Mathematisches Forschungsinstitut Oberwolfach

Report No. 30/2017

DOI: 10.4171/OWR/2017/30

Geometric Structures in Group Theory

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25 June – 1 July 2017

Abstract. Geometric group theory has natural connections and rich interfaces with many of the other major fields of modern mathematics. The basic motif of the field is the construction and exploration of actions by infinite groups on spaces that admit further structure, with an emphasis on geometric structures of different sorts: one usually seeks actions in order to illuminate the structure of groups of particular interest, but one also explores actions in order to understand the underlying spaces. The dramatic growth of the field in the late twentieth century was closely associated with the study of generalized forms of non-positive and negative curvature, and classically the spaces at hand were cell complexes with some additional structure. But the scope of the field, the range of groups embraced by its techniques, and the nature of the spaces studied, have expanded enormously in recent years, and they continue to do so. This meeting provided an exciting snapshot of some of the main strands in the recent development of the subject.

Mathematics Subject Classification (2010): 20Exx, 20Fxx, 20Jxx, 57Exx, 57Fxx, 57Mxx.

Introduction by the Organisers

This meeting explored the rapidly expanding range of ways in which ideas developed in the field of geometric group theory are finding new applications, both to wider classes of groups and to more general spaces.

At its inception and during a period of rapid growth in the late twentieth century, geometric group theory was focussed on trying to understand the role that non-positive curvature plays in the theory of infinite groups, with an emphasis on finitely generated groups acting on cell complexes that carry metrics with non-positive (or strictly negative) curvature. Here notions of "curvature" are needed which can be applied to singular spaces; those most commonly used are defined by either local conditions or coarse global conditions. One can import and adapt much of the classical geometry of non-positively curved Riemannian manifolds (such as symmetric spaces of non-compact type) to this more general setting. But the flexibility of allowing singular spaces leads to striking advances in situations where the techniques of Riemannian or differential geometry no longer apply.

A long-term trend in the field is to expand the class of actions governed by the above philosophy to include discrete groups of particular intrinsic interest. Many such groups arise as automorphism groups of fundamental objects. These include the mapping class group $\mathrm{MCG}(\Sigma)$ of a closed surface Σ and its close cousin, the group $\mathrm{Out}(F_n)$ of outer automorphisms of the free group on n letters F_n. These groups cannot be realized as lattices in any Lie group, so are not covered by the classical theory of actions on Riemannian symmetric spaces.

The first step towards a fruitful exploration of these groups is to construct suitable spaces with nice geometric properties, which admit actions adapted to the geometry. Ideally, these actions should be balanced, in the sense that they are both proper and cocompact, but one often has to be prepared to trade some amount of niceness for existence. A guiding paradigm is the action of a lattice Γ in a non-compact simple Lie group G on the corresponding Riemannian symmetric space G/K, but the main focus now is on non-classical examples, e.g. where Γ is replaced by $\mathrm{MCG}(\Sigma)$ or $\mathrm{Out}(F_n)$, and in each case a range of challenging questions arises which are specific to the particular example being studied.

Because the groups studied are so closely related to the spaces on which they act, one expects to find deep connections with algebraic topology, and this is indeed the case. Constructing suitable singular metric spaces X, such as the Culler-Vogtmann Outer space CV_n, leads to determining subtle homological finiteness properties for the acting group (i.e. for $\mathrm{Out}(F_n)$ when $X = \mathrm{CV}_n$). Other homological theories are focused on the behavior of spaces or groups at infinity.

The central ideas of geometric group theory can be adapted to situations where curvature conditions are relaxed, so that more groups can be taken into account. In particular the notion of hyperbolic spaces was generalized to that of relatively hyperbolic spaces, in analogy to the passage from uniform to non-uniform lattices in the setting of semisimple Lie groups. Relatively hyperbolic groups are a much larger class than strictly hyperbolic groups, but are still susceptible to analysis using hyperbolic methods.

A particularly striking feature of geometric group theory is its demonstrated usefulness for solving long-standing questions in other fields, perhaps most notably 3-dimensional topology and geometry. For example, CAT(0) cubical complexes, first introduced for the rich interplay between geometric and combinatorial considerations they admit, were a crucial tool for answering the remaining questions in Thurston's program on 3-dimensional geometry.

A further direction the field is taking is adapting its techniques to apply to large groups that need not be discrete – for example locally compact groups. Here cell complexes are not necessarily well-suited to the groups at hand – this is certainly the case for totally disconnected locally compact (but no longer discrete) groups, TDLC groups for short. The prototype of such situations is the action of a non-Archimedean simple algebraic group on a Euclidean building, as provided by classical Bruhat-Tits theory. Recently, the theory has become much richer by allowing groups that no longer admit any matrix interpretation (typically, non linear groups containing non-residually finite discrete subgroups). The first step of this generalization still deals with groups acting on buildings, but the most recent work may very well lead to a much more general theory of TDLC groups.

The ideas and topics mentioned above were all important themes of the workshop. A final topic that was covered is a new frontier in geometric topology, in which the profinite completion of the fundamental group emerges as a key invariant of low-dimensional manifolds and is shown to encode a remarkable amount of the geometry of the manifold. This exciting area is likely to engage many colleagues from geometric group theory in the near future.

We had 53 participants from a wide range of countries, and 22 official lectures. The staff in Oberwolfach was—as always—extremely supportive and helpful. We are very grateful for the additional funding for 5 young PhD students and recent postdocs through Oberwolfach-Leibniz-Fellowships. We think that this provided a great opportunity for these students. We feel that the meeting was highly successful. The quality of the lectures was outstanding, and outside of lectures there was a constant buzz of intense mathematical conversations. We are confident that this conference will lead to new and exciting mathematical results and to new mathematical collaborations.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.

