proposition 2. The canonical restriction functor $\mathrm{H}\mathscr{M}^{\mathrm{tr}}_k \to \mathrm{HI}(k), F_* \mapsto F_0$ has a right adjoint Σ^∞ which is fully faithfull.

We deduce from this that there exists a structure of symetric monoidal category on $\mathrm{H}\mathscr{M}^{\mathrm{tr}}_k$ such that the functor Σ^∞ is strict monoidal, and $\Sigma^\infty S^1_t$ is invertible.

2. Generic motives

We denote by $\mathscr{E}_k$ the category of finitely generated extension of k.

In the following, we will associate to each extension of finite type of the base field k a pro-object which represents a fibre functor for the Nisnevich topology. From the point of view of topos theory, it is thus a point in the sense of Grothendieck.

2.1. **Certain Nisnevich points.**

Definition 2. Let $E/k \in \mathscr{E}_k$.

(1) We define an ordered set associated to E/k

$$\mathcal{M}^{lis}(E/k) = \left\{ A \subset E \mid \begin{array}{l} A \text{ is a sub } k\text{-algebra of } E \\ \operatorname{Spec}(A) \in \mathscr{S}m_k \end{array} \right\}$$

(2) As $\mathcal{M}^{lis}(E/k)$ is right filtering, we define a pro-object of $\mathscr{S}m_k$

$$\begin{array}{rcl} (E) : \mathcal{M}^{lis}(E/k)^{op} & \to & \mathscr{S}m_k \\ A & \mapsto & \operatorname{Spec}(A) \end{array}$$

If $F \in \mathrm{HI}(k)$, and $E/k \in \mathscr{E}_k$, we simply put

$$F(E) = \varinjlim_{A \in \mathcal{M}^{lis}(E/k)} F(\operatorname{Spec}(A)).$$

Remark.– One can see easily that the pro-objets of the form (E) pro-represent a fiber functor for the Nisnevich topology. More precisely, if $X \in \mathscr{S}m_k$ is an integral scheme, with generic point x and fraction field E, there is a canonical isomorphism of pro-objects

$$(E) \simeq \left(\begin{array}{rcl} \mathcal{V}_x(X) & \to & \mathscr{S}m_k \\ V & \mapsto & V \end{array} \right).$$

From the geometric point of view, the pro-objects of the form (E) correspond to the localisation of smooth schemes at a generic point.

From results of V. Voevodsky, we obtain the following proposition :

Proposition 3. The functor

$$\begin{array}{rcl} \mathrm{HI}(k) & \to & (\mathscr{A}b)^{\mathscr{E}_k} \\ F & \mapsto & (E/k \mapsto F(E)). \end{array}$$

is exact and conservative.

2.2. **Generic motives.**

Definition 3. For all $E/k \in \mathscr{E}_k$ and for all integer $n \in \mathbb{Z}$, we define the following pro-object of $\mathrm{H}\mathscr{M}_k^{\mathrm{tr}}$

$$M(E)\{n\} = (\Sigma^\infty S_t^1)^{\otimes^{Htr},n} \otimes^{Htr} \Sigma^\infty h_0 \mathrm{L}[(E)]$$

and call it the generic motive of E/k of weight n.

We denote by $DM_{gm}^{(0)}(k)$ the subcategory of pro$-\mathrm{H}\mathscr{M}_k^{\mathrm{tr}}$ generated by the generic motives.

Remark.– The notation is a bit tricky. But indeed, if we do the same construction as in the previous definition in the derived category of mixed motives $DM_{gm}(k)$ instead of the category $\mathrm{H}\mathscr{M}_k^{\mathrm{tr}}$, we obtain a canonically equivalent category.

2.3. **Morphisms.** The category previously defined is the analog of the category of orbits from the equivariant stable homotopy category. In particular, following this analogy, we konw that morphisms are particularly important in this category. Indeed we obtain four type of morphisms :

Let $E/k, L/k \in \mathscr{E}_k$:

- **D1**$^\sharp$: (corestriction) If $E \xrightarrow{\varphi} L$ is a k-morphism, $\varphi^\sharp : M(L) \to M(E)$
- **D2**$^\sharp$: (restriction) If $E \xrightarrow{\varphi} L$ is a finite morphism, $\varphi_\sharp : M(E) \to M(L)$
- **D3**$^\sharp$: (action of units) If $\sigma \in K_n^M(E)$, $\gamma_\sigma : M(E) \to M(E)\{n\}$
- **D4**$^\sharp$: (residue) If v is a k-valuation of E such that $\mathcal{O}_v$ is essentially of finite type over k, $\partial_v : M(\kappa(v))\{1\} \to M(E)$

The second type of morphism is obtained directly through finite correspondances, because one can transpose the graph of a finite dominant morphism.

The third one uses the canonical isomorphism

$$H^n(\mathrm{Spec}\,(E)\,; \mathbb{Z}(n)) \simeq K_n^M(E).$$

The last one relies exaclty on the Gysin triangle in the dervied category of motives. Recall that if X is a smooth scheme, and Z a closed subscheme of X which is smooth and everywhere of codimension 1, we have a canonical distinguished triangle

$$M(Z)\,(1)[1] \xrightarrow{\partial_{X,Z}} M(X-Z) \to M(X) \to M(Z)\,(1)[2]$$

If we consider $X = \mathrm{Spec}\,(\mathcal{O}_v)$ and $Z = \mathrm{Spec}\,(\kappa(v))$ (these schemes are only essentially of finite type over k in general), the residue is induced by the first morphism in this triangle.

3. Generic transform

3.1. **Definition.** Let F_* be a homoriented module.

It thus induces a canonical contravariant functor

$$\begin{array}{rcl} \hat{F}_* : DM_{gm}^{(0)}(k) & \to & \mathscr{A}b \\ M(E)\,\{n\} & \mapsto & F_{-n}(E). \end{array}$$

Additionaly, as part of the structure, we have the functoriality corresponding to the morphisms between geneirc motives. This drives us to the notion of cycle module.

3.2. **Cycle modules.**

Proposition 4. The dual relations of the relations of cycle premodule R^* are true in the category of generic motives.

As a corollary, we obtain now easily that for $F_* \in \mathrm{H}\mathscr{M}_k^{\mathrm{tr}}$, the functor $\hat{F}_*$ is a cycle premodule.

3.3. **The equivalence of category.** As claimed at the beginning of this report, we come to the following theorem, which can be tought of as an analogue of the isomorphisms between the heart of the equivariant stable homotopy category and the Mackey functors :

Theorem 5. (k is a perfect field)

(1) Let M be a cycle module. There is a canonical structure of homoriented module on the sheaf $A^0(.;M)$. This homotopy module with transfers is functorial in M.

(2) Let F_* be a homoriented module. Denote by $\bar{F}_*$ the presheaf on pro$-DM(k)$ represented by the constant pro-objet of value F_*. The restriction of F_* to the category of generic motives defines a cycle pre-module $\hat{F}_*$ which is a cycle module. This cycle module is functorial in F_*.

(3) The functors defined above

$$\begin{array}{rcl} \mathrm{H}\mathscr{M}_k^{\mathrm{tr}} & \leftrightarrow & \mathscr{M}Cycl_k \\ F_* & \mapsto & \hat{F}_* \\ \mathrm{A}^0(.;M) & \leftarrow\!\shortmid & M \end{array}$$

are mutually inverse equivalences of categories.

