Mathematisches Forschungsinstitut Oberwolfach

Report No. 2/2016

DOI: 10.4171/OWR/2016/2

Graph Theory

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10 January – 16 January 2016

Abstract. This workshop focused on recent developments in graph theory. These included in particular recent breakthroughs on nowhere-zero flows in graphs, width parameters, applications of graph sparsity in algorithms, and matroid structure results.

Mathematics Subject Classification (2010): 05C.

Introduction by the Organisers

The aim of the workshop was to offer a forum to communicate recent developments in graph theory and discuss directions for further research in the area. The atmosphere of the workshop was extremely lively and collaborative. On the first day of the workshop, each participant introduced her/himself and briefly presented her/his research interests, which helped to establish a working atmosphere right from the very beginning of the workshop. The workshop program consisted of 9 long more general talks and 21 short more focused talks. The talks were complemented by six evening workshops to discuss particular topics at a larger depth; these were held in ad hoc formed groups in the evenings.

The workshop focused on four interlinked topics in graph theory, which have recently seen new exciting developments. These topics were

- nowhere-zero flows and the dual notion of graph colorings,
- sparsity of graphs and its algorithmic applications,
- width parameters and graph decompositions, and
- results following the proof of Rota's conjecture in matroid theory.

The four main themes of the workshop were reflected in the selection of the workshop participants and in the choice of the talks for the program. However, space was also allocated for talks covering additional recent results that the workshop can touch all major developments in graph theory.

The first day of the workshop was designed to ignite the working atmosphere for the rest of the week. Most of the day was devoted to 5-minute minipresentations of workshop participants. Each participant was asked to briefly introduce her/himself and to present some of her/his research interests. This was complemented by having a problem session in the evening. The program of the remaining four days of the workshop consisted of talks scheduled in the mornings and late afternoons, and the rest of the afternoons reserved for work in groups. The exception was the afternoon on Wednesday when the workshop trip took place. In addition, there were six evening workshops organized by workshop participants on the following topics:

- information theory and graph limits,
- width parameters and their duality,
- graph immersions,
- Albertson's conjecture on the crossing and chromatic numbers of graphs,
- structure of matroids excluding a uniform matroid, and
- flows in graphs.

Most of the evening workshops consisted of few short presentations followed by an in-depth discussion of the topic.

One of the greatest recent breakthroughs in graph theory is the proof of the weak 3-flow conjecture. The three conjectures of Tutte on nowhere-zero flows are among the most fundamental open problems in graph theory. The recent breakthrough by Thomassen who proved that every 8-edge-connected graph has a nowhere-zero 3-flow, which was followed by the improvement to 6-edge-connected graphs by Lovasz Jr., Thomassen, Wu and Zhang, was one of the main themes of the workshop. In the very first lecture of the workshop, Thomassen briefed the participants on the state of the art in the area and presented many related open problems. The original proof of Thomassen proceeds by reformulating the problem as a question on covering graphs by stars. Barát and Thomassen conjectured that the result can be generalized to covering graphs by trees; the proof of this conjecture was the subject of Thomassé's talk on the second day of the workshop.

A large portion of the workshop was also devoted to graph colorings, which is the dual notion of nowhere-zero flows. Graph theory has seen exciting developments on colorings of graphs with forbidden induced subgraphs. The celebrated Strong Perfect Graph Theorem characterizes perfect graphs, i.e., graphs where the chromatic number and the clique number of every induced subgraph are equal. A conjecture of Gyárfás asserts that the chromatic number of a graph with no odd hole is bounded by a function of its clique number. The conjecture has recently been proven by Seymour and Scott. During his presentation, Scott updated the participants on the state of the art in the area and presented results related to

stronger conjectures of Gyárfás. Other related results were presented in the talks of Charbit and Chudnovsky.

Sparsity of graphs and its algorithmic applications formed another of the main themes of the workshop. Ossona de Mendez presented results on the structure of classes of sparse graphs based on his joint work with Nešetřil. On the algorithmic side, this was complemented by talks of Grohe, Marx and Schweitzer. In particular, Grohe and Schweitzer presented a polynomial time algorithm for isomorphism testing of graphs with bounded clique-width. It should be mentioned that the problem of isomorphism testing of general graphs is one of the main open problems in algorithmic graph theory. Indeed, the recent breakthrough of Babai, who designed a quasipolynomial time algorithm for the problem, attracted a very substantial attention in the discrete mathematics and computer science community. Babai's result was announced only few weeks before the workshop took place but it was one of the results that also resonated throughout the workshop.

The algorithm of Grohe and Schweitzer is based on tree-like decomposition of graphs, which together with width parameters formed another theme of the workshop. In addition to having a focused evening workshop on the subject, the workshop participants were updated on the progress related to the structure of graphs excluding certain graphs as minors in the talks of Chuzhoy, Kreutzer and Wollan. In particular, Chuzhoy reported in her talk on her work leading to obtaining a polynomial bound in the Grid Minor Theorem.

Matroid theory is another area that has seen a rapid development in the last several years. The origin of the Matroid Minor Project can be traced back to a workshop in Oberwolfach in 1999. The Matroid Minor Project of Geelen, Gerards, and Whittle led to the structural characterization of matroids with forbidden minors, which is analogous to the existing results on graphs with forbidden minors. Most of the results of the project have not been published yet. The project also led to developing tools that allowed proving Rota's conjecture, whose proof was announced in 2014 and is currently being written up. Recent results in the area were covered in the talks by Geelen and Nelson, and also in the evening workshop on the structure of matroids forbidding a uniform matroid as a minor.

The atmosphere during the workshop was extremely collaborative. The talks were often followed by in-depth discussions in small groups in the afternoons and evenings. It happened several times that small groups formed immediately after the end of a talk to discuss different approaches to the open problems mentioned in the talk. The evening workshops allowed more focused discussions of particular problems. With most of the participants staying till Friday evening or Saturday morning, the whole allocated week for the workshop was fully utilized. The workshop was significant for the graph theory community to stay updated on recent important developments, and it also significantly contributed to the professional development of its junior participants.

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Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, "US Junior Oberwolfach Fellows".

Workshop: Graph Theory

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Abstracts

The weak 3-flow conjecture and applications.

Carsten Thomassen

This talk is about the interplay between graph colorings, flows and decompositions. I shall point out that Dehn's result from 1916 about claw-decompositions of planar triangulations can be thought of as a consequence of the list-color version of Grötzsch's Theorem using a connection between group-coloring and group-connectivity established by Jaeger, Linial, Payan and Tarsi. Tutte's 3-flow conjecture from the beginning of the 1970ies is equivalent to Grötzsch's Theorem when restricted to planar graphs and is still open. The weak version, proposed by Jaeger, is now a theorem, and can be thought of as a result about claw decomposition. I shall discuss other applications, namely group flow (where the flow values are allowed to be elements in a specified subset of a specified group), graph factors modulo k (where we seek a subgraph with prescribed degrees modulo a fixed natural number k) and finally the so-called 1-2-3-conjecture saying that we can give each edge a weight 1 or 2 or 3 such that the weighted degrees form a proper vertex-coloring. If the edge-connectivity is large compared to the chromatic number, then such a weighting is possible with weights $1, 2$ only.