Workshop: Geometric Structures in Group Theory

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Abstracts

Connectivity of outer space at infinity revisited

Kai-Uwe Bux
(joint work with Karen Vogtmann, Peter Smillie)

In [1], M. Bestvina and M. Feighn have introduced a bordification of outer space in the spirit of the Borel-Serre compactification of the symmetric space. We give an alternative construction, defining instead a deformation retract of outer space. This is the analogue of Grayson's approach to arithmetic groups.

Fix a free group F_n of rank n and a rose R_n with n petals. Then F_n can naturally be identified with the fundamental group of R_n. A *marked graph* is a finite graph Γ together with a homotopy equivalence $\rho : R_n \to \Gamma$. In our graphs, we do not allow separating edges or vertices of degrees 1 and 2. Let $\mathcal{G}$ be the category of such graphs where morphisms are collapses of subforests. For each marked graph $\Gamma \in \mathcal{G}$ put:

$$\tilde{\sigma}_\Gamma := \left\{ v : \text{Edges}(\Gamma) \to [0,1] \;\middle|\; \sum_e v(e) = 1, \bigcup_{v(e)=0} e \text{ is a forest} \right\}$$

I.e., $\tilde{\sigma}_\Gamma$ consists of the volume one metrics on Γ that are degenerate only on a subforest. The set

$$\sigma_\Gamma := \left\{ v : \text{Edges}(\Gamma) \to (0,1] \;\middle|\; \sum_e v(e) = 1 \right\} \subseteq \tilde{\sigma}_\Gamma$$

of nowhere degenerate volume one metrics on Γ is an open simplex inside $\tilde{\sigma}_\Gamma$, and the closed simplex

$$\overline{\sigma}_\Gamma := \left\{ v : \text{Edges}(\Gamma) \to [0,1] \;\middle|\; \sum_e v(e) = 1 \right\} \supseteq \tilde{\sigma}_\Gamma$$

bounds $\tilde{\sigma}_\Gamma$ from above. The closed simplex $\overline{\sigma}_\Gamma$ contains also faces where the metric v degenerates on a non-forest subgraph. We consider these faces to *lie at infinity.* Thus, we obtain $\tilde{\sigma}_\Gamma$ from the open simplex σ_Γ by adding its finite faces.

The assignment

$$\mathbf{M} : \Gamma \mapsto \tilde{\sigma}_\Gamma$$

is a contravariant functor from $\mathcal{G}$ to the category of spaces. If $\Delta = \Gamma/\Xi$, then $\tilde{\sigma}_\Delta$ can be naturally identified with the subspace of Γ of those metrics $v : \text{Edges}(\Gamma) \to [0,1]$ that vanish on Ξ. This way, we may consider *outer space* $\mathbf{X}$ as the colimit of the functor $\mathbf{M}$, i.e., $\mathbf{X}$ is the union of the $\tilde{\sigma}_\Gamma$ with the subspace $\tilde{\sigma}_\Delta$ being identified in all $\tilde{\sigma}_\Gamma$ where Γ is a blow up of Δ. The group $\text{Out}(F_n)$ acts on $\mathcal{G}$ by changing the marking. This induces an action on outer space.

The action of $\text{Out}(F_n)$ on outer space has finite stabilizers but the orbit space is not compact. To establish connectivity at infinity, one needs a cocompact action with finite stabilizers. Note that the colimit $\overline{\mathbf{X}}$ of the contravariant functor $\Gamma \mapsto \overline{\sigma}_\Gamma$ is a bordification of outer space with compact quotient mod $\text{Out}(F_n)$, however the faces at infinity introduce infinite stabilizers.

In [1], Bestvina and Feighn constructed a different bordification $\mathbf{Z}$ of outer space with a compact quotient and finite stabilizers. Their construction can be regarded as a Borel-Serre bordification. They give a functorial way to add faces at infinity to $\tilde{\sigma}_\Gamma$. One may think of their cells Σ_Γ as blow ups (in the sense of algebraic geometry) of the closed simplices $\overline{\sigma}_\Gamma$. In Σ_Γ a point at infinity also records the direction in which it may be approached from the finite realm (in fact, even higher order derivatives are recorded). They use $\mathbf{Z}$ to show that $\mathrm{Out}(F_n)$ is $(2n-5)$-connected at infinity and that it is a virtual duality group.

Following Grayson's alternative to the Borel-Serre bordification, we provide a construction of an equivariant deformation retract $\mathbf{J}$ inside outer space that has a compact quotient mod $\mathrm{Out}(F_n)$. In a functorial way, we define a polyhedral cell

$$J_\Gamma \subseteq \tilde{\sigma}_\Gamma \subseteq \overline{\sigma}_\Gamma$$

by "shaving off" infinite faces of $\overline{\sigma}_\Gamma$. The higher the codimension of the infinite face, the larger is the amount of shaving. This way, we ensure that each infinite face of $\overline{\sigma}_\Gamma$ contributes a facet of the *jewel* J_Γ. The bordification $\mathbf{J}$ of outer space is then obtained as the colimit over the jewels. Noteworthy properties of this construction are:

(1) If Γ is a rose, the jewel J_Γ is a permutahedron whose vertices are in one-to-one correspondence with the orderings of the petals in Γ.
(2) For any graph Γ, the jewel $J_\Gamma \subset \overline{\sigma}_\Gamma$ is the convex hull (taken inside $\overline{\sigma}_\Gamma$) of the jewels $J_\Delta \subseteq \overline{\sigma}_\Delta \subseteq \overline{\sigma}_\Gamma$ for all rose blow downs of Γ. In particular, the vertices of $\mathbf{J}$ are in one-to-one correspondence with ordered marked roses.

Using the last property, we establish an equivariant homotopy equivalence of $\mathbf{J}$ to *rose complex*, i.e., the simplicial complex whose vertices are the marked roses where roses span a simplex if they have a common blow up. Rose complex $\mathbf{R}$ is contractible, the action of $\mathrm{Out}(F_n)$ on $\mathbf{R}$ has finite stabilizers and compact quotient. Hence, rose complex can be used to establish connectivity at infinity. We prove:

Theorem 1. *Rose complex* $\mathbf{R}$ *is* $(2n-5)$*-connected at infinity.*

It follows that $\mathbf{J}$ is $(2n-5)$-connected at infinity, as well.