(4) Let M be a cycle module, and $\mathcal{M}$ be its associated homoriented module. For a smooth scheme X, there is a canonical isomorphism

$$H^p(X;\mathcal{M}) = A^p(X;M).$$

Thus, because the original category of homotopy invariant sheaves with transfers of V.Voevodsky is a full sub-category of the category of homotopy modules with transfers, it is also canonically a full sub-category of the category of cycle modules of M.Rost.

Moreover, this proves that the category of cycle module is abelian, and gives it a canonical symmetric monoidal structure such that Milnor K-theory is the neutral object.

This theorem can be used to give another proof of the following fact due to V.Voevodsky :

Theorem 6. (k is a perfect field) Let F be homotopy invariant sheaf with transfers.

Then, the cohomology of F is a homotopy invariant presheaf on the category of smooth schemes.

In fact, M.Rost showed that the presheaf $A^p(.;M)$ is homotopy invariant. One can prove 5 direclty without using the category of motivic complex, and without unsing the theorem stated above. The theorem below is thus prooved for homotopy modules. But using results of V.Voevodsky, one can prove that for all homotopy invariant sheaf with transfers F, there is a canonical homotopy module $\Omega^\infty(F)$ equipped with a canonical monomorphism

$$F \to \Omega^\infty(F)\,.$$

This last fact allows to deduce the theorem for F from the theorem for $\Omega^\infty(F)$.

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Schur functors and motives

Carlo Mazza

In this talk we study the class of Schur-finite motives, that is, motives which are annihilated by a Schur functor. We compare this notion to a similar one due to Kimura and, independently, O'Sullivan. In particular, we can prove that the motive of any curve is Kimura-finite. This last result has also been obtained by V. Guletskiĭ.

Let $\mathcal{M}_r$ be the category of $\mathbb{Q}$-linear motives modulo rational equivalence. Following [Kim], we say that a motive M is "finite dimensional" if there is a decomposition $M = M_+ \oplus M_-$ such that $Sym^i(M_+) = 0$ and $\Lambda^j(M_-) = 0$ for some i and j.

Theorem 0.1. (Gulestkiĭ-Pedrini) Let M be the motive of a smooth projective surface with $p_g = 0$. Then M is "finite dimensional" if and only if it satisfies Bloch's conjecture (i.e., the kernel of the Albanese map vanishes).

Kimura showed in *loc. cit.* that the "if" part holds. However, we have very few examples of "finite dimensional" motives. This is partly because the definition of "finite dimensial" motive is rather rigid. Following [Del02], we define a new notion, which has the same combinatorial flavor, but which is more flexible than Kimura's one.

1. Basic definition and properties

Most of the basic properties can be defined in a more general setting, as follows.

Definition 1.1. We say that a symmetric monoidal category $\mathcal{A}$ is a **$\mathbb{Q}$-linear tensor category** if it is additive, pseudo-abelian, $\mathbb{Q}$-linear, and $\otimes$ is $\mathbb{Q}$-bilinear.

Let $F : \mathcal{A} \to \mathcal{B}$ be a functor between two $\mathbb{Q}$-linear tensor categories. We say that F is a **$\mathbb{Q}$-linear tensor functor** if it is $\mathbb{Q}$-linear and it respects the symmetric monoidal structures.

Recall that for every partition λ of n there is an idempotent $c_\lambda \in \mathbb{Q}[\Sigma_n]$ called the Young symmetrizer. If Σ_n acts on an object A of $\mathcal{A}$, then there is an algebra map $\mathbb{Q}[\Sigma_n] \to \mathrm{End}(A)$. Since $c_\lambda^2 = c_\lambda$ and $\mathcal{A}$ is pseudo-abelian, $c_\lambda(A)$ is a direct summand of A.

Definition 1.2. Let $\mathcal{A}$ be a $\mathbb{Q}$-linear tensor category. The symmetric group Σ_n acts on $X^{\otimes n}$ for every X. For every partition λ of $n > 0$, we define $S_\lambda(X) = c_\lambda(X^{\otimes n})$. This assignment makes $S_\lambda(-)$ into a functor, which we call the **Schur functor** of λ. In particular, we define $Sym^n(X) = S_{(n)}(X)$ and $\wedge^n X = S_{(1,\dots,1)}(X)$.

The following definitions are extracts from [Del02] and [Kim].

Definition 1.3. An object X of $\mathcal{A}$ is called **Schur-finite** if there is an integer n and a partition λ of n such that X is annihilated by the Schur functor of λ, i.e., $S_\lambda(X) = 0$.

An object X of $\mathcal{A}$ is called **even** (respectively, **odd**) if there is an n so that $\Lambda^n X = 0$ (respectively, $Sym^n X = 0$). An object X is called **Kimura-finite** if there is a decomposition $X = X_+ \oplus X_-$ such that X_+ is even and X_- is odd.

We will say that the category $\mathcal{A}$ is Schur-finite (respectively, Kimura-finite) if all objects of $\mathcal{A}$ are Schur-finite (respectively, Kimura-finite).

Remark 1.4. The same notion of Kimura-finiteness was introduced independently by P. O'Sullivan, who studied its properties mainly from the point of view of category theory.

Lemma 1.5. Kimura-finiteness and Schur-finiteness are closed under direct sums and tensor products. Moreover, every Kimura-finite object is Schur-finite.

Lemma 1.6. Let $F : \mathcal{A} \to \mathcal{B}$ be a $\mathbb{Q}$-linear tensor functor. If an object X of $\mathcal{A}$ is Schur-finite, so is $F(X)$. If F is also faithful, then the converse holds, i.e., if $F(X)$ is Schur-finite, then so is X.

Example 1.7. (B. Kahn) Consider the category of bounded chain complexes of R-modules, where $R = \mathbb{Q}[x]$. This is clearly a $\mathbb{Q}$-linear tensor category. Let $RMod_{fg}$ be the category of finitely generated R-modules. Then $RMod_{fg}$ is Kimura-finite and so is the super category $RMod^{\pm}_{fg}$. By 1.6, $Ch^b(RMod_{fg})$, the category of bounded chain complexes, is Schur-finite.

Let M be the Schur-finite complex $R \to R$, where the map is multiplication by x. This complex is irreducible, and is not Kimura-finite because $Sym^n M \cong M$ and $\wedge^n M \cong M[n-1]$. Notice also that the image of M in $RMod^{\pm}_{fg}$ is Kimura-finite, and so 1.6 is false for Kimura-finiteness.

2. Applications to motives

For any adequate equivalence relation (see [Jan00]), we can construct a category of $\mathbb{Q}$-linear motives. They are all $\mathbb{Q}$-linear tensor categories and therefore the notions of Schur-finiteness and Kimura-finiteness make sense. Kimura-finiteness has been studied in [Kim], [AK02], [GP02], and [GP] among others.

Let $\mathcal{M}_h$ be the category of $\mathbb{Q}$-linear motives modulo homological equivalence, for a fixed Weil cohomology H. By the Künneth formula, the cohomology yields a faithful $\mathbb{Q}$-linear tensor functor $H : \mathcal{M}_h \to Vect^{\pm}_{\mathbb{Q}}$. Since $Vect^{\pm}_{\mathbb{Q}}$ is Schur-finite, $\mathcal{M}_h$ is Schur-finite by 1.6.