Edge partitioning highly connected graphs into trees

Stéphan Thomassé

(joint work with J. Bensmail, A. Harutyunyan, T.-N. Le, M. Merker.)

In his 2012 paper, Carsten Thomassen provided a solution to one of the most intriguing open question in graph theory raised by Tutte: highly connected (here 8-connected) graphs indeed have 3-flows. The existence of 3-flows was proved few years before equivalent to edge-decomposition of graphs into 3-stars, and Carsten 2012' paper included the following result:

Theorem 1. Every highly connected graph G has an edge decomposition into k-stars if the number of edges of G is divisible by k, where highly relates to k.

I will present in this talk a proof of the following result conjectured by Barát and Thomassen in 2006:

Theorem 2: (BHLMT 2015) For every fixed tree T, there exists t such that every t-connected graph G has an edge decomposition into copies of T if the number of edges of G is divisible by the number of edges of T.

The case of paths was proved in 2015 by Botler, Mota, Oshiro and Wakabayashi. And it turned out later on that arbitrary connectivity is not needed:

Theorem 3: (BHLT 2015) Every 24-edge connected graph with degree $f(t)$ can be edge decomposed into paths of length t, provided that its number of edges is divisible by t.

The proof of Theorem 2 was obtained by merging the ideas of the proof of Theorem 3 together with the work of M. Merker on decomposing a graph into

homomorphic copies of tree. Let us however emphasize that even if our result includes the case of stars (and hence implies that highly connected graphs have 3-flows), we rely on Thomassen's result in our proof, and therefore Theorem 2 does not shed new light on 3-flow.

A work in progress with the additional collaboration of T. Klimosova seems to decrease the 24 of Theorem 3 to only 3, which would be best possible for paths.

I will conclude this talk by discussing an exciting question of Jaeger, Linial, Payan and Tarsi stated in the language of subset sums which is a far reaching extension of 3-flows, and could be a new direction for future research on the topic.

Colouring quadrangulations of projective spaces

Tomáš Kaiser
(joint work with Matěj Stehlík)

By a well-known result of Youngs (1996), every quadrangulation of the projective plane P^2 is either bipartite or 4-chromatic. (Recall that a *quadrangulation* of a surface is an embedded graph with all faces of size 4.)

We extend the definition of quadrangulation to higher-dimensional spaces in place of surfaces, in such a way that the chromatic number of any quadrangulation of the n-dimensional projective space P^n is either 2, or at least $n + 2$, extending Youngs' theorem. The proof uses the topological method.

At the same time, the family of quadrangulations of projective spaces is quite rich: it includes all (generalised) Mycielski graphs, all complete graphs and certain graphs homomorphic to Schrijver graphs. The latter fact provides an alternative proof of the Lovász–Kneser theorem on the chromatic number of Kneser graphs.

How many colors can be saved?

Luke Postle
(joint work with Marthe Bonamy, Da Qi Chen, Thomas Perrett)

In 1998, Reed proved that the chromatic number of a graph is bounded away from its maximum degree and towards its clique number. Alternatively, one can save a number of colors proportional to the difference between the clique number and maximum degree. Here we discuss various generalizations of this result. First, we examine extensions of Reed's result for list-coloring and for a generalization of list coloring called correspondence coloring, first introduced by Dvořak. We apply these generalizations to progress on a conjecture of Erdős and Nešetřil that the strong chromatic index of a graph is at most 1.25 times the maximum degree squared. Second, we examine generalizations of Reed's result where the list, sizes degree sizes and clique numbers are allowed to vary locally over the vertices. Third, we apply these local list versions to show that the chromatic number of a graph is bounded away from its maximum average degree and toward its clique number.

Equivalently, we give a multiplicative improvement in the ratio of edges to vertices in a k-critical graph without a clique of size pk for all $p < 0.99$.

Complete graph immersions and minimum degree

Zdeněk Dvořák
(joint work with Liana Yepremyan)

An *immersion* of a graph H in a graph G is a function θ that assigns pairwise distinct vertices of G to vertices of H and pairwise edge-disjoint paths in G to edges of H, such that for each $uv \in E(H)$, $\theta(uv)$ joins $\theta(u)$ with $\theta(v)$. The immersion is *strong* if the paths intersect $\theta(V(H))$ only in their endpoints.

Note that if G contains H as a topological minor, then G also contains H as a strong immersion; the converse is false in general, and the (strong) immersion containment is incomparable with minor containment.

Motivated by Hadwiger's conjecture, Abu-Khzam and Langston [1] conjectured the following.

Conjecture 1. *Every graph without immersion of K_{n+1} has chromatic number at most n.*

This was proved for $n \leq 6$ by DeVos et al. [3], but the claim is open in general. Recently, Kawarabayashi proved that a smallest counterexample to this conjecture for any given n must have treewidth bounded by a function of n.

DeVos et al. [3] in fact proved the following stronger result.

Theorem 2. *For $n \leq 6$, every graph of minimum degree at least n contains an immersion of K_{n+1}.*

An analogue of this theorem for any $n \geq 7$ is known to be false—there exist constructions of graphs of minimum degree exactly n not containing an immersion of K_{n+1}. Somewhat surprisingly, no better lower bound is known, motivating the following question.

Question 3. *Is it true that for all $n \geq 1$, every graph of minimum degree at least n contains an immersion of K_n?*

It seems somewhat unlikely that this would be the case; on the other hand, the affirmative answer to Question 3 would still imply Conjecture 1, by a Kempe chain argument.

DeVos et al. [2] made a significant progress towards resolving Question 3 by proving that minimum degree at least $200n$ implies a strong immersion of K_n. This contrasts sharply with the known superlinear bounds for minors and topological minors. Recently, in a joint work with Liana Yepremyan, we improved this result as follows.

Theorem 4. *Let t be a positive integer. If a graph G has minimum degree at least $11t + 7$, then G contains K_t as a strong immersion.*

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Packing and Covering Immersions in 4-Edge-Connected Graphs

Chun-Hung Liu

(joint work with coauthors)

A classical result of Erdős and Pósa states that for every integer k, there exists N such that every graph either contains k disjoint cycles or contains a set of N vertices intersecting all cycles. Robertson and Seymour generalized this result to provide another characterization of planar graphs: a graph H is planar if and only if for every integer k there exists $f(k)$ such that every graph either contains k subgraphs each containing H as a minor or contains a set of $f(k)$ vertices intersecting all subgraphs of G containing H as a minor. We say that H has the Erdős-Pósa property with respect to the minor containment if the mentioned condition holds.

The topological containment and immersion containment are two graph containment relations closely related with the minor relation. A complete characterization for the graphs H in which H has the Erdős-Pósa property with respect to the topological containment was proved by Postle, Wollan and I. The description of the characterization is complicated, but it is unlikely to have a clean statement as we also proved that testing whether a graph H has the Erdős-Pósa property with respect to the topological containment is NP-hard.