The proof of Theorem 1 uses combinatorial Morse theory, as do Bestvina and Feighn. However, it is remarkable that the Morse function used to analyze rose complex $\mathbf{R}$ is pretty much the classical height of roses used in the Culler-Vogtmann proof [2] for contractibility of outer space. See [4] for a modern exposition of this method. Also, the analysis of ascending links in rose complex is done purely by combinatorial topology of finite complexes.

Furthermore, we can show that the jewel J_Γ is homeomorphic to the Bestvina-Feighn cell Σ_Γ and the resulting bordifications $\mathbf{J}$ and $\mathbf{Z}$ are equivariantly homeomorphic. This corresponds to the fact that Grayson's construction yields a space homeomorphic to the Borel-Serre compactification.

Finally, we can also understand the *boundary* of $\mathbf{J}$, i.e., the union of all facets of jewels that arise from shaving off faces at infinity. No two of these facets are ever fully identified when jewels are glued together in **J**. We can understand the boundary $\partial(\mathbf{J})$ in terms of sphere systems as introduced in [3]. Let M be the double of a genus n handle body. Then, sphere systems in M give rise to graphs in the following way: each complementary component yields a vertex and each sphere yields an edge connecting the components on either side of the sphere. If no sphere separates M and all complementary components are simply connected (a *simple* sphere system), the resulting graph lies in $\mathcal{G}$ and the sphere system corresponds to a simplex σ_Γ in outer space. If some complementary component has non-trivial fundamental group (a *non-simple* sphere system), the resulting graph has smaller fundamental group and the sphere system corresponds to a face at infinity. We find:

Theorem 2. *The boundary $\partial(\mathbf{J})$ is homotopy equivalent to the complex of non-simple systems of non-separating spheres in M.*

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Kac–Moody symmetric spaces

Ralf Köhl

(joint work with Walter Freyn, Max Horn, Tobias Hartnick)

Kac–Moody groups over a local field $\mathbb{K}$ are infinite-dimensional generalizations of the groups of $\mathbb{K}$-points of (split) semisimple algebraic groups. From a geometric point of view, semisimple groups over local fields arise as subgroups of the isometry groups of Riemannian symmetric spaces (in the Archimedean case) and Euclidean buildings (in the non-Archimedean case). It is thus natural to ask whether Kac–Moody groups over local fields admit a similar geometric interpretation.

For Kac–Moody groups over non-Archimedean local fields such a geometric interpretation has been described by Rousseau who discusses the notion of an ordered affine hovel. Hovels are certain generalizations of Euclidean buildings that admit an action by a Kac–Moody group over a non-Archimedean local field $\mathbb{K}$, generalizing the notion of a Bruhat–Tits building endowed with the action of the $\mathbb{K}$-points of a split semisimple group.

In the present research we investigate the Archimedean situation, focussing on the split real case. We introduce a generalization of Riemannian symmetric spaces, which we call Kac–Moody symmetric spaces and on which split real Kac–Moody groups act in a way that generalizes the action of semisimple split real Lie groups on their Riemannian symmetric spaces. It turns out that in this setting one can observe both phenomena that one is familiar with from the finite-dimensional theory and phenomena that are specific to the infinite-dimensional situation; some of these infinite-dimensional phenomena in fact have non-Archimedean analogs in the theory of hovels.

A key structural problem that one has to face when generalizing the notion of a Riemannian symmetric space, is that the latter is originally defined in terms of a smooth Riemannian metric on a manifold; we are unaware of any reasonable notion of smoothness on the kind of homogeneous spaces on which a (non-spherical and non-affine) real Kac–Moody group naturally acts, nor are these spaces metrizable with respect to their natural topologies. Our starting point is thus an alternative characterization of affine symmetric spaces, due to Loos.

Theorem (Loos)**.** *Let $\mathcal{X}$ be an affine symmetric space, and given $x, y \in \mathcal{X}$ denote by $x \cdot y$ the point reflection of y at x. Then $\mu : \mathcal{X} \times \mathcal{X} \to \mathcal{X}$, $\mu(x, y) := x \cdot y$ is a C^1-map satisfying the following axioms:*

Axioms 1. *RS*
(RS1) *for any $x \in \mathcal{X}$ we have $x \cdot x = x$,*
(RS2) *for any pair of points $x, y \in \mathcal{X}$ we have $x \cdot (x \cdot y) = y$,*
(RS3) *for any triple of points $x, y, z \in \mathcal{X}$ we have $x \cdot (y \cdot z) = (x \cdot y) \cdot (x \cdot z)$,*
(RS4$_{\text{loc}}$) *every $x \in \mathcal{X}$ has a neighbourhood U such that $x \cdot y = y$ implies $y = x$ for all $y \in U$.*

Conversely, if $\mathcal{X}$ is a smooth manifold and $\mu : \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ is a C^1-map subject to (RS1)–(RS4$_{\text{loc}}$) above, then $\mathcal{X}$ is an affine symmetric space, and $\mu(x, y)$ is the point reflection of y at x. If $\mathcal{X}$ is a Riemannian symmetric space, then the isometries of $\mathcal{X}$ are exactly the C^1-maps $\alpha : \mathcal{X} \to \mathcal{X}$ satisfying $\alpha(x \cdot y) = \alpha(x) \cdot \alpha(y)$. If $\mathcal{X}$ is moreover of the non-compact type, then instead of the local condition (RS4$_{\text{loc}}$) it satisfies the global condition

Axioms 2. *RS*
(RS4) *$x \cdot y = y$ implies $y = x$ for all $y \in \mathcal{X}$.*