Let $\mathcal{M}_n$ be the category of $\mathbb{Q}$-linear motives modulo numerical equivalence. Since we have a $\mathbb{Q}$-linear tensor functor from $\mathcal{M}_h$ to $\mathcal{M}_n$ and $\mathcal{M}_h$ is Schur-finite, $\mathcal{M}_n$ is Schur-finite by 1.6.

Kimura conjectured in [Kim] that the category $\mathcal{M}_r$ of $\mathbb{Q}$-linear motives modulo rational equivalence is Kimura-finite. This conjecture combined with 1.6 implies that $\mathcal{M}_h$ is Kimura-finite.

Theorem 2.1. *The category $\mathcal{M}_n$ is super-Tannakian, i.e., there exists a field K of characteristic zero and a faithful fibre functor from $\mathcal{M}_n$ to $Vect^{\pm}_K$.*

Let $\mathbf{DM} = \mathbf{DM}^{eff,-}_{Nis}(k,\mathbb{Q})$ be the tensor triangulated category of $\mathbb{Q}$-linear motives, i.e., the localization by $\mathbb{A}^1$-weak equivalences of the derived category of (cochain) complexes of Nisnevich sheaves $\mathbf{D}^- = \mathbf{D}^-(Sh_{Nis}(Cor_k,\mathbb{Q}))$ (see [Voe00] and [MVW]). Both $\mathbf{D}^-$ and $\mathbf{DM}$ are $\mathbb{Q}$-linear tensor categories.

Proposition 2.2. *Schur-finiteness has the "two out of three" property in $\mathbf{DM}$, i.e., let $A \to B \to C \to A[1]$ be a triangle in $\mathbf{DM}$. If two objects are Schur-finite, then so is the third. Moreover, if A and C are even (respectively, odd) then B is even (respectively, odd).*

Corollary 2.3. *The subcategory of $\mathbf{DM}$ consisting of Schur-finite objects is thick and closed under twists.*

The following theorem can be proved using 2.2.

Theorem 2.4. *The motive of any curve is Kimura-finite.*

Remark 2.5. V. Guletskiĭ has independently obtained 2.4 in his preprint [Gul].

After posting the preprint, O'Sullivan provided an example of a smooth surface U whose motive is not Kimura-finite.

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Motivic localisation of Bloch's conjecture

Vladimir Guletskii

Let CHM be the category of Chow-motives over a field with coefficients in $\mathbb{Q}$, $M \in CHM$, and let $f : M \to M$ be a homologically trivial endomorphism of M. If M is finite dimensional in the sense of S.-I. Kimura, see [7], then f is nilpotent in the associative ring $\mathrm{End}(M)$, Prop. 7.5 loc. cit. It follows that, given a smooth projective surface X over $\mathbb{C}$ with $p_g(X) = 0$, finite dimensionality of the motive $M(X)$ implies the triviality of the Albanese kernel for X, see [7] or [4]. In [5] we have shown the inverse: if Bloch's conjecture, [1], holds for X, then $M(X)$ is finite dimensional in Kimura's sense.

In [8] Mazza introduced more flexible notion of motivic finite dimensionality using Schur-functors in tensor $\mathbb{Q}$-linear categories, and proved that it has 2-of-3 property in distinguished triangles in $\mathbb{Q}$-localized Voevodsky's triangulated category DM, [9]. In [3] we proved the following result, which can be considered as a Schur-analog of Kimura's nilpotency theorem:

Theorem 1. Let X be a smooth projective variety over a field, such that its Chow-motive $M(X)$ is Schur-finite. Then any numerically trivial correspondence of degree zero from X to X is nilpotent in the ring $\mathrm{End}(M(X))$.

This result has at least two applications. Let us recall the Sign Conjecture: for any smooth projective variety X over a field its diagonal class Δ_X can be decomposed (modulo an adequate equivalence relation) into two orthogonal projectors $\Delta_X = \pi_+ + \pi_-$, such that π_+ carries the even part of the Weil cohomology groups $H^*(X)$, and π_- carries the odd part of $H^*(X)$.

Corollary 1. (Uwe Jannsen) Assuming the sign conjecture modulo rational equivalence relation, the motive $M(X)$ for a smooth projective variety X over a field is Schur finite dimensional if and only if $M(X)$ is finite dimensional in Kimura's sense.

The second corollary says that, in some sense, Bloch's conjecture on Albanese kernel can be motivically localized. This correlates with the following well known Chow-localization: if X is a smooth projective irreducible surface defined over an algebraically closed field k, then its Albanese kernel is trivial if and only if there exists a closed subscheme $Z \hookrightarrow X$ of dimension one, such that $CH^d(X-Z)\otimes\mathbb{Q} = 0$, see [2] or [6], Prop. 1.6.

Corollary 2. Let X be a smooth projective surface over $\mathbb{C}$ with $p_g = 0$ and of general type. Then the following items are equivalent:

- Bloch's conjecture holds for X;
- the motive $M(U)$, considered in Voevodsky's category DM, is Kimura-finite for any Zariski open U in X;
- there exists a Zariski open U in X, such that $M(U)$ is Kimura-finite in DM.

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A spectral sequence for equivariant K-theory

Christian Serpé
(joint work with Marc Levine)

If X is a smooth scheme of finite type over a field, then there is a strongly convergent spectral sequence

$$E_2^{p-q} = H_{\mathcal{M}}^{p-q}(X, \mathbb{Z}(-q)) \Rightarrow K_{-p-q}(X)$$

from the motivic cohomology to the algebraic K-theory of X (compare [1], [2] [3] and [5]). Here the motivic cohomology groups can be either defined by Bloch's higher Chow groups or by the motivic cohomology groups of Voevodsky. The aim of this talk is to give an analogon of this in an equivariant situation.

For that let G be a finite group and X be a smooth G–scheme of finite type over a field k with $\frac{1}{\sharp G} \in k$. We denote by $K(G,X)$ the equivariant algebraic K–theory spectrum of X. For a closed G–invariant subset $W \subset X \times \Delta^r$ we define

$$K^W(G, X \times \Delta^r) := \text{hofib}(K(G, X \times \Delta^r) \to K(G, (X \times \Delta^r) \setminus W).$$

Let $K^{(p)}(G,X,r)$ be the homotopy colimit of all $K^W(G, X \times \Delta^r)$ where $W \subset X \times \Delta^r$ has at least codimension p and has good intersection with all faces of Δ^r. With that we get an equivariant homotopy coniveau tower

$$\cdots \to K^{(p+1)}(G,X,-) \to K^{(p)}(G,X,-) \to \cdots \to K^{(0)}(G,X,-) \simeq K(G,X)$$

of simplicial spectra $K^{(p)}(G,X,\_)$. We denote by $K^{(p/p+1)}(G,X,-)$ the homotopy cofiber of the map $K^{(p+1)}(G,X,-) \to K^{(p)}(G,X,-)$. So we get:

Proposition 1. There is a strongly convergent spectral sequence

$$E_1^{p,q} = \pi_{-p-q}(K^{(p/p+1)}(G,X,-)) \Rightarrow K_{-p-q}(G,X).$$

To get a better description of the starting term of the spectral sequence we denote by

$$X_G^{(p)}(r) := \{[x] \in (X \times \Delta_k^r)^{(p)}/G| \begin{array}{c} \text{for all faces } F \subset \Delta_k^r \\ \text{codim}(\overline{G \cdot x} \cap X \times F, X \times F) \geq q \end{array} \}$$

and define

$$z^p(G,X,r) := \bigoplus_{[x] \in X_G^{(p)}(r)} K_0(G_d(x), Spec(\kappa(x)))$$

where $G_d(x) := \{x \in G | gx = x\}$ is the decomposition group of x. This is an simplicial abelian group and an analogon of Bloch's higher cycle complex in the equivariant situation. Therefore we define the equivariant higher Chow groups (of Bredon type):

$$CH^p(G,X,r) := \pi_r(z^p(G,X,-)).$$

We get a natural cycle class map

$$cl : K^{(p/p+1)}(G,X,-) \to z^p(G,X,-),$$

where we consider both sides as simplicial spectra.