We will address the immersion containment in this talk. Due to the nature of the immersion relation, it is more reasonable to consider an edge-variant of the Erdős-Pósa property. We say that a graph H has the edge-variant of the Erdős-Pósa property with respect to the immersion containment if for every integer k, there exists a number $f(k)$ such that every graph either contains k pairwise edge-disjoint subgraphs each immersing H or contains a set of $f(k)$ edges intersecting all subgraphs immersing H. The complete characterization for the graphs H with the edge-variant Erdős-Pósa property with respect to the immersion containment is considered to be as complicated as the topological containment. However, in this talk, we will prove that no condition for H is needed if we restrict the host graphs to be 4-edge-connected. More precisely, we prove that for every graph H, there exists a function f such that every 4-edge-connected graph G either contains k edge-disjoint subgraphs each immersing H or contains a set of $f(k)$ edges intersecting all subgraphs of G immersing H. This result is best possible in the sense that the 4-edge-connectivity cannot be replaced by the 3-edge-connectivity.

Explicit bounds for the graph minor structure theorem

Paul Wollan
(joint work with Ken Kawarabayashi, Robin Thomas)

Robertson and Seymour proved that every graph excluding a fixed H as a minor has a tree decomposition where each bag of the decomposition nearly embeds in a surface of bounded genus. The technical definition of "nearly embedding in a surface" includes a parameter k which measures how close the near-embedding is to a genuine embedding with no crossing edges (when $k = 0$, a k-near embedding is the same as an embedding without crossing edges). The theorem shows that the parameterized k-near embedding exists where the parameter k is bounded by a function on H.

The original proof of Robertson and Seymour was existential and gave no explicit bound on k in terms of H. Recent work by Geelen, Huynh, and Richter gives an explicit bound for the existential step in the Robertson-Seymour proof, and so together, there is a constructive bound on k. However, the length and technicality of the original proof meant no bound on k was known, although it had been estimated to be iterated tower functions.

We present a new and simpler proof of the graph minor structure theorem. A consequence of the shorter proof is that it is now possible to give an explicit bound on all the parameters in the theorem.

The Directed Grid Theorem

Stephan Kreutzer
(joint work with Ken-ichi Kawarabayashi)

Structural graph theory has proved to be a powerful tool for coping with computational intractability. It provides a wealth of concepts and results that can be used to design efficient algorithms for hard computational problems on specific classes of graphs occurring naturally in applications. Of particular importance is the concept of *tree width*, introduced by Robertson and Seymour as part of their seminal graph minor series [28][1]. Graphs of small tree width can recursively be decomposed into subgraphs of constant size which can be combined in a tree like way to yield the original graph. This property allows to use algorithmic techniques such as dynamic programming, divide and conquer etc. to solve many hard computational problems efficiently on graphs of small tree width. In this way, a huge number of problems has been shown to become tractable, e.g. solvable in linear or polynomial time, on graph classes of bounded tree width. See e.g. [2, 1, 3, 9] and references therein. But methods from structural graph theory, especially graph minor theory, also provide a powerful and vast toolkit of concepts and ideas to handle graphs of large tree width and to understand their structure.

[1]Strictly speaking, Halin [14] came up with the same notion in 1976, but it went unnoticed until it was rediscovered by Robertson and Seymour [25] in 1984.

One of the most fundamental theorems in this context is the *grid theorem*, proved by Robertson and Seymour in [25]. It states that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that every graph of tree width at least $f(k)$ contains a $k \times k$-grid as a minor. The known upper bounds on this function $f(k)$, initially being enormous, have subsequently been improved and are now polynomial [4]. The grid theorem is important both for structural graph theory as well as for algorithmic applications. For instance, algorithmically it is the basis of an algorithm design principle called *bidimensionality theory*, which has been used to obtain many approximation algorithms, PTASs, subexponential algorithms and fixed-parameter algorithms on graph classes excluding a fixed minor. See [5, 6, 7, 8, 11, 10] and references therein.

Furthermore, the grid theorem also plays a key role in Robertson and Seymour's graph minor algorithm and their solution to the disjoint paths problem [26] (also see [17]) in a technique known as the *irrelevant vertex technique*. Here, a problem is solved by showing that it can be solved efficiently on graphs of small tree width and otherwise, i.e. if the tree width is large and therefore the graph contains a large grid, that a vertex deep in the middle of the grid is irrelevant for the problem solution and can therefore be deleted. This yields a natural recursion that eventually leads to the case of small tree width. Such applications also appear in some other problems, see [13, 18, 21].

Furthermore, with respect to graph structural aspects, the excluded grid theorem is the basis of the seminal structure and decomposition theorems in graph minor theory such as in [27].

The structural parameters and techniques discussed above all relate to undirected graphs. However, in various applications in computer science, the most natural model are directed graphs. Given the enormous success width parameters had for problems defined on undirected graphs, it is natural to ask whether they can also be used to analyse the structure of digraphs and the complexity of NP-hard problems on digraphs. In principle it is possible to apply the structure theory for undirected graphs to directed graphs by ignoring the direction of edges. However, this implies an information loss and may fail to properly distinguish between simple and hard input instances (for example, the disjoint paths problem is NP-complete for directed graphs even with only two source/terminal pairs [12], yet it is solvable in polynomial time for any fixed number of terminals for undirected graphs [17, 26]). Hence, for computational problems whose instances are digraphs, methods based on undirected graph structure theory may be less useful.

As a first step towards a structure theory specifically for directed graphs, Reed [23] and Johnson, Robertson, Seymour and Thomas [15] proposed a concept of *directed tree width* and showed that the k-disjoint paths problem is solvable in polynomial time for any fixed k on any class of graphs of bounded directed tree width. Reed [22] and Johnson et al. [15] also conjectured a directed analogue of the grid theorem.

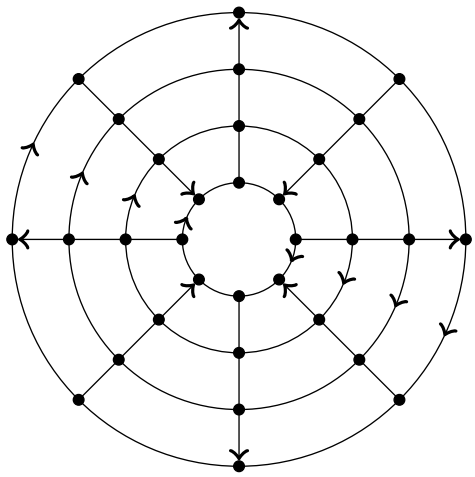

FIGURE 1. Cylindrical grid G_4.

Conjecture 1. (Reed and Johnson, Robertson, Seymour, Thomas) *There is a function $f : \mathbb{N} \to \mathbb{N}$ such that every digraph of directed tree width at least $f(k)$ contains a cylindrical grid of order k as a butterfly minor*

Actually, according to [15], this conjecture was formulated by Robertson, Seymour and Thomas, together with Alon and Reed at a conference in Annecy, France in 1995. Here, a *cylindrical grid* consists of k concentric directed cycles and $2k$ paths connecting the cycles in alternating directions. See Figure 1 for an illustration. A *butterfly minor* of a digraph G is a digraph obtained from a subgraph of G by contracting edges which are the only outgoing edge of their tail or the only incoming edge of their head.