Since we are interested in generalizations of Riemannian symmetric spaces of non-compact type, we define the following:

Definition. A pair $(\mathcal{X}, \mu)$ is called a *topological symmetric space* provided $\mathcal{X}$ is a topological space and $\mu : \mathcal{X} \times \mathcal{X} \to \mathcal{X}$, $\mu(x, y) := x \cdot y$ is a continuous map subject to the axioms (RS1)–(RS4) above. The *automorphism group* $\mathrm{Aut}(\mathcal{X}, \mu)$ of $(\mathcal{X}, \mu)$ is defined as

$$\mathrm{Aut}(\mathcal{X}, \mu) := \{\alpha : \mathcal{X} \to \mathcal{X} \mid \alpha \text{ homeomorphism}, \alpha(x \cdot y) = \alpha(x) \cdot \alpha(y)\}.$$

Loos' theorem strongly uses the differentiability of μ, and not much is known about general topological symmetric spaces without any smoothness assumption. For example, it is not even known to us whether a topological symmetric space which is homeomorphic to a finite-dimensional manifold necessarily arises from an affine symmetric space.

We thus pursue three goals in our research:

(1) to develop a basic theory of topological symmetric spaces in the absence of any smoothness assumption;
(2) to associate a topological symmetric space to a large class of Kac–Moody groups over an Archimedean local field (focusing on the split real case for simplicity);
(3) to develop the structure theory of such Kac–Moody symmetric spaces, studying their geodesics, maximal flats, (local and global) automorphisms, causal structures and boundaries.

Our results concerning (1) might actually be of interest beyond Kac–Moody theory.

The three concepts of flats, geodesics and one-parameter subgroups of the isometry group are of fundamental nature in the study of Riemannian symmetric spaces. The former two are usually defined using the curvature tensor, and the existence of the latter is derived from an existence theorem for ordinary differential equations. In our topological setting we need to define flats and geodesics without reference to the curvature tensor, and to establish the existence of one-parameter subgroups without analytic tools.

Based on preliminary work by the third-named author we observe that in a non-spherical Kac–Moody symmetric space there exist pairs of points that do not lie on a common geodesic; however, any two points can be connected by a chain of geodesic segments. We moreover classify maximal flats in Kac–Moody symmetric spaces and study their intersection patterns, leading to a classification of global and local automorphisms. Some of our methods apply to general topological reflection spaces beyond the Kac–Moody setting.

Unlike Riemannian symmetric spaces, non-spherical non-affine irreducible Kac–Moody symmetric spaces also admit an invariant causal structure. For causal and anti-causal geodesic rays with respect to this structure we find a notion of asymptoticity, which allows us to define a future and past boundary of such Kac–Moody symmetric space. We show that these boundaries carry a natural simplicial structure and are simplicially isomorphic to the halves of the geometric realization of the twin buildings of the underlying split real Kac–Moody group. We also show that every automorphism of the symmetric space is uniquely determined by the induced simplicial automorphism of the future and past boundary.

The invariant causal structure on a non-spherical non-affine irreducible Kac–Moody symmetric space gives rise to an invariant pre-order on the underlying space, and thus to a subsemigroup of the Kac–Moody group. For many Kac–Moody symmetric spaces including the E_n-series, $n \geq 10$, we establish that this pre-order is actually a partial order. The case of general Kac–Moody symmetric spaces remains open.

We conclude that while in some aspects Kac–Moody symmetric spaces closely resemble Riemannian symmetric spaces, in other aspects they behave similarly to ordered affine hovels, their non-Archimedean cousins.

Action dimensions of some simple complexes of groups

Michael Davis
(joint work with Giang Le, Kevin Schreve)

The *geometric dimension* of a discrete torsion-free group G is the smallest dimension of a model for BG by a CW complex. This number equals $\operatorname{cd} G$, the cohomological dimension of G, provided $\operatorname{cd} G \neq 2$. The *action dimension* of G, denoted $\operatorname{actdim} G$ is the smallest dimension of a model for BG by a manifold with boundary (possibly empty boundary). In other words, $\operatorname{actdim} G$ is the minimum dimension of a thickening of a model for BG to a manifold. It follows that $\operatorname{gdim} G \leq \operatorname{actdim} G$ with equality if and only if BG is homotopy equivalent to a closed manifold. On general principles, $\operatorname{actdim} G \leq 2 \operatorname{gdim} G$.

In [2], Bestvina, Kapovich, and Kleiner defined a number which is a lower bound for the action dimension of G called its "obstructor dimension," denoted $\operatorname{obdim} G$. It is based on the classical van Kampen obstruction for embedding a simplicial complex K into a euclidean space of given dimension. Let $\mathcal{C}(K)$ denote the configuration space of unordered pairs of distinct points in K, i.e., if $\Delta \subset K \times K$ is the diagonal, then $\mathcal{C}(K)$ is the quotient of $(K \times K) - \Delta$ by the free involution which interchanges the factors. Let $c : \mathcal{C}(K) \to \mathbf{R}P^\infty$ classify the double cover and let w_1 be the generator of $H^1(\mathbf{R}P^\infty; \mathbb{Z}/2)$. The *van Kampen obstruction* of K in degree m is the cohomology class $\operatorname{vk}^m(K) := c^*(w_1)^m \in H^m(\mathcal{C}(K); \mathbb{Z}/2)$. It is an obstruction to embedding K in $\mathbf{R}^m$. Let $\operatorname{Cone} K$ stand for the cone of infinite radius with base K, i.e., $K \times [0, \infty)/K \times 0$. The simplicial complex K is an *m-obstructor* for G if $\operatorname{Cone} K$ coarsely embeds in EG (or, more precisely, in some contractible geodesic space on which G acts properly) and if $\operatorname{vk}^m(K) \neq 0$. Define $\operatorname{obdim} G = 2 + \max\{m \mid \text{there is an } m\text{-obstructor } K \text{ for } G\}$. The main result of [2] is that $\operatorname{obdim} G \leq \operatorname{actdim} G$.