Using localization techniques of [4] and results from [5] one can show that in convenient situations the morphism cl is a weak equivalence. So all together we have the following main result.

Theorem 1. Let G be a finite group and X be a smooth G–scheme of finite type over a field k with $\frac{1}{\sharp G} \in k$. Then there is a strongly convergent spectral sequence

$$E_1^{p,q} = CH^p(G,X,-p-q) \Rightarrow K_{-p-q}(G,X)$$

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On the Chow motive attached to the product of two surfaces.

Jacob Murre

Let $V = V(k)$ be the category of smooth,projective varieties defined over a field k. Let $CHM(k)$ be the category of Chow motives and $M = M_h(k)$ the category of homology motives with respect to some Weil cohomology theory (see for instance [4] for the definitions). There are contravariant functors $ch : V^{opp} \to CHM$ given by $ch(X) = (X, \Delta(X), 0)$ and similarly $h : V^{opp} \to M$.

We say ([3]) that X has a Chow-Kuenneth decomposition (C-K for short) if the Kuenneth components of the diagonal of X are algebraic and if we can lift them to a set of orthogonal projectors $\pi_i(X) \in CH^d(X \times X; Q)$, where $0 \leq i \leq 2d$ with $d = dimX$, summing up to $\Delta(X)$. It is conjectured ([3]) that every $X \in V$ has a Chow-Kuenneth decomposition (after, if necessary, a finite extension of the base field k). The conjecture is known to be true for curves, surfaces, abelian varieties, hypersurfaces and certain classes of threefolds. Also if X and Y have a C-K decomposition then $X \times Y$ has a C-K decomposition .

In [2] we have proved that for surfaces $ch(S) = \oplus_{i=0}^4 ch^i(S)$ with $ch^i(S) = (S, \pi_i(S), 0)$. One can split further $ch^2(S) = ch^2_{alg}(S) \oplus ch^2_{tr}(S)$, i.e., in the "algebraic" and the "transcendental" part. For the transcendental part we have $CH(ch^2_{tr}(S)) = T(S)$, the Albanese kernel of S, and $H(ch^2_{tr}(S)) = H^2(S)_{tr}$, the transcendental cycles of S.

In [3] we have formulated a set of conjectures describing the action of the (conjectural !) C-K projectors $\pi_i(X)$ on the Chow groups $CH(X; Q)$. These conjectures imply the existence of a filtration on the Chow groups (or better on the "Chow vectorspaces"). Uwe Jannsen has proved [1] that this conjectural filtration is the conjectural Bloch-Beilinson filtration.

Now consider the case $X = S \times S'$, with S and S' surfaces. From the above it follows that such X have a C-K decomposition and in fact we have $\pi_m(X) = \oplus_{r+s=m} \pi_r(S) \times \pi_s(S')$. In this lecture we investigate how far we can prove these conjectures for such X , and more in particular how far we can prove unconditionally a related crucial proposition of Uwe Jannsen ("proposition 5.8 " of [1])for $S \times S'$. The results are the following:

Prop. 1. $Hom_{CHM}(ch^i(S), ch^j(S')) = 0$ for $i < j$.

Prop. 2. $Hom_{CHM}(ch^i(S), ch^i(S')) \cong Hom_M(h^i(S), h^i(S'))$ for $i = 0, 1, 3$ and 4.

We are not able to prove a similar statement for $i = 2$. In fact this statement for $i = 2$ would imply the famous Bloch conjecture for surfaces, saying that for a surface S with $H^2(S)_{tr} = 0$ we also should have $T(S) = 0$.

For $i = 2$ we have (only) the following result. Let $CH^2_{\equiv}(S \times S')$ be the subgroup of $CH^2(S \times S')$ generated by the correspondences supported on subvarieties of type $Y \times S'$ and of type $S \times Y'$, with Y a curve on S and Y' a curve on S'. Then:

Prop. 3 . Under the map $T \to \pi_2^{tr}(S') \bullet T \bullet \pi_2^{tr}(S)$ we have $CH^2(S \times S')$ mod $CH^2_{\equiv}(S \times S') \cong Hom_{CHM}(ch^2_{tr}(S), ch^2_{tr}(S'))$.

In the proofs of the above propositions we use the explicit construction of the Chow-Kuenneth components for surfaces and their properties ([2]) and results from [3].

Proposition 3 is entirely analogous to what happens for curves. Namely if C and C' are smooth projective curves and writing $CH^1_{\equiv}(C \times C')$ for the subgroup generated in $CH^1(C \times C')$ by the "vertical" and "horizontal" divisors, we have $CH^1(C \times C')$ mod $CH^1_{\equiv}(C \times C') \cong Hom_{CHM}(ch^1(C), ch^1(C'))$ (for curves this follows almost immediately from the definitions) . However now we have for $ch^1(C)$,resp. $ch^1(C')$, the "realization" (in a strong sense) as the Jacobian variety $J(C)$, resp. $J(C')$. Therefore instead of proposition 3 there holds the following theorem ([5], Thm.22, chap.6, section 43) :

Theorem .(Weil). $CH^1(C \times C')$ mod $CH^1_{\equiv}(C \times C') \cong Hom_{AV}(J(C), J(C'))$, where J(C) and J(C') are the Jacobian varieties and the $Hom_{AV}(-,-)$ stands for the homomorphisms as abelian varieties.

Due to the theory of abelian varieties one can draw strong conclusions from this theorem. However in our case the insight in the "true nature" of motives of type $ch^2_{tr}(S)$ is still missing, preventing us from proving proposition 2 for the case $i = 2$.

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Birational motives

R. Sujatha

The material presented in the talk is joint work with Bruno Kahn. The purpose of this project is to construct a category of birational motives and study the properties of such a category, bearing in mind the constructions and properties of the category $DM^{\rm eff}_{\rm gm}(F)$ of effective geometric motives constructed by Voevodsky [V].

For simplicity of exposition, we assume that F is a base field of characteristic 0. Let $SmProj(F)$ denote the category whose objects are smooth integral varieties over F and morphisms are the usual morphisms. Recall from [V] that $SmCor_{\rm proj}(F)$ is the additive category whose objects are smooth projective varieties over F and morphisms are finite correspondences. Let $Chow^{\rm eff}(F)$ denote the category of effective Chow motives [V]. the following tensor triangulated categories were constructed by Voevodsky:
i) $DM^{\rm eff}_{\rm gm}(F)$: Category of effective geometric motives over F,
ii) $DM^{\rm eff}_{-}(F)$: Category of effective motivic complexes.
Further, there are the following functors between these categories:

$$SmCor_{\rm proj}(F) \xrightarrow{h} Chow^{\rm eff}(F) \xrightarrow{\Phi} DM^{\rm eff}_{gm}(F) \xrightarrow{i} DM^{\rm eff}_{-}(F) \quad (1)$$

with Φ and i fully faithful. further i has dense image and the property that the image consists precisely of compact objects.