In an unpublished manuscript, Johnson et al. [16] proved the conjecture for planar digraphs. In [20], this result was generalised to all classes of directed graphs excluding a fixed undirected graph as an undirected minor. This includes classes of digraphs of bounded genus. Another related result was established in [19], where a half-integral directed grid theorem was proved. More precisely, it was shown that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that every digraph G of directed tree width at least $f(k)$ contains a half-integral grid of order k. Here, essentially, *a half-integral grid* in a digraph G is a cylindrical grid in the digraph obtained from G by duplicating every vertex, i.e. adding for each vertex an isomorphic copy with the same in- and out-neighbours.

The main result of this talk, building on [24, 20, 19], is to finally solve this long standing open problem.

Theorem 2. *There is a function $f : \mathbb{N} \to \mathbb{N}$ such that every digraph of directed tree width at least $f(k)$ contains a cylindrical grid of order k as a butterfly minor.*

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Excluded Grid Theorem: Improved and (somewhat) Simplified

Julia Chuzhoy

We study the Excluded Grid Theorem of Robertson and Seymour — a fundamental and widely used result in graph theory. Informally, the theorem states that for every undirected graph G, if the treewidth of G is large, then G contains a large grid as a minor. Graph treewidth is an important and extensively used graph parameter, that, intuitively, measures how close a given graph G is to being "tree-like". For example, the treewidth of a tree is 1; the treewidth of the $(g \times g)$-grid is $\Theta(g)$; and the treewidth of an n-vertex constant-degree expander is $\Theta(n)$. Many combinatorial optimization problems that are hard on general graphs, have efficient algorithms on trees, often via the dynamic programming technique. Such algorithms can frequently be extended to bounded-treewidth graphs, usually by applying the dynamic programming-based algorithms to the bounded-width tree-decomposition of G. However, for large-treewidth graphs, a different toolkit is often needed. The Excluded Grid Theorem provides a useful insight into the structure of such graphs, by showing that every large-treewidth graph must contain a large grid as a minor. Formally, the Excluded Grid Theorem is the following.

Theorem 1. *[Robertson and Seymour] There is some function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$, such that for every integer $g \geq 1$, every graph of treewidth at least $f(g)$ contains the $(g \times g)$-grid as a minor.*

The Excluded Grid Theorem plays an important role in Robertson and Seymour's seminal Graph Minor series, and it is one of the key elements in their efficient algorithm for the Node-Disjoint Paths problem (where the number of the demand pairs is bounded by a constant). It is also widely used in Erdos-Posa-type results and in Fixed Parameter Tractability; in fact the Excluded Grid Theorem is the key tool in the bidimentionality theory.

It is therefore important to study the best possible upper bounds on the function f, for which Theorem 1 holds. Besides being a fundamental graph-theoretic question in its own right, better upper bounds on f immediately result in faster algorithms and better parameters in its may applications. Until recently, the best upper bound on $f(g)$ was super-exponential in g, while the best lower bound, due to Robertson, Seymour and Thomas is $f(g) = \Omega(g^2 \log g)$; the also conjecture that this value is sufficient. In this talk we discuss a recent series of results that

have lead to a polynomial upper bound on $f(g)$, with the best current bound being $f(g) = O(g^{19} \operatorname{poly} \log g)$. In addition to improved bounds, we describe a number of new techniques, including a conceptually simple and almost entirely self-contained proof of the theorem that achieves a polynomial bound on $f(g)$. We hope that this approach will eventually lead to tighter bounds on the theorem.

Computing with Tangles

Martin Grohe
(joint work with Pascal Schweitzer)

Tangles of graphs have been introduced by Robertson and Seymour in the context of their graph minor theory. Tangles may be viewed as describing "k-connected components" of a graph (though in a twisted way). They play an important role in graph minor theory. An interesting aspect of tangles is that they cannot only be defined for graphs, but more generally for arbitrary connectivity functions (that is, integer-valued submodular and symmetric set functions).

However, tangles are difficult to deal with algorithmically. To start with, it is unclear how to represent them, because they are families of separations and as such may be exponentially large. Our first contribution is a data structure for representing and accessing all tangles of a graph up to some fixed order.

Using this data structure, we can prove an algorithmic version of a very general structure theorem due to Carmesin, Diestel, Hamann and Hundertmark (for graphs) and Hundertmark (for arbitrary connectivity functions) that yields a canonical tree decomposition whose parts correspond to the maximal tangles. (This may be viewed as a generalisation of the decomposition of a graph into its 3-connected components.)

Isomorphism Testing for Graphs of Bounded Rank Width

Pascal Schweitzer
(joint work with Martin Grohe)

Decompositions play a central role in graph theory in particular in the structural theory of graphs with a forbidden minor. A general structure theorem due to Carmesin, Diestel, Hamann and Hundertmark (for graphs) and Hundertmark (for arbitrary connectivity functions) yields a canonical decomposition whose parts correspond to the maximal tangles. These tangles may be viewed as describing objects resembling "k-connected components". We describe algorithmic versions of such theorems.

Using the notion of treelike decompositions, it is then sometimes possible to further decompose said parts in a canonical fashion. Such canonical decompositions find their application for example in graph isomorphism testing. We describe how they can be constructed and used to test isomorphism of graphs of bounded rank width (or equivalently of bounded clique width) in polynomial time.

All infinite graphs have tree-decompositions displaying their topological ends

Johannes Carmesin

In 1964 Halin conjectured that every infinite connected graph has an end-faithful spanning tree. Roughly, such a spanning tree is one that has the same boundary at infinity as the graph. This was disproved by Seymour and Thomas and independently by Carsten Thomassen in the 1990s. I will explain how this conjecture can be repaired.

This talk will be self-contained; in particular I will not assume any special knowledge about infinite graphs.

Packing and covering in finite and infinite matroids

Nathan Bowler

The most important unsolved problem in the theory of infinite matroids remains the Packing/Covering conjecture. I'll introduce the conjecture, briefly discuss which special cases are known, and outline some consequences for infinite graphs and finite matroids.

Optimal Electrical Networks

Mark Walters

Given a graph on n vertices with m edges, each of unit resistance, how small can the average resistance between pairs of vertices be? There are two very plausible extremal constructions – graphs like a star, and graphs which are close to regular – with the transition between them occuring when the average degree is 3. However, one of our main aims in this talk is to show that there are significantly better constructions for a range of average degree including average degree near 3.

A key idea is to link this question to an analogous question about rooted graphs – namely 'which rooted graph minimises the average resistance to the root?'. The rooted case is much simpler to analyse that the unrooted, and one of our main results is that the two cases are asymptotically equivalent.

Joins of cubic graphs

Alexander Schrijver

Although our results are purely combinatorial, it will be convenient to consider cubic graphs as compact 1-dimensional topological spaces such that the neighbourhood of any point is either a line or a 3-star. (So the vertexless loop ◯ counts as cubic graph.) Removing a vertex results in a graph with three 'loose ends'.

Let k be a natural number and let G and H be cubic graphs with at least k vertices. The *k-join* $G \overset{k}{\vee} H$ of G and H is the random graph obtained as follows. Choose randomly k distinct vertices $u_1, \ldots, u_k$ of G and k distinct vertices $v_1, \ldots, v_k$ of H. For each $i = 1, \ldots, k$, remove (topologically) the vertex u_i from G and the vertex v_i from H; the three open ends thus arising in G are connected randomly to the three loose ends thus arising in H. (So for each i, there exist 3! possible connections.)