A *simple complex of groups* over a poset $\mathcal{Q}$ is a functor $G\mathcal{Q}$ from $\mathcal{Q}$ to the category of groups and monomorphisms. The group G_σ associated to the element $\sigma \in \mathcal{Q}$ is called the *local group* at σ. One can glue together the classifying spaces BG_σ, with $\sigma \in \mathcal{Q}$, to form a space $BG\mathcal{Q}$ called the *aspherical realization* of $G\mathcal{Q}$. Assume the geometric realization of $\mathcal{Q}$ is simply connected. Then van Kampen's Theorem implies that $\pi_1(BG\mathcal{Q})$ is the direct limit G of $G\mathcal{Q}$. The $K(G,1)$-Question for $G\mathcal{Q}$ asks if $BG\mathcal{Q}$ is aspherical, i.e., if it is homotopy equivalent to BG. If, whenever $\tau < \sigma$, we have $\dim BG_\tau < \dim BG_\sigma$, then $\dim BG\mathcal{Q} = \max\{\dim BG_\sigma \mid \sigma \in \mathcal{Q}\}$. If the answer to the $K(G,1)$-Question is affirmative, then, supposing each BG_σ has minimum dimension $\operatorname{gdim} G_\sigma$, we see $\operatorname{gdim} G = \max\{\operatorname{gdim} G_\sigma \mid \sigma \in \mathcal{Q}\}$. Given models for the BG_σ by manifolds with boundary M_σ and "dual disks" D_σ, one can glue together the $M_\sigma \times D_\sigma$ to get a manifold model for $BG\mathcal{Q}$. Hence, when the $K(G,1)$-Question has a positive answer, we get a manifold model for BG and hence, an upper bound for $\operatorname{actdim} G$. Note that these methods only work when the $K(G,1)$-Question has a positive answer.

In most applications $\mathcal{Q}$ will be a poset of the form $\mathcal{S}(L)$, where L is a simplicial complex and $\mathcal{S}(L)$ means the poset of simplices in L (including the empty simplex).

Our first two examples of such simple complexes of groups are "Artin complexes" and "graph product complexes".

One associates to a Coxeter system (W, S) a simplicial complex L called its "nerve". The vertex set of L is S and its simplices are the subsets of S which generate finite subgroups (such subgroups are said to be "spherical" Coxeter groups). There is an Artin group A associated to (W, S) and for each $\sigma \in \mathcal{S}(L)$ there is an associated "spherical" Artin group A_σ. The functor $\sigma \mapsto A_\sigma$ defines a simple complex of groups $A\mathcal{S}(L)$ called the *Artin complex.* The direct limit of $A\mathcal{S}(L)$ is the *Artin group* A associated to (W, S). The $K(\pi, 1)$-Question for $A\mathcal{S}(L)$ was raised in [4]. Although the answer is not known in general, it was proved in [4] that the answer is positive whenever L is a flag complex. In particular, since L is a flag complex whenever A is right-angled (i.e., when A is a "RAAG"), the $K(\pi, 1)$-Question has a positive answer for RAAGs.

Given a simplicial complex L of dimension d, there is another simplicial complex OL of the same dimension called the *octahedralization* of L. It is formed formed by replacing each simplex of L by a join of 0-spheres. In the case of a RAAG A associated to a flag complex L, OL is the link of a vertex in the standard model for BA as a union of tori.

Lemma. ([1, Theorems 5.1 and 5.2]). *Suppose L is a flag complex of dimension d. Then $\mathrm{vk}^{2d}(OL) \neq 0$ if and only if $H_d(L; \mathbb{Z}/2) \neq 0$. (Hence, OL is a $2d$-obstructor for the associated* RAAG*.)*

In the case of a general Artin group A, the relevant obstructor is defined by the simplicial complex $L_\oslash$ of standard abelian subgroups of A (cf. [6]). It is a certain subdivision of L. Moreover, the cone on its octahedralization coarsely embeds in EA. By the Lemma, if $H_d(L; \mathbb{Z}/2) \neq 0$, then $\mathrm{vk}^{2d}(OL_\oslash) \neq 0$ and hence, $\operatorname{obdim} A = 2d + 2$. Using this and the gluing method in Le's thesis [9] we get the following.

Theorem 1. *Suppose the $K(\pi, 1)$-Question has a positive answer for the Artin complex $A\mathcal{S}(L)$. Let d denote the dimension of L.*

1. *(Proved in* [6]*). If $H_d(L; \mathbb{Z}/2) \neq 0$, then* $\operatorname{actdim} A = \operatorname{obdim} A = 2d + 2 = 2 \operatorname{gdim} A$.
2. *(Proved in* [9]*). If L embeds is some d-dimensional contractible complex (this implies $H_d(L; \mathbb{Z}/2) \neq 0$), then* $\operatorname{actdim} A \leq 2d + 1$.

Suppose $\{G_v\}_{v \in \mathrm{Vert}\, L^1}$ is a collection of groups indexed by the vertex set of a simplicial graph L^1. Let L be the flag complex (i.e., the clique complex) of L^1. For each $\sigma \in \mathcal{S}(L)$, G_σ denotes the product of the G_v, with $v \in \mathrm{Vert}\, \sigma$. There is simple complex of groups $G\mathcal{S}(L)$ over $\mathcal{S}(L)$, called the *graph product complex*, defined by $\sigma \mapsto G_\sigma$. The group $G := \lim G\mathcal{S}(L)$ is the *graph product* of the G_v. Note that G_v and G_w commute in G if and only if $\{v, w\}$ spans an edge of L^1. When each G_v is equal to $\mathbb{Z}/2$, then G is the *right-angled Coxeter group* associated to L^1. When each G_v is infinite cyclic, G is the RAAG associated to L^1.