Let S_r denote the set of stably rational morphisms (i.e morphisms $X \to Y$ between smooth projective varieties over F such that the function field $F(X)$ is a pure transcendental extension of $F(Y)$, we then have the localised category $S_r^{-1}SmProj(F)$ which renders all the arrows of S_r invertible. We construct the following categories:
(a) $Chow^o(F)$: Symmetric monoidal category of pure birational Chow motives.
(b): $DM^o_{\rm gm}(F)$: Tensor triangulated category of birational motives.
(c): $DM^o_{-}(F)$: Tensor triangulated category of birational motivic complexes.
We also construct functors

$$S_r^{-1}SmCor(F) \xrightarrow{h^0} Chow^o(F) \xrightarrow{\Phi^0} DM^o_{\rm gm}(F) \xrightarrow{i^0} DM^0_{-}(F) \quad (2)$$

such that Φ^0 is fully faithful, i^0 is fully faithful whose image is dense and consists of compact objects. Further, we show that there are natural functors from

(1) to (2) making the corresponding diagram commutative. By its very construction, two stably birational smooth projective varieties have isomorphic motives in $DM^o_{\mathrm{gm}}(F)$. We also study the adjoints of several of the functors above.

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Explicit construction of Borel-Moore objects for finite type schemes

Joël Riou

1. Introduction

Theorem 1 (Poincaré duality). Let X be a connected oriented differentiable manifold of dimension d. Then, for any $i \in \mathbb{Z}$, the $\cup$-product

$$H^i_{\mathrm{dR}}(X) \times H^{d-i}_{\mathrm{dR}}(X) \to H^d_{\mathrm{dR,c}}(X) = \mathbb{R}$$

induces an isomorphism between $H^i_{\mathrm{dR}}(X)$ and the $\mathbb{R}$-dual of $H^{d-i}_{\mathrm{dR}}(X)$, where $H^\star_{\mathrm{dR}}$ denote de Rham cohomology groups and $H^\star_{\mathrm{dR,c}}$ de Rham cohomology groups with compact supports.

There is a version of this theorem with $\mathbb{Z}$-coefficients: $H^i_c(X;\mathbb{Z}) = H_{d-i}(X;\mathbb{Z})$. In algebraic geometry, one usually defines cohomology groups rather than homology groups. It turns out that there is a dual version of the previous isomorphism: $H^{d-i}(X;\mathbb{Z}) = H^{BM}_i(X;\mathbb{Z})$ where the latter group is a Borel-Moore homology group.

In the étale formalism, if $f\colon X \to \mathit{Spec}\,k$ denotes a morphism of finite type and k an algebraically closed field, then one may consider four natural objets in the derived category of $\mathbb{Z}/n$-modules where n is an integer that is prime to the characteristic exponent of k: $f_*f^*\mathbb{Z}$ (étale cohomology), $f_!f^*\mathbb{Z}/n$ (étale cohomology with compact supports), $f_!f^!\mathbb{Z}/n$ (étale homology) and $f_*f^!\mathbb{Z}/n$ (étale Borel-Moore homology). Now, when f is smooth of relative dimension d, there is a canonical isomorphism $f^!\mathbb{Z}/n \simeq \mathbb{Z}/n(d)[2d]$ so that

$$H^{BM}_i(X/k;\mathbb{Z}/n) = H^{2d-i}(X;\mathbb{Z}/n(d))$$

Consequently, for smooth varieties, Borel-Moore homology groups and cohomology groups coincide up to a twist.

When one wants to extend a cohomology theory from smooth schemes over k to arbitrary ones, one may consider the following situation: let X be a smooth scheme over k, Z a closed subscheme of X and U the open complement. When Z is also smooth over k, there is a well known Gysin sequence that links cohomology groups of X, U and Z, but it is not true when Z is not smooth. Alternatively, if one considers cohomology with compact supports, there is a long exact sequence:

$$\dots \to H^i_c(U;\mathbb{Z}/n) \to H^i_c(X;\mathbb{Z}/n) \to H^i_c(Z;\mathbb{Z}/n) \to H^{i+1}_c(U;\mathbb{Z}/n) \to \dots$$

As Borel-Moore homology groups are dual to cohomology groups with compact supports, there is a dual long exact sequence:

$$\dots \to H^{BM}_i(Z;\mathbb{Z}/n) \to H^{BM}_i(X;\mathbb{Z}/n) \to H^{BM}_i(U;\mathbb{Z}/n) \to H^{BM}_{i-1}(Z;\mathbb{Z}/n) \to \dots$$

So using the Poincaré duality isomorphism for X and U, one can give an expression of the Borel-Moore homology of Z involving twisted cohomologies of X and U.

2. Setting

Let S be any noetherian scheme. We will use the homotopy theory of schemes introduced by Morel and Voevodsky (see [9] and [10]). Roughly speaking, one starts with pointed simplicial sheaves on the site of smooth schemes over S endowed the Nisnevich topology. Then, we define (see [5]) the notion of simplicial weak equivalence (which corresponds to the notion of quasi-isomorphism if we replace simplicial sheaves by complexes of abelian sheaves). Then we may localise with respect to the map $(\mathbb{A}^1 \times X)_+ \to X_+$ for any smooth S-scheme X, where Y_+ denotes the disjoint union of some space Y and a base-point ; we get the notion of $\mathbb{A}^1$-weak equivalence. The homotopy category $H_\bullet(S)$ is defined by formally inverting $\mathbb{A}^1$-weak equivalences in the category of pointed spaces. This homotopy category is endowed with a $\wedge$-product that maps two pointed spaces (X,x) and (Y,y) to the quotient $X \wedge Y$ of $X \times Y$ by the subsheaf $X \times y \cup x \times Y$.

$SH(S)$ is a stable version of this construction: we invert the $\wedge$-product with the space $(\mathbb{P}^1,\infty)$ in some sense. See [6] for more details about this construction. This category $SH(S)$ is a triangulated category endowed a $\wedge$-product. We define a functor $-(1)\colon SH(S) \to SH(S)$ by the formula $- \wedge (\mathbb{P}^1,\infty) = -(1)[2]$.

Now, a cohomology theory on smooth S-scheme will be an object $\mathbf{E}$ of $SH(S)$. Then, for any tuple (p,q) of integers and any smooth S-scheme X, one can define homology and cohomology groups associated to this spectrum $\mathbf{E}$:

$$\begin{aligned} \mathbf{E}^{p,q}(X/S) &= \mathrm{Hom}_{SH(S)}(X_+, \mathbf{E}(q)[p]) \\ \mathbf{E}_{p,q}(X/S) &= \mathrm{Hom}_{SH(S)}(S^0, X_+ \wedge \mathbf{E}(-q)[-p]) \end{aligned}$$

Many usual cohomology theories are represented in this sense by spectra: motivic cohomology (provided S is the spectrum of a perfect field), usual singular cohomology of real or complex points of varieties, étale cohomology with $\mathbb{Z}/n$-coefficients (provided n is invertible in S), continuous étale cohomology with $\mathbb{Z}_\ell$-coefficients (ℓ invertible in S), algebraic $\mathbf{K}$-theory (S regular), algebraic de Rham cohomology ($S = Spec\, k$ with k a field of characteristic 0, see [2] and [3])...