A function f on {cubic graphs} is a *partition function* (cf. [3]) if their exists a natural number k and a symmetric function $b : \{1, \ldots, k\}^3 \to \mathbb{R}$ such that for each cubic graph G:

$$f(G) = \sum_{\phi : E(G) \to \{1,\ldots,k\}} \prod_{v \in V(G)} b(\phi(\delta(v))),$$

where $\delta(v)$ is the set of edges incident with v, thus $\phi(\delta(v))$ is the multiset of 'colours' from $\{1, \ldots, k\}$ given to the edges incident with v. Then $b(\phi(\delta(v)))$ is the 'value' of this multiset. Counting perfect matchings or 3-edge-colorings can be described as partition functions.

A function f : {cubic graphs} $\to \mathbb{R}$ is called *multiplicative* if $f(\emptyset) = 1$ and $f(G \cup H) = f(G)f(H)$ for all $G, H \in$ {cubic graphs}. (Here $\emptyset$ is the graph with no vertices and edges, and $\sqcup$ denotes disjoint union.)

In [6] we proved that any real-valued function f on {cubic graphs} is a partition function if and only if f is multiplicative and for each k the matrix

$$(f(G \overset{k}{\vee} H))_{G,H \in \{\text{cubic graphs with at least } k \text{ vertices}\}}$$

is positive semidefinite. Here for any random graph γ, $f(\gamma)$ is the expected value of $f(\gamma)$. The proof is based on the first fundamental theorem (FFT) of invariant theory and on a theorem of Procesi and Schwarz [4]. It extends a theorem of Balázs Szegedy [7].

With Guus Regts and Bart Sevenster we proved in [5] an analogue of this theorem for *cyclic* cubic graphs — that is, to cubic graphs in which for each vertex v a cyclic order of the three edges incident with v is prescribed. Then in the join of u_i and v_i we must respect the cyclic orders at u_i and v_i. (So now there are 3 possible connections.) If we restrict b to cyclically invariant functions (i.e., $b(h, i, j)$ is invariant under cyclic permutations of h, i, j), the above characterization is maintained. Proving it requires consideration of the Young tableaux associated to certain representations of the symmetric group and a theorem of Hanlon and Wales [2].

We apply the characterization to weight systems (that is, functions satisfying the IHX relation) as occurring in the Vassiliev knot invariant theory (cf. [1]).

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Holes in graphs of large chromatic number

Alexander Scott

(joint work with Paul Seymour, Maria Chudnovsky)

A "hole" in a graph is an induced cycle of length at least 4. The perfect graph theorem says that if a graph has no odd holes and no odd antiholes (the complement of a hole), then its chromatic number equals its clique number. But what happens if we only exclude odd holes? Is chromatic number at most some function of the clique number? More generally, for what classes of graphs does a bound on clique number imply some (larger) bound on chromatic number?

Gyarfas proposed several conjectures of this form in 1985, and recently there has been significant progress on them. We survey this and several related results.

Chi Bounding Families of Oriented Graphs

Pierre Charbit

(joint work with P. Aboulker, J. Bang-Jensen, N. Bousquet, F. Havet, F. Maffray, S.Thomassé and J.Zamora)

Several results have been obtained, including recently, on the following topic: "if a graph does not contain a fixed clique as a subgraph but nevertheless has very big chromatic number, what other induced structures must it contain?". A famous conjecture of Gyárfás and Sumner states for any tree T and integer k, if $\chi(G)$ is big enough, either the graph contains a clique of size k or it contains T as an induced subgraph (by a classical result of Erdős, such a result is impossible if one

replaces T by any graph that contain a cycle). In this talk I will discuss some results and open problems about extensions of this conjecture to oriented graphs.

Coloring (some) perfect graphs

Maria Chudnovsky
(joint work with Irene Lo, Frederic Maffray, Nicolas Trotignon and Kristina Vuskovic, and Aurelie Lagoutte, Sophie Spirkl and Paul Seymour)

Grötschel, Lovász and Schrijver gave an algorithm that uses the ellipsoid method to find an optimal coloring of a perfect graph in polynomial time. However, no purely combinatorial algorithm is known for this problem, in spite of our fairly deep understanding of the structure of perfect graphs. This talk will describe recent results on the problem, namely present combinatorial coloring algorithms for a few different subclasses of perfect graphs.

Group-Walk Random Graphs

Agelos Georgakopoulos

The study of random graphs is currently one of the most active branches of graph theory. By far the most studied random graph model is that of Erdős & Rényi (ER), in which every pair of vertices is joined with an edge with the same probability, and independently of each other pair.

In recent years, many models of *geometric* random graphs have been emerging [2, 4]. The idea now is to embed the set of vertices (possibly randomly) into a geometric space —usually the euclidean or hyperbolic plane, and their higher-dimensional analogs— and then to independently join each pair of vertices with a probability that decays as the distance between the vertices in the underlying space grows.

One advantage of these geometric random graphs compared to the ER model is that they can approximate real-life networks much more realistically, but they are also of great theoretical interest given the impact of the ER model. A disadvantage is that there is an infinity of such models, obtained by varying the underlying geometry, the way the points are embedded, and the connection probability as a function of distance, and no canonical choice is available.

In [1] I introduce a model of (finite) geometric random graphs, called Group-Walk Random Graphs (GWRG), that arose from the study of the space of Dirichlet harmonic functions on an infinite group. This model uses an infinite Cayley graph as the underlying geometry in which the vertex set is embedded. The probability to join a pair of vertices with an edge is determined by the behaviour of random walk, and hence indirectly by their distance.

As an example, let G be an infinite homogeneous tree, rooted at a vertex o. In general, we allow G to be an arbitrary locally finite Cayley graph, or even a more

general graph. Let $G_n := G[\{v \in V(G) \mid d(v,o) \leq n\}]$ be the ball of radius n centered at o, and define the boundary ∂G_n to be the set $\{v \in V(G) \mid d(v,o) = n\}$ of vertices at distance exactly n from o.

We construct a random graph R_n as follows. The vertex set of R_n is the deterministic set ∂G_n. The edge set of R_n is constructed by the following process. We start an independent simple random walk in G_n from each vertex $v \in \partial G_n$, and stop it upon its first return to ∂G_n, letting $v^\dagger$ denote the vertex in ∂G_n where this random walk was stopped. We then put an edge in R_n joining v to $v^\dagger$ for each $v \in \partial G_n$.

We could stop the construction here and declare R_n to be our GWRG, but it is more interesting to consider the following evolution: let $R_n^1 := R_n$, and for $i = 2, 3, \ldots$ let R_n^i be the union of R_n^{i-1} with an independent sample of R_n^1; or in other words, R_n^i is the random graph obtained as above when we start i independent particles at each vertex in ∂G_n. Alternatively, rather than starting a fixed number i of particles at each vertex, we could start a random number of particles independently at each vertex, according to the Poisson distribution with mean i.