Consider the case, where each BG_v is a closed aspherical m-manifold M_v. Provided that we also assume that the ideal boundary of the universal cover of each

M_v is S^{m-1}, then the appropriate candidate for an obstructor is the complex $\tilde{O}L$ formed by taking the polyhedral join of $(m-1)$-spheres over L. Its dimension is $\tilde{O}L = m(d+1)-2$. We show that $\tilde{O}L$ is an $((m+1)(d+1)-2)$-obstructor for G if and only if $H_d(L;\mathbb{Z}/2) \neq 0$. This gives the following.

Theorem 2. *Suppose G is a graph product of closed m-manifold groups as above. Then* $\operatorname{gdim} G = m(d+1)$. *Moreover,*

(1) If $H_d(L;\mathbb{Z}/2) \neq 0$, then $\operatorname{actdim} A = \operatorname{obdim} A = (m+1)(d+1)$.
(2) If L embeds is some d-dimensional contractible complex, then $\operatorname{actdim} G \leq (m+1)(d+1)-1$.

Details for these theorems and related results will appear in [7].

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Convexity of parabolic subgroups of Artin groups

Ruth Charney
(joint work with Luis Paris)

Let Γ be a finite simplicial graph with vertex set $S = \{s_1, s_2, \dots s_n\}$ and with each edge $e(s_i, s_j)$ labelled by an integer $m_{i,j} \geq 2$. The associated *Artin group* is the group with presentation

$$A_\Gamma = \langle s_1, \dots, s_n \mid \underbrace{s_i s_j s_i \dots}_{m_{ij}} = \underbrace{s_j s_i s_j \dots}_{m_{ij}} \rangle$$

with a relation for each edge $e(s_i, s_j)$ in Γ. Adding relations of the form $s_i^2 = 1, \forall i$ gives rise to a *Coxeter group*

$$W_\Gamma = \langle s_1, \dots, s_n \mid (s_i)^2 = 1,\ (s_i s_j)^{m_{ij}} = 1 \rangle.$$

Interest in Artin groups originally stems from their geometric interpretation. Coxeter groups can be realized as reflection groups acting on $\mathbb{C}^n$. Removing the hyperplanes in $\mathbb{C}^n$ fixed by some reflection, one obtains a space $\mathcal{H}_\Gamma$ whose fundamental group is the pure Artin group, that is, the kernel of the projection map $A_\Gamma \to W_\Gamma$. Modding out by the action of W_Γ, one obtains a space with fundamental group A_Γ.

Artin groups have been extensively studied and some classes of Artin groups (such as right-angled and spherical type Artin groups) are well understood. On the other hand, very little is known about Artin groups in general. For example, some long-standing open conjectures include,

- A_Γ has solvable word problem,
- A_Γ is torsion-free,
- A_Γ has trivial center (providing W_Γ is infinite),
- A_Γ has a finite K(A_Γ,1)-space,
- $W_\Gamma \backslash \mathcal{H}_\Gamma$ is a K(A_Γ,1)-space.

For a discussion of partial results on these conjectures see [2].

Given a subset T of the generating set S, the subgroup A_T generated by T is called a parabolic subgroup of A_Γ. It was shown by van der Lek [1] that this subgroup is naturally isomorphic to the Artin group A_Δ, where Δ is the subgraph of Γ spanned by T. In this talk, we discuss the geometry of the inclusion map $A_T \hookrightarrow A_\Gamma$. In particular, we prove that the following useful fact holds for arbitrary Artin groups [3].

Theorem. *For any Artin group A_Γ and any $T \subset S$, the inclusion of Cayley graphs, $Cay(A_T, T) \hookrightarrow Cay(A_\Gamma, S)$ is convex. That is, every geodesic in $Cay(A_\Gamma, S)$ joining two points in $Cay(A_T, T)$ lies entirely in $Cay(A_T, T)$.*

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Canonical attracting trees and polynomial invariants for free-by-cyclic groups

Spencer Dowdall

(joint work with Ilya Kapovich, Christopher J. Leininger)

A free-by-cyclic group $G = F_n \rtimes_\varphi \mathbb{Z}$, where φ is an automorphism of the rank–n free group F_n, can often be expressed as a free-by-cyclic group in infinitely many ways. Accordingly, one may try to study and relate these various splittings. Here we focus on the case that the automorphism φ is both atoroidal and fully irreducible;

this means that no power φ^k, $k \geq 1$, preserves the conjugacy class of any proper free factor or nontrivial element of F_n and implies (by work of Bestvina–Feighn [1] and Brinkmann [3]) that G is hyperbolic.

The given splitting $G = F_n \rtimes_\varphi \mathbb{Z}$ of the group determines (via quotienting out the normal subgroup $F_n \lhd G$) an epimorphism $G \to \mathbb{Z} \cong G/F_n$ onto the infinite cyclic group. Conversely, every epimorphism $u\colon G \to \mathbb{Z}$ induces a split (since $\mathbb{Z}$ is free) short exact sequence

$$1 \longrightarrow \ker(u) \longrightarrow G \xrightarrow{u} \mathbb{Z} \longrightarrow 1$$

and a corresponding splitting $G = \ker(u) \rtimes_{\varphi_u} \mathbb{Z}$ of the group in which the monodromy automorphism $\varphi_u\colon \ker(u) \to \ker(u)$ is given by conjugating the normal subgroup $\ker(u) \lhd G$ by an element $g \in G$ for which $u(g) = 1 \in \mathbb{Z}$. In this setting, it is known (e.g. [8]) that $\ker(u)$ is a free group whenever it is finitely generated. Thus the [finitely-generated free]-by-cyclic splittings of G are in bijective correspondence with the epimorphisms $G \to \mathbb{Z}$ with finitely generated kernel. Further, there is an open, $\mathbb{R}_+$–invariant set $\Sigma(G) \subset \mathrm{Hom}(G,\mathbb{R}) = H^1(G;\mathbb{R})$, the *Bieri–Neumann–Strebel invariant* [2], such that a nontrivial homomorphism $u \in \mathrm{Hom}(G,\mathbb{Z})$ has $\ker(u)$ finitely-generated if and only if $u \in \Sigma(G) \cap (-\Sigma(G))$. Thus, since $\Sigma(G)$ is open, we see that G has infinitely many free-by-cyclic splittings whenever $\mathrm{rank}(H^1(G;\mathbb{R})) \geq 2$.