Now the idea is to construct, for any morphism $f\colon X \to S$ of finite type an object $\mathrm{BM}(X/S)$ in $SH(S)$ so that we may define, for any spectrum $\mathbf{E}$ in $SH(S)$, the Borel-Moore homology (and the cohomology with compact supports) of X/S with coefficients in $\mathbf{E}$ in the following way:

$$\begin{aligned} \mathbf{E}^{\mathrm{BM}}_{p,q}(X/S) &= \mathrm{Hom}_{SH(S)}(\mathrm{BM}(X/S), \mathbf{E}(-q)[-p]) \\ \mathbf{E}^{p,q}_{\mathrm{c}}(X/S) &= \mathrm{Hom}_{SH(S)}(S^0, \mathrm{BM}(X/S) \wedge \mathbf{E}(q)[p]) \end{aligned}$$

When X/S is projective and smooth, the Spanier-Whitehead duality should imply that $\mathrm{BM}(X/S)$ is the S^0-dual of X_+: in topology, there is the following theorem:

Theorem 2. Let X be a smooth compact differentiable manifold, $i\colon X \to \mathbb{R}^n$ an embedding, N a (closed) tubular neighbourhood and ∂N its border. Then there are stable maps

$$S^0 \to (N/\partial N)[-n] \to X_+ \wedge (N/\partial N)[-n] \text{ and } X_+ \wedge (N/\partial N)[-n] \to S^0$$

that define a Spanier-Whitehead duality between X_+ and $(N/\partial N)[-n]$.

Let ν be the normal bundle of the immersion $i\colon X \to \mathbb{R}^n$. By definition, $N/\partial N$ is the Thom space of this bundle ν over X. In some sense that will be clarified in the next section, the Spanier-Whitehead dual of X_+ is the Thom space of the virtual bundle $\nu - \varepsilon^n$ where ε is the trivial bundle of rank 1. Using the decomposition $\nu \oplus TX \simeq \varepsilon^n$, it turns out that the dual of X_+ is the Thom space of the virtual bundle $-TX$ where TX is the tangent bundle X.

3. Construction of Borel-Moore objects

First, we start with the study of Thom spaces. Recall that if F is a vector bundle over some noetherian scheme X, we define the Thom space $\mathrm{Th}_X(F)$ of F as the quotient sheaf $F/(F - s_0(X))$ in $H_\bullet(X)$ (or in $SH(X)$) where $s_0\colon X \to F$ denotes the zero section of the bundle.

Lemma 1. Let X be a noetherian scheme that admits an ample family of line bundles (see [4]). Let $0 \to F' \to F \to F'' \to 0$ be a short exact sequence of vector bundles on X. Then, there exists a *canonical* isomorphism in $H_\bullet(X)$:

$$\mathrm{Th}_X(F) \simeq \mathrm{Th}_X(F' \oplus F'')$$

where $\mathrm{Th}_X(F' \oplus F'') = \mathrm{Th}_X(F) \wedge \mathrm{Th}_X(F'')$.

The proof involves Jouanolou's trick (see [7]). One can also prove that the Thom space of any vector bundle on X admits an inverse for the $\wedge$-product on $SH(X)$, so that this lemma implies that there is a map (of commutative monoids) from $\mathbf{K}_0(X)$ to the class of isomorphism classes of objects in $SH(X)$ that maps the class of a vector bundle to its Thom spaces.

Definition 1. Let $f\colon X \to S$ be a quasi-projective morphism between noetherian schemes. We choose a closed embedding $X \to Y$ of X into a smooth S-scheme $g\colon Y \to S$, we define the Borel-Moore object of X/S in $SH(S)$ by the formula:

$$\mathrm{BM}(X/S, i) = g_\sharp((Y/Y - i(X)) \wedge_Y \mathrm{Th}_Y(-TY))$$

(We assume that Y admits an ample family of line bundles.)

Using the previous lemma, we can prove that if X itself is smooth, then

$$\mathrm{BM}(X/S, i) \simeq f_\sharp \,\mathrm{Th}_X(-TX)\,.$$

One can construct some piece of functoriality for these objects (contravariance for projective morphisms) using two cases: closed immersions and projection from a projective space. The former is easy, the ladder is harder (see [11, section 2]). Unfortunately, we were not able to prove directly that these Borel-Moore objects do not depend on the choice we had to make during the construction.

Theorem 3 (Voevodsky, Ayoub [1])**.** For any quasi-projective morphism $f\colon X \to S$ of finite type between noetherian schemes (that admit an ample family of line bundles), there exist natural functors $f_\star, f_!\colon SH(X) \to SH(S)$ and $f^\star, f^!\colon SH(S) \to SH(X)$, and a natural transformation $f_! \to f_\star$ that is an isomorphisme whenever f is projective.

A corollary of their construction is that there is an isomorphism $\mathrm{BM}(X/S, i) \simeq f_! S^0$. As a result, these Borel-Moore objects are well defined and admits a contravariant functoriality for projective morphisms.

As a conclusion, one can note that we can do the same construction of Borel-Moore objects in the category of geometric motives $DM_{\mathrm{gm}}(k)$ for any perfect field k (alternatively, one may use the canonical functor $SW(k) \to DM_{\mathrm{gm}}(k)$, where $SW(k)$ is the triangulated category of finitely presented objects in $SH(k)$). It enables us to define the Borel-Moore motive $M_{\mathrm{BM}}(X)$ of any quasi-projective scheme X over k and to construct a localisation triangle (which obviously also exists in $SH(S)$) :

$$M_{\mathrm{BM}}(X - Z) \to M_{\mathrm{BM}}(X) \to M_{\mathrm{BM}}(Z) \xrightarrow{+}$$

for any closed subscheme Z in X. Note that this construction does not require the use of resolution of singularites whereas the dual construction of motives with compact supports requires it.

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p-adic Étale Tate twists and arithmetic duality

Kanetomo Sato

Let X be a proper smooth variety over a finite field $\mathbb{F}_q$, let m a positive integer with $(m, \mathrm{ch}(\mathbb{F}_q)) = 1$, and let μ_m be the étale sheaf of mth roots of unity on X. Then for a non-negative integer n with $0 \le n \le d := \dim(X)$, we have the following non-degenerate pairing of finite groups (Poincaré-Pontryagin duality):

$$\mathrm{H}^i_{\text{ét}}(X, \mu_m^{\otimes n}) \times \mathrm{H}^{2d+1-i}_{\text{ét}}(X, \mu_m^{\otimes d-n}) \longrightarrow \mathrm{H}^{2d+1}_{\text{ét}}(X, \mu_m^{\otimes d}) \simeq \mathbb{Z}/m\mathbb{Z}.$$

Using logarithmic Hodge-Witt sheaves $W_r\Omega^*_{X,\log}$ of Illusie, Milne proved a p-primary variant of the above duality (cf. [Mi]; see also [Mo] for a further generalization). We like to generalize these duality facts to proper arithmetic schemes.