The original motivation for introducing the model was to try to find connections between the properties of the group of the underlying Cayley graph G and the statistics of the corresponding GWRGs, and hence use the latter as a tool for studying groups. Since then connections to other objects have emerged, including Sznitman's random interlacements [5]. Moreover, in the special case where G is the 2-dimensional square grid $\mathbb{Z}^2$, it turns out that R_n^i converges in the Benjamini-Schramm sense, as $n \to \infty$, to a well-studied model of long-range percolation [3]. I hope that other known models of geometric random graphs will turn out to be special cases of GWRGs.

Finally, when G is a homogeneous tree, R_n^i has a very interesting Benjamini-Schramm limit which is almost surely finite for every i, and is the object of ongoing research joint with O. Angel, J. Haslegrave and G. Ray.

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Accessibility in transitive graphs

Matthias Hamann

We look for connections between the cycle space and the cut space of infinite transitive graphs. The *cut space* of a graph G is the set of all *cuts* $E(A,B)$ for bipartitions $\{A,B\}$ of $V(G)$ seen as a $GF(2)$-vector space and the *cycle space* is

the set of all finite sums over $GF(2)$ of edge sets of finite cycles. We call the vector spaces $\mathrm{Aut}(G)$-*modules*, as the automorphisms of G act canonically on them. They are *finitely generated* $\mathrm{Aut}(G)$-module if they have a generating set that consists of only finitely many $\mathrm{Aut}(G)$-orbits. In [4], we obtain the following theorem.

Theorem 1. *Let G be a 2-edge-connected transitive graph. If its cycle space is a finitely generated* $\mathrm{Aut}(G)$*-module, then the same is true for its cut space.*

Thomassen and Woess [6] showed that locally finite graphs whose cut spaces are generated by cuts of bounded size are *accessible*, i. e. there exists some $n \in \mathbb{N}$ such that any two ends can be separated by removing at most n edges of the graph. Theorem 1 implies the following.

Theorem 2. *Every locally finite transitive graphs whose cycle space is generated by cycles of bounded length is accessible.*

Theorem 2 has various applications. As locally finite transitive planar graphs have the property that their cycle space is generated by cycles of bounded length, see [5], we obtain as a corollary of Theorem 2 that these graphs are accessible, a theorem of Dunwoody [2] which we proved combinatorially.

Another class of graphs whose cycle spaces are generated by cycles of bounded length are hyperbolic graphs. As another corollary, we can confirm a conjecture of Dunwoody [1] that locally finite transitive hyperbolic graphs are accessible.

Accessibility is also a notion in group theory, which corresponds to the graph theoretical notion in a natural way, see [6]. Therefore, we can apply Theorem 2 to obtain a combinatorial proof of Dunwoody's theorem [3] that finitely presented groups are accessible.

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Cycles in Directed 3-Graphs

Imre Leader

It is trivial that if a tournament (complete directed graph) is not transitive then it contains a directed 3-cycle, and that in general a directed graph that has a cycle may have its shortest cycle of length n. We consider the analogous questions for 3-graphs: a directed 3-graph consists of some 3-sets, each of which has been given

one of its two possible cyclic orientations, and a 'directed cycle' means a positive sum of some of the 3-sets that gives weight zero to each 2-set.

We show that there are directed 3-graphs whose shortest cycle has length about $n^2/2$, which is best possible, and that there are 3-tournaments whose shortest cycle has length about $n^2/3$.

Chromatic threshold and critical threshold

Peter Nelson

The *critical number* of a GF(q)-representable matroid M, viewed as a subset of a projective geometry G over GF(q), is the codimension of the largest flat of G that does not intersect M. This is a geometric analogue of chromatic number. In this context, I will discuss surprising geometric analogues of well-studied problems in graph theory, including the Erdős-Stone Theorem, the chromatic number of triangle-free graphs, and the more general 'chromatic threshold' problem.

Generating k-connected matroids and graphs

Jim Geelen
(joint work with Bert Gerards, Tony Huynh, and Stefan van Zwam)

Tutte's Wheels and Whirls Theorem states that, if M is a 3-connected matroid with $|M| \geq 4$ and M is neither a wheel nor a whirl, then there is an element e such that $M\backslash e$ or M/e is 3-connected. This implies that, if M is a 3-connected matroid with $|M| \neq 0$, then M has a 3-connected minor N with $|N| = |M| - 1$ or $|M| - 2$.

These results do not extend to 4-connectivity; there exist 4-connected matroids where the distance to the largest 4-connected proper minor can be made arbitrarily large (consider the cycle-matroid of a toroidal grid). Moreover, while Tutte's notion of 3-connectivity is very natural for graphs, the notion of 4-connectivity is not. The cycle matroid of a graph G is k-connected exactly when the graph is k-connected and has girth at least k. So $M(G)$ is 3-connected if and only if G is simple and 3-connected, which is very natural, however, the cycle-matroids of complete graphs are not 4-connected, which is not very satisfactory to graph theorists. We overcome both of these issues by relaxing notion of k-connectivity.

Let $L = (l_1, l_2, \ldots, l_{k-1})$ be a sequence of non-negative integers. A matroid M is *L-connected* if for each t-separation (A, B) of M with $t < k$ either $|A| \leq l_t$ or $|B| \leq l_t$. Thus Tutte's version of k-connectivity corresponds with $(0, 1, \ldots, k-2)$-connectivity. In order for the cycle-matroids of complete graphs to be L-connected we need that $(l_1, l_2, \ldots, l_{k-1})$ grows quadratically, but then the cycle-matroids of grids will also be L-connected.

In joint work with Bert Gerards, Tony Huynh, and Stefan van Zwam we prove the following result:

Theorem 1. *Let $L = (l_1, l_2, \ldots, l_{k-1})$ be a sufficiently fast growing sequence of non-negative integers. If M is a sufficiently large L-connected matroid then M has an L-connected minor N with $|N| = |M| - 1$ or $|M| - 2$.*

We can say a lot about the structure of M when we require $|N| = |M| - 2$; in particular, we can avoid this outcome when M is the cycle matroid of a graph.

The theorem arose out of work on Rota's Conjecture and played a central role in proving that conjecture.

Structural Sparsity (Graph Theory meets Model Theory)

Patrice Ossona de Mendez
(joint work with Jaroslav Nešetřil)

The structural properties of sparse graphs has a long history, from the study of structural properties of graphs with bounded degrees and Robertson–Seymour graph structure theorem for graphs excluding a minor [28] to the structure theorems for graphs excluding a topological minor [10, 2] and of graphs excluding a strong immersion [6].

Although the notion of sparsity may appear as elusive, the authors introduced a general dichotomy for (monotone) graph classes [19, 18, 21] between *nowhere dense* and *somewhere dense* classes of graphs, and the division of nowhere dense classes between classes with bounded expansions and those with unbounded expansion [15]. Several structural decomposition tools [14, 16, 22, 25] opened the way to efficient algorithms for classes with bounded expansion [17, 3, 5] and for nowhere dense classes [9].

Nowhere dense classes and classes with bounded expansion received several non-trivially equivalent characterizations [22, 26] (see also [8]).