Previous work with Kapovich and Leininger [6, 4, 5, 7] has begun to explore this family of free-by-cyclic splittings of G. Our results are most succinctly captured by a new polynomial $\mathfrak{m} \in \mathbb{Z}[H_1(G;\mathbb{Z})/\text{torsion}]$ that we constructed in the spirit of McMullen's Teichmüller polynomial for fibered hyperbolic 3–manifolds [9]. This polynomial has the form $\mathfrak{m} = a_1h_1 + \ldots + a_kh_k$ for some $a_i \in \mathbb{Z}$ and $h_i \in H_1(G;\mathbb{Z})/\text{torsion}$ and is computed in terms of a train-track graph map $f\colon \Gamma \to \Gamma$ representing $\varphi\colon F_n \to F_n$. Further, $\mathfrak{m}$ explicitly determines 3 pieces of information:

(1) an open, convex, finite-sided polyhedral cone $\mathcal{C}_\mathfrak{m} \subset H^1(G;\mathbb{R})$,
(2) a specialized Laurent polynomial $\mathfrak{m}_u(t) = a_1t^{u(h_1)} + \ldots + a_kt^{u(h_k)}$ in $\mathbb{Z}[t,t^{-1}]$ for each $u \in \mathcal{C}_\mathfrak{m} \cap H^1(G;\mathbb{Z})$, and
(3) a convex, real-analytic, homogeneous of degree -1 function $\mathfrak{h}_\mathfrak{m}\colon \mathcal{C}_\mathfrak{m} \to \mathbb{R}$ that tends to ∞ at $\partial\mathcal{C}_\mathfrak{m}$ and is defined on integral classes $u \in \mathcal{C}_\mathfrak{m} \cap H^1(G;\mathbb{Z})$ by $\mathfrak{h}_\mathfrak{m}(u) = \log(\text{largest root of } \mathfrak{m}_u)$.

At base, the polynomial $\mathfrak{m}$ records algebraic relations in a certain dynamical system built from the train-track map f, but in so doing it also encapsulates information about the free-by-cyclic splittings of G. Specifically, we show that:

(a) The cone $\mathcal{C}_\mathfrak{m}$ is a connected component of the BNS-invariant $\Sigma(G)$.
(b) For every epimorphism $u\colon G \to \mathbb{Z}$ in $\mathcal{C}_\mathfrak{m}$ with $\ker(u)$ finitely generated (i.e., every free-by-cyclic splitting $G = \ker(u) \rtimes_{\varphi_u} \mathbb{Z}$ in $\mathcal{C}_\mathfrak{m}$), the algebraic stretch factor of the monodromy $\varphi_u\colon \ker(u) \to \ker(u)$ is equal to $e^{\mathfrak{h}_\mathfrak{m}(u)}$.

Thus our polynomial both identifies an interesting family of splittings of G, by calculating a component of $\Sigma(G)$, and gives detailed dynamical information about all of the splittings in this family.

One downside of $\mathfrak{m}$ is that it relies on *choosing* a train-track graph map representing φ and is therefore not canonical. In this work, we introduce a variation of the polynomial that enjoys all the properties listed above but has the advantage of being a canonical invariant. This canonicity is obtained by using attracting trees rather than train-track representatives: To each fully irreducible automorphism ϕ of a finite-rank free group F_k, one may associate a well-defined $\mathbb{R}$–tree equipped with an isometric action of F_k on which ϕ lifts to an F_k–equivariant homothety. This tree T_ϕ^+ is a canonical invariant of the automorphism and called the *attracting tree* of ϕ.

A main result from our prior work [7] is that for every splitting $G = \ker(u) \rtimes_{\varphi_u} \mathbb{Z}$ covered by (b) above, the monodromy φ_u is a fully irreducible automorphism of the free group $\ker(u)$. Therefore each such u has a well-defined attracting tree $T_u = T_{\varphi_u}^+$. Here we show that these $\mathbb{R}$–trees all have the same underlying topological tree. That is, there is a single topological tree T such that for every integral $u \in \mathcal{C}_\mathfrak{m}$ with $\ker(u)$ finitely generated, T is homeomorphic to the $\mathbb{R}$–tree T_u equipped with its observers topology. This topological tree T is therefore a canonical invariant for all the splittings in the cone $\mathcal{C}_\mathfrak{m}$.

Moreover, since φ_u acts equivariantly on T_u as a $e^{\mathfrak{h}_\mathfrak{m}(u)}$–homothety, it is possible to extend the given action of $\ker(u)$ on $T = T_u$ to an action of the entire group $G = \ker(u) \rtimes_{\varphi_u} \mathbb{Z}$ on T. One may then use this action $G \curvearrowright T$ to define a new polynomial by considering the free $\mathbb{Z}[G]$–module on the set of embedded arcs in T with endpoints at branchpoints and imposing a natural subdivision relation. By passing to an appropriate quotient, we obtain a finitely presented $\mathbb{Z}[H_1(G;\mathbb{Z})/\text{torsion}]$–module whose Fitting ideal determines a canonical polynomial $\mathfrak{T} \in \mathbb{Z}[H_1(G;\mathbb{Z})/\text{torsion}]$ that behaves similarly to $\mathfrak{m}$. In particular, this new polynomial $\mathfrak{T}$ calculates both the component of the BNS-invariant $\Sigma(G)$ containing $G \to G/F_n$ as well as the algebraic stretch factors of all splittings in this component.