Let X be a regular scheme which is flat over an algebraic integer ring $\mathfrak{O}$, and let p be a prime number. Let n and r be integers with $n \ge 0$ and $r > 0$. We like to find an object $\mathcal{K} \in D^b(X_{\text{ét}}, \mathbb{Z}/p^r\mathbb{Z})$ satisfying the following four properties:

- **T1:** There exists an isomorphism $t : \mathcal{K}|_V \simeq \mu_{p^r}^{\otimes n}$, where $V := X[1/p]$.
- **T2:** $\mathcal{K}$ is concentrated in $[0, n]$
- **T3:** For a locally closed regular subscheme $i : Z \to X$ of characteristic p, we have a Gysin isomorphism:

$$\mathrm{Gys}^n_i : W_r\Omega^{n-c}_{Z,\log}[-n-c] \xrightarrow{\simeq} \tau_{\le n+c} Ri^!\mathcal{K} \quad \text{in} \quad D^b(Z_{\text{ét}}, \mathbb{Z}/p^r\mathbb{Z})$$

  with $c := \mathrm{codim}_X(Z)$, where $W_r\Omega^{n-c}_{Z,\log}$ means the zero sheaf if $n < c$.
- **T4:** A compatibility property between Gysin maps and boundary maps of Galois cohomology groups defined by Kato [KCT].

These properties are $\mathbb{Z}/p^r\mathbb{Z}$-coefficient variant of axioms of Beilinson and Lichtenbaum (cf. [Li]) on the conjectural étale motivic complex $\mathbb{Z}(n)$. We have the following fundamental result:

Theorem 1. Assume that X is smooth or semistable over $\mathrm{Spec}(\mathfrak{O})$ around the fibers of characteristic p. Then there exists an object $\mathcal{K} \in D^b(X_{\text{ét}}, \mathbb{Z}/p^r\mathbb{Z})$ that satisfies **T1**–**T4**. Furthermore, a pair $(\mathcal{K}, t)$ satisfying **T2**–**T4** is unique up to a unique isomorphism.

In what follows, assume X to satisfy the assumption in this theorem. We fix a pair $(\mathcal{K}, t)$ as in this theorem and define the p-adic étale Tate twist $\mathbb{Z}/p^r\mathbb{Z}(n)$ as $\mathcal{K}$; the isomorphism t plays the role of a 'trivialization'. In case X is smooth, the above definition of $\mathbb{Z}/p^r\mathbb{Z}(n)$ is originally due to Schneider [Sch], §7, and it is closely related to the syntomic complex $\mathrm{S}_r(n)$ of Fontaine-Messing ([FM], [Ku]). However, $\mathbb{Z}/p^r\mathbb{Z}(n)$ is *not* a log-syntomic complex defined by Kato and Tsuji for intermediate n. The main results concerning p-adic étale Tate twists are the following:

Theorem 2. For non-negative integers m and n, there uniquely exists a morphism

$$\mathbb{Z}/p^r\mathbb{Z}(m) \otimes^{\mathbb{L}} \mathbb{Z}/p^r\mathbb{Z}(n) \longrightarrow \mathbb{Z}/p^r\mathbb{Z}(m+n) \quad \text{in} \quad D^-(X_{\text{ét}}, \mathbb{Z}/p^r\mathbb{Z})$$

that extends the natural isomorphism $\mu_{p^r}^{\otimes m} \otimes \mu_{p^r}^{\otimes n} \simeq \mu_{p^r}^{\otimes m+n}$ on $V_{\text{ét}}$.

Theorem 3 (Jannsen-Saito-S.)**.** Suppose that X is integral, and put $d := \dim(X)$.

(1) There exists a canonical trace isomorphism $\mathrm{H}_c^{2d+1}(X, \mathbb{Z}/p^r\mathbb{Z}(d)) \simeq \mathbb{Z}/p^r\mathbb{Z}$.

(2) For a constructible $\mathbb{Z}/p^r\mathbb{Z}$-sheaf $\mathcal{F}$ on $X_{\text{ét}}$ and an integer i, the natural pairing

$$\mathrm{H}_c^i(X, \mathcal{F}) \times \mathrm{Ext}_{X, \mathbb{Z}/p^r\mathbb{Z}}^{2d+1-i}(\mathcal{F}, \mathbb{Z}/p^r\mathbb{Z}(d)) \longrightarrow \mathbb{Z}/p^r\mathbb{Z}$$

is a non-degenerate pairing of finite $\mathbb{Z}/p^r\mathbb{Z}$-modules.

Theorem 4. Suppose that X is integral and proper over $\mathrm{Spec}(A)$. Then for integers i and n with $0 \le n \le d$, the natural pairing

$$\mathrm{H}_c^i(X, \mathbb{Z}/p^r\mathbb{Z}(n)) \times \mathrm{H}_{\text{ét}}^{2d+1-i}(X, \mathbb{Z}/p^r\mathbb{Z}(d-n)) \longrightarrow \mathbb{Z}/p^r\mathbb{Z}$$

is a non-degenerate pairing of finite $\mathbb{Z}/p^r\mathbb{Z}$-modules.

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On the category of the p-adic coefficients on curve

Zoghman Mebkhout

This lecture is an introduction to the category of p-adic coefficients on a smooth curve leading to a proof of the mixeness of the p-adic de Rham cohomology of a smooth affine variety over a finite field. There have been five parts.

In the first part we recalled the mains results of ℓ-adic global theory:

1) The finetness of the ℓ-adic cohomology of compact support of an algebraic variety with values in a constructible coefficient [SGA$_4$], the finetness of the ℓ-adic cohomology of an algebraic variety with values in a constructible coefficient [SGA$_{4,1/2}$],

2) The Grothendieck trace formula over a finite field [SGA$_5$],

3) The mixeness of the ℓ-adic cohomology of compact support of an algebraic variety with values in a mixe coefficient [D], [L],

4) The motivic property of an irreducible smooth ℓ-sheaf over a smooth curve over a finite field [La].

In the second part we recalled the local ℓ-adic theory of the Galois representations of a local fonction field.

The ℓ-theories must p-adic analogues.

In the third part we recalled the definition of the p-adic de Rham cohomology of a smooth affine variety $H^\bullet_{dR}(X/K)$ with coefficient in a discrete complete field [M-W], [A].

In the four part we recalled the local p-adic theory of the differential systems [CM$_1$], [CM$_2$], [CM$_3$], [CM$_4$]. This theory provide us with the abelian category $\mathrm{MLS}(\mathcal{R}_\mathrm{K}(1), \mathbb{Q})$ of finite modules over the ring $\mathcal{R}_K(1)$ of the analytic fonctions on the boundary of the unit disque, soluble and having rational p-adic exponents. This category is the p-adic fonction fields analogue of the local ℓ-adic Galois representations $\mathrm{Rep}_\ell(\mathrm{G}_{\mathrm{k}((\mathrm{x}))})$ and of the local p-adic number field category $\mathrm{Rep}_{\mathrm{dR}}(\mathrm{G}_\mathrm{K})$ of Fontaine de Rham representations. The tree categories have a monodromy theorem. The properties of the category $\mathrm{MLS}(\mathcal{R}_\mathrm{K}(1), \mathbb{Q})$ lead the the proof of the finetness of the de Rham cohomology $H^\bullet_{dR}(X/K)$ [M$_1$].

In the five part we recalled the global p-adic theory of coefficients on curve [M_1] leading to the proof lo the mixeness of of the p-adic de Rham cohomology $H^{\bullet}_{dR}(X/K)$ of a smooth affine variety over a finite field. More precisly we stated the following theorem [M_2]:

Theorem. Let K be complete discrete valuation field of caracteristic zero with a finite residu field k and X a smooth affine variety over k then the p-adic de Rham cohomology $H^i_{dR}(X/K)$ is mixed of weigth $\geq i$ for each $i, 0 \leq i \leq \dim X$.