In order to stress the variety of the applications, we mention four results:

- the boundedness, for each odd integer p, of the exact p-distance chromatic number of graphs within a class with bounded expansion (which already for $p = 3$ and the class of planar graphs leads to interesting developments), and how this property characterizes bounded expansion classes;
- the possibility to find large induced sunflower-like substructures witnessing a large induced logarithmic density of copies of a fixed graph [20];
- the characterization by means of r-neighborhood covers of nowhere dense classes and of classes with bounded expansion [8];
- the existence of fixed parameter model checking algorithms [4, 5, 9, 11].

Among the many characterizations of nowhere dense classes and of classes with bounded expansion, we focus on structural characterizations based on excluded substructures. This study leads to the conjecture [22, 24] that every monotone nowhere dense class of graphs either has bounded expansion of contains for some $k \in \mathbb{N}$ the k-th subdivisions of graphs with arbitrarily large girth and chromatic number. Note that this conjecture would follow from a conjecture of Erdős and Hajnal [7], and also from a conjecture of Thomassen [31].

We say that a structure is *structurally sparse* if it may be reconstructed from a small amount of information, in particular if it can be obtained as an interpretation of a sparse structure. The notions of sparse class and structurally sparse class are obviously linked, and closely related to the general model theoretic notions of *independence property* introduced by Shelah [30] — which expresses the possibility of encoding a random bipartite graph with a definable edge relation — and the notion of VC dimension, which arose in probability theory in the work of Vapnik and Chervonenkis [32]. These notions are deeply linked as, as observed by Laskowski [12], a complete first-order theory does not have the independence property if and only if, in each model, each definable family of sets has finite VC dimension. This link is made clear by a theorem by Adler and Adler [1], which states that for a monotone class of graphs, it is equivalent that the class is nowhere dense, that first-order formulas have bounded VC-dimension on the class (NIP), and that first-order formulas have bounded order property on the class (stability). These last properties are defined as the possibility to construct some special generic structures (bipartite random graph, point-set graph, half graphs, transitive tournaments) by means of first-order interpretations.

We show two examples of direct applications:

- the proof of existence of functions f, g such that every graph G that can be oriented in such a way that some subset X of vertices of order $f(n, r)$ has the property that any two of its elements are linked by a directed path of length r in exactly one direction contains an exact $g(r)$-subdivision of K_n;
- the proof that the category of graphs cannot be represented by any monotone nowhere dense class of graphs with homomorphisms (answering a question by J. Nešetřil).

Further investigating these connections, we note that the *shatter function* (used to define VC-dimension) can be used to coin yet another characterization of nowhere dense and bounded expansion classes: it follows from the characterization theorem of Adler and Adler [1], Sauer–Shelah lemma [29, 30], and the characterization of bounded expansion classes by the boundedness of neighborhood complexity by Reidl [27] that the shatter function of the family of all r-balls of the graphs in a monotone class is linear (for each r) if and only if the class has bounded expansion, polynomial (for each r) if and only if the class is nowhere dense, and exponential (for sufficiently large r) otherwise.

We also mention some connections with graph limits (characterization of random-free hereditary classes by Lovász and Szegedy [13], and the conjecture on existence of modeling limits for structural limits of graphs in a nowhere dense class [23]), and with class entropy [26].

In conclusion, it appears that connections to finite model theory can give most valuable hints on the reasons why excluding special generic substructures may imply strong global properties.

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High-dimensional combinatorics – Hypertrees

Nati Linial

There is a major ongoing research effort to develop *high dimensional combinatorics*. It starts from the observation that most basic combinatorial constructs are, in a well-defined sense, one-dimensional. It is extremely interesting to investigate the higher-dimensional counterparts of these objects.

Specifically, we study high-dimensional analogs of: Permutations, Graphs (simplicial complexes), tournaments and more. Here we discuss high-dimensional analogs of trees. There are several ways to define (one-dimensional) trees, and we pick a definition that nicely adapts to higher dimensions. Let A be the one-dimensional *boundary operator*, i.e., the $n \times \binom{n}{2}$ incidence matrix of some orientation of the complete graph K_n. It is easy to see that $\mathrm{rank}_{\mathbb{R}}(A) = n-1$, that a set of linearly independent columns in A is synonymous with a forest and column bases correspond to trees.

The rows and columns of the d-dimensional boundary operator A are indexed by $\binom{[n]}{d}$ resp. $\binom{[n]}{d+1}$. For every $S = \{x_1, \ldots, x_{d+1}\} \in \binom{[n]}{d+1}$ where the x_i appear in order, and for every i, the $(S \setminus \{x_i\}, S)$-entry of A is $(-1)^i$. All other entries of A are zero. It is easy to show that $\mathrm{rank}_{\mathbb{R}}(A) = \binom{n-1}{d}$. We only consider d-dimensional simplicial complexes X with a full $(d-1)$-skeleton. When the columns of A that correspond to the d-faces of X are linearly independent, we say that X is *acyclic*, and when they form a column basis, we say that X is a *d-dimensional hypertree*. **G. Kalai** who introduced this notion in 1983 proved some striking

results about hyeprtrees, but we still do not know: **How many d-dimensional n-vertex hypertrees are there**?

In an elementary 1-collapse we remove from a graph G a vertex of degree one and the single edge incident with it. We say that G is 1-collapsible if it is possible to eliminate all its edges by a series of elementary 1-collapses. Clearly, G is collapsible iff it is acyclic. Likewise, in an elementary d-collapse we remove from a d-complex X a $(d-1)$-face τ and a d-face σ provided that σ is the unique d-face that contains τ. We say that X is *d-collapsible* if we can eliminate all its d-faces in a series of such steps. It is easy to see that a collapsible complex is acyclic, but in dimensions $d \geq 2$ the reverse implication fails. In fact, Kalai posed **Conjecture: The fraction of d-hypertrees that are collapsible tends to zero when n grows**. This is still open.

Can we at least find *many* non-collapsible hypertrees? The following construction works in all dimensions $d \geq 2$, but for simplicity of notation I only state it for $d = 2$: Let n be prime and let $a, b, c \in \mathbb{F}_n$. Create the 2-hypertree $X_{a,b,c}$ where $\{x, y, z\}$ is a 2-face iff $x + y + z \in \{a, b, c\}$. **Theorem: [Linial, Meshulam, Rosenthal] The complex $X_{a,b,c}$ is a 2-hypertree. It is 2-collapsible iff a, b, c is an arithmetic triple**.

This is only one of several aspects in which the one and high-dimensional situations differ. Let F be an n-vertex forest. We say that an edge $e \notin F$ is in the *shadow* of F if $F \cup \{e\}$ contains a cycle. It is easy to see that if F is "almost a tree", i.e., it has $n-2$ edges, then at least $\lfloor \frac{n^2}{4} \rfloor$ edges are in its shadow and this is tight. If X is an acyclic d-complex, we say that a d-face not in X is in X's *shadow* if the column corresponding to σ is linearly spanned by the columns of X. Let n be prime for which 2 is a primitive element (A conjecture of Artin states that a positive fraction of all primes satisfy this condition). Consider the simplicial complex Z_n whose 2-faces consist of all arithmetic triples $\bmod n$. **Theorem: [Linial, Yuval Peled] The complex Z_n is acyclic, it has $\binom{n-1}{2} - 1$ 2-faces and its shadow is empty**.