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How to quickly generate hyperbolic elements

Koji Fujiwara
(joint work with Emmanuel Breuillard)

In the paper [2], we recast the notion of joint spectral radius in the setting of groups acting by isometries on non-positively curved spaces and give geometric versions of results of Berger-Wang and Bochi valid for δ-hyperbolic spaces, symmetric spaces and Bruhat-Tits buildings. This method produces nice hyperbolic elements in many classical geometric settings.

I discussed one application of our work to uniform growth, which is a quick proof of a generalization of the following theorem by Besson-Courtois-Gallot, [1].

Theorem 1. *Let X be a d-dimensional, simply connected Riemannian manifold with curvature $-a^2 \leq K \leq -1$. Let $\Gamma = \langle S \rangle$ be a finitely generated discrete subgroup of $Isom(X)$ with $S = S^{-1}$. Then either Γ is virtually nilpotent, or S^N contains two generators of free semigroup, in particular, $h(S) \geq \frac{1}{N} \log 2$, where the constant N depends only on d and a.*

Our theorem is the following.

Theorem 2. *Given P, there is $N(P)$ with the following property. Let X be a geodesic δ-hyperbolic space, with $\delta > 0$, such that every ball of radius 2δ is covered by at most P balls of radius δ.*

Let S be a finite subset in $Isom(X)$ with $S = S^{-1}$ and assume that $\Gamma = \langle S \rangle$ is a discrete subgroup of $Isom(X)$. Then either Γ is virtually nilpotent, or S^N contains two generators of a free semi-group, and in particular:

$$h(S) \geq \frac{1}{N} \log 2.$$

Moreover, if Γ is virtually nilpotent, then either (i) Γ is finite, (ii) fixes a point in ∂X, or (iii) Γ is virtually cyclic and contains a hyperbolic isometry g such that $Fix(g)$ in ∂X is invariant by Γ.

One key ingredient of the proof is a generalization of the Margulis Lemma to the setting of singular spaces with curvature bounded from below obtained in [3].

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Hyperbolic structures on groups

Denis Osin
(joint work with Carolyn R. Abbott, Sahana H. Balasubramanya)

For every group G, we introduce the set of *hyperbolic structures* on G, denoted $\mathcal{H}(G)$, which consists of equivalence classes of (possibly infinite) generating sets of G such that the corresponding Cayley graph is hyperbolic; two generating sets of G are *equivalent* if the corresponding word metrics on G are bi-Lipschitz equivalent. Alternatively, one can define hyperbolic structures in terms of cobounded G-actions on hyperbolic spaces. We are especially interested in the subset $\mathcal{AH}(G) \subseteq \mathcal{H}(G)$ of *acylindrically hyperbolic structures* on G, i.e., hyperbolic structures corresponding to acylindrical actions.

Elements of $\mathcal{H}(G)$ can be ordered in a natural way according to the amount of information they provide about the group G. The main goal of this paper is to initiate the study of the posets $\mathcal{H}(G)$ and $\mathcal{AH}(G)$ for various groups G.

We begin by discussing the cardinality of $\mathcal{H}(G)$ and $\mathcal{AH}(G)$. It turns out that $\mathcal{H}(G)$ can have any non-zero cardinality, while $\mathcal{AH}(G)$ always has cardinality 1, 2, or at least continuum. We then study basic properties of these posets such as height, width, existence of extremal elements, etc. We also obtain several results about hyperbolic structures induced from hyperbolically embedded subgroups of G. Finally, we study the question to what extent a hyperbolic structure is determined by the set of loxodromic elements and their translation lengths.

Tame automorphism group

Piotr Przytycki
(joint work with Stéphane Lamy)

The *tame automorphism group* of the affine space $\mathbf{k}^3$, over a base field $\mathbf{k}$ of characteristic zero, is the subgroup of the polynomial automorphism group $\mathrm{Aut}(\mathbf{k}^3)$ generated by the affine and elementary automorphisms:

$$\mathrm{Tame}(\mathbf{k}^3) = \langle A_3, E_3 \rangle,$$

where

$$A_3 = \mathrm{GL}_3(\mathbf{k}) \ltimes \mathbf{k}^3, \text{ and}$$
$$E_3 = \{(x_1, x_2, x_3) \mapsto (x_1 + P(x_2, x_3), x_2, x_3) \mid P \in \mathbf{k}[x_2, x_3]\}.$$

There is a natural homomorphism Jac: $\mathrm{Tame}(\mathbf{k}^3) \to \mathbf{k}^*$ given by the Jacobian determinant. The kernel $\mathrm{STame}(\mathbf{k}^3)$ of this homomorphism is the *special tame automorphism group.* It is a natural question whether $\mathrm{STame}(\mathbf{k}^3)$ is a simple group. We prove that $\mathrm{STame}(\mathbf{k}^3)$ is not simple (and indeed very far from being simple). Our strategy is to use an action of $\mathrm{Tame}(\mathbf{k}^3)$ on a Gromov-hyperbolic triangle complex.

For any $1 \le r \le 3$, an *r-tuple of components* is a polynomial map

$$f\colon \mathbf{k}^3 \to \mathbf{k}^r$$
$$x = (x_1, \dots, x_3) \mapsto (f_1(x), \dots, f_r(x))$$

that can be extended to a tame automorphism $f = (f_1, \dots, f_3)$ of $\mathbf{k}^3$. We consider the orbits of the action of the affine automorphism group $A_r = \mathrm{GL}_r(\mathbf{k}) \ltimes \mathbf{k}^r$ on the r-tuples of components:

$$[f_1, \dots, f_r] = A_r(f_1, \dots, f_r) = \{a \circ (f_1, \dots, f_r) \mid a \in A_r\}$$

A *vertex of type r* of $\mathcal{C}$ is such an orbit $[f_1, \dots, f_r]$. The vertices $[f_1], [f_1, f_2]$ and $[f_1, f_