The proof was know to us for a long time [M_1], is reduced the the proof of the mixeness of the p-adic the de Rham cohomology of the punctured plane with values in the exponent differentials module $M_{f,n,m}$. This proof has been proposed to S. Rozensztajn has a dissertation in 2000.

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On non-commutative twisting in étale and motivic cohomology

Guido Kings

(joint work with Jens Hornbostel)

In this talk we described a result about the construction of étale cohomology classes as twists of units in certain towers of number fields. The idea of twisting in étale cohomology was introduced by Iwasawa, Tate and with great succes by Soulé [So]. Building on ideas of Kato [Ka] a non-abelian Iwasawa Main Conjecture was formulated in [Hu-Ki]. As a consequence one sees that certain étale cohomology classes should arise as twists of units.

More precisely let K be a number field, p a prime number, T_p a finitely generated $\mathbb{Z}_p$-module with a continuous action of $\mathrm{Gal}(K_S/K)$, where S is a finite set of places and K_S is the maximal outside of S unramified field extension of K. Let $\mathcal{O}_K$ be the ring of integers of K. Then we consider T_p as an étale sheaf on $\mathcal{O}_K[1/S]$. Let $\mathcal{G}$ be the image of

$$\mathrm{Gal}(K_S/K) \to \mathrm{Aut}(T_p)$$

and let K_∞/K be the associated field extension with $\mathrm{Gal}(K_\infty/K) \cong \mathcal{G}$. The Iwasawa algebra is $\Lambda(\mathcal{G}) := \varprojlim \mathbb{Z}_p[\mathrm{Gal}(L/K)]$, where L runs through all field extensions $K \subset L \subset K_\infty$, which are finite over K. By Shapiro's lemma we get

$$\varprojlim_{K\subset L\subset K_\infty} H^j(\mathcal{O}_L[1/S], T_p(n)) \cong H^j(\mathcal{O}_K[1/S], \Lambda(\mathcal{G}) \otimes T_p(n)).$$

Now as $\mathrm{Gal}(K_S/K)$-module

$$\Lambda(\mathcal{G}) \otimes T_p(n) \cong \Lambda(\mathcal{G}) \otimes T_p(n)^{triv}$$

where $T_p(n)^{triv}$ means that we forget the $\mathrm{Gal}(K_S/K)$-action. Thus

$$H^j(\mathcal{O}_K[1/S], \Lambda(\mathcal{G}) \otimes T_p(n)) \cong H^j(\mathcal{O}_K[1/S], \Lambda(\mathcal{G})(1)) \otimes T_p(n-1)^{triv}.$$

By Kummer theory $H^1(\mathcal{O}_K[1/S], \Lambda(\mathcal{G})(1)) \cong \varprojlim \mathcal{O}_L[1/S]^* \otimes \mathbb{Z}_p$. Define the twisting map Tw as the composition

$$H^1(\mathcal{O}_K[1/S], \Lambda(\mathcal{G})(1)) \otimes T_p(n-1)^{triv} \cong H^1(\mathcal{O}_K[1/S], \Lambda(\mathcal{G}) \otimes T_p(n)) \to$$
$$\to H^1(\mathcal{O}_K[1/S], T_p(n))$$

where the last map is induced by the augmentation $\Lambda(\mathcal{G}) \to \mathbb{Z}_p$. The main result of [Ho-Ki] is

Theorem 3.0.1. Suppose that the Iwasawa μ-invariant of K (of the cyclotomic extension) is zero and that $\mathcal{G}$ is pro-p. Then for $n >> 0$ the twisting map

$$H^1(\mathcal{O}_K[1/S], \Lambda(\mathcal{G})(1)) \otimes T_p(n-1)^{triv} \xrightarrow{Tw} H^1(\mathcal{O}_K[1/S], T_p(n))$$

has finite cokernel.

Using the "Bloch-Kato-conjecture" announced by Voevodsky, Geisser proves in [Ge] that

$$H^i_{mot}(X, \mathbb{Z}/p^r(n)) \cong H^i_{et}(X, \mathbb{Z}/p^r(n))$$

if $i \leq n$. As the Hochschild-Serre spectral sequence is compatible with cup-product, it is possible to define a twisting map in motivic cohomology as well and we get the same result as before (for details see [Ho-Ki]).

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Riemann-Roch theorem for oriented cohomology on algebraic varieties

Ivan Panin

The talk is my report on my joint work with A.Smirnov [P1]. It is well-known that the classical Riemann-Roch theorem is one of the major technical tool in algebraic geometry and algebraic topology. We gave a far generalization of that classical theorem. It gives a new insight on Riemann-Roch, covers a lot of previous Riemann-Roch type theorems and gives new ones. For instance, we proved a motivic version of the famous Wu-formulae in topology.

For that following Adams [Ad1] we introduced the notion of an oriented cohomology pretheory on algebraic varieties and prove a Riemann-Roch theorem for ring morphisms between oriented pretheories. An explicit formula for the Todd genus related to a ring morphism is given. The theory is illustrated by classical and other examples.

An oriented cohomology theory comes equipped with an isomorphism $A(pt)[[t]] = A(\mathbf{P}^\infty)$ taking the variable t to the Euler class ξ_A of the tautological line bundle $\mathcal{O}(-1)$ on $\mathbf{P}^\infty$. A ring morphism $\varphi : A \to B$ of oriented cohomology theories gives rise to an inverse Todd series $itd_\varphi(t)$ which is just the ratio $\varphi(\xi_A)/\xi_B \in B(\mathbf{P}^\infty)$. If $\varphi(\xi_A)$ is a local parameter in $B(\mathbf{P}^\infty)$ then the ratio $td_\varphi(t) = \xi_B/\varphi(\xi_A) \in B(\mathbf{P}^\infty)$ is called the Todd series of φ. These series gives rise in a standard manner to the inverse Todd genus $itd_\varphi(E) \in B(X)$ of a vector bundle E over X and to the Todd genus $td_\varphi(E) \in B(X)$.

3.0.1. **Theorem.** *Let $\varphi : A \to B$ be a ring morphism of the oriented cohomology pretheories and let $i : Y \hookrightarrow X$ be a closed imbedding of smooth varieties with the normal bundle N. Then for each element $\alpha \in A(Y)$ one has the relation in $B(X)$*

$$i_B(\varphi(\alpha) \cup itd_\varphi(N)) = \varphi(i_A(\alpha)).$$

3.0.2. **Theorem.** *Let $\varphi : A \to B$ be a ring morphism of the oriented cohomology pretheories. Suppose that the series $itd_\varphi(t)$ is invertible in $\bar{B}[[t]]$. Then for each projective morphism of smooth projective varieties $f : Y \to X$ and each element $\alpha \in A(Y)$ one has the relation in $B(X)$*

$$f_B(\varphi(\alpha) \cup td_\varphi(T_Y)) = \varphi(f_A(\alpha)) \cup td_\varphi(T_X)$$

where T_Y (resp. T_X) is the tangent bundle to Y (resp. to X).

The classical Riemann-Roch-Hirzebruch theorem in the form of A.Grothendieck [BS], Riemann-Roch theorem of H.Gillet for higher K-theory, Riemann-Roch-Adams theorem of J.F.Adams and Ch.Soule [Ad1], [So], Baum-Fulton-MacPherson theorem [BFM], Wu-formulae in topology and some others are various particular cases of the two mentioned results.

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