Subexponential parameterized algorithms on planar graphs via low-treewidth covering families

Dániel Marx

(joint work with Fedor Fomin, Daniel Lokshtanov, Marcin Pilipczuk, Michał Pilipczuk, and Saket Saurabh)

Our goal is to prove the existence of algorithms with running time $2^{O(\sqrt{k}\cdot \mathrm{polylog} k)} \cdot n^{O(1)}$ for various problems on planar graphs that involve finding a connected subgraph on k vertices. Our main technical result is the following: given a planar graph G on n vertices and an integer k, we can construct in time $2^{O(\sqrt{k}\cdot \mathrm{polylog} k)} \cdot n^{O(1)}$ a family $G_1, \ldots, G_m$ of $2^{O(\sqrt{k}\cdot \mathrm{polylog} k)} \cdot n^{O(1)}$ subgraphs of G such that

(1) each G_i has treewidth $O(\sqrt{k} \cdot \mathrm{polylog} k)$,

(2) for every connected subgraph $H \subseteq G$ on k vertices, there is at least one G_i that contains H as a subgraph.

When putting together this result with known algorithms having running time $2^{O(w \cdot \mathrm{polylog} w)} \cdot n^{O(1)}$ on graphs of treewidth w, we immediately obtain algorithms of running time $2^{O(\sqrt{k} \cdot \mathrm{polylog} k)} \cdot n^{O(1)}$ for various problems. The resulting algorithms are very robust in the sense that it is easy to generalize them to weighted, colored, or directed settings. For example, we can achieve this running time for the following problems, where no such algorithms were known previously:

- finding a path of length k in a directed graph,
- finding a path of length k and minimum weight in a weighted undirected or directed graph,
- finding an s-t path of length exactly k in an undirected or directed graph,
- finding a cycle of length exactly k in a directed or undirected graph,
- finding a subgraph isomorphic to a given connected bounded-degree graph H on k vertices.

Decompositions of large graphs into small subgraphs

Daniela Kühn, Deryk Osthus
(joint work with Ben Barber, Stefan Glock, Allan Lo, Richard Montgomery and Amelia Taylor)

A fundamental theorem of Wilson states that, for every graph F, every sufficiently large F-divisible clique has an F-decomposition. Here a graph G has an F-decomposition if the edges of G can be covered by edge-disjoint copies of F and F-divisibility is a trivial necessary condition for this. We extend Wilson's theorem to graphs which are allowed to be far from complete. In particular, this yields a "graph theoretical" rather than "algebraic" proof of Wilson's theorem. Our main contribution is a general "iterative absorption" method which turns an approximate or fractional decomposition into an exact one. This is also connected to the question of when a partially completed Latin square can be extended to a full one. (This covers joint work with Ben Barber, Stefan Glock, Allan Lo, Richard Montgomery and Amelia Taylor.)

Tilings in graphons

Jan Hladký
(joint work with Ping Hu, Diana Piguet)

Tools emerged with the theory of dense graph limits (and the related theory of flag algebras) have been immensely useful in connection with the part of extremal graph theory that deals with relations between subgraph density. A prominent example in this direction is a result of Razborov [3] who determined an optimal

function $f : [0,1] \to [0,1]$ such that if an n-vertex graph G contains at least $\alpha\binom{n}{2}$ edges then

$$G \text{ contains at least } (f(\alpha) + o(1)) \binom{n}{3} \text{ triangles,} \tag{1}$$

thus resolving an old question of Lovász and Simonovits.

However, many problems in extremal graph theory concern other quantities. For example, Allen, Bötcher, Hladký, and Piguet [1], established — as a counterpart to (1) — the optimal function $g : [0,1] \to [0,1]$ such that if an n-vertex graph G contains at least $\alpha\binom{n}{2}$ edges then

$$G \text{ contains at least } (g(\alpha) + o(1)) \tfrac{n}{3} \text{ vertex-disjoint triangles,} \tag{2}$$

In the reported project we studied in general the underlying concept of F*-tilings*, by which we mean a collection of vertex-disjoint copies a fixed graph F. Note that when $F = K_2$, we are dealing with matchings.

The main achievement of the project is that we can define a counterpart of F-tilings for graphons. We illustrate the notion on the simplest case when $F = K_2$.

Definition 1. *Suppose that $W : \Omega^2 \to [0,1]$ is a graphon. We say that $f \in \mathcal{L}^1(\Omega^2)$ is a* matching *in W if*

(i) $f \geq 0$,
(ii) $\mathrm{supp}(f) \subset \mathrm{supp}(W)$,
(iii) for almost every $x \in \Omega$, we have $\int_y f(x,y) + \int_y f(y,x) \leq 1$.

The above properties are limit counterparts to fractional matchings in finite graphs. Indeed, any fractional matching in a finite graph G can be viewed as a function $h \in \mathcal{L}^1(V(G)^2)$ such that (i) $h \geq 0$, (ii) h is supported only on pairs of vertices that form an edge, and (iii) for each $x \in V(G)$, we have $\sum_y h(x,y) + \sum_y h(y,x) \leq 1$.

Let us emphasize that while we call the objects in Definition 1 *(integral) matchings*, motivation for them comes from fractional matchings of finite graphs. Roughly speaking, the Regularity lemma which is inherent in any graphon allows to view many fractional objects as integral ones. Also, observe that the notion a matching (or an F-tiling in general) does not depend on the values of the graphon, but only on its support. This is because the Blow-up lemma tells us that sufficiently regular pair of any positive density is a suitable object for tiling.

In particular, we can introduce a graphon parameter which is the supremum of the sizes over all such F-tilings in that graphon. We give a transference statement that allows us to switch between the finite and limit notion, and derive several favourable properties, including the LP-duality counterpart to the classical relation between the fractional vertex covers and fractional matchings/tilings.

We give two applications of this theory. Firstly, we determine the asymptotically almost sure F-tiling number of inhomogeneous random graphs $\mathbb{G}(n, W)$. Secondly, we give a short proof of graphon version of a tiling theorem of Komlós [2] which in this context extends the Hajnal–Szemerédi Theorem. This version of Komlós's theorem together with the transference result implies the original result. The

advantage of the current proof is that it is, unlike the original one, completely pedestrian.

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Quasi-random hypergraphs

Christian Reiher
(joint work with Vojtěch Rödl and Mathias Schacht)

In recent work with Rödl and Schacht, we found a new proof of a conjecture due to Erdős and Sós stating that large uniformly dense 3-uniform hypergraphs of density greater than 1/4 contain four vertices spanning at least three hyperedges. This was proved earlier by Glebov, Král' and Volec with the help of flag algebras and computers. The new proof is based on the hypergraph regularity method and gave rise to further developments in this field that are surveyed in this talk.

Packing induced subgraphs

Jacob Fox
(joint work with Hao Huang and Choongbum Lee)

Packing problems have been studied for more than four centuries, and have connections to diverse areas of pure and applied mathematics. In this talk, we focus on two well-studied combinatorial packing problems, Pippenger and Golumbic's graph inducibility problem from 1975 and (time permitting) Wilf's permutation packing problem from 1992. We discuss recent joint work with Hao Huang and Choongbum Lee which solves these problems in almost all instances.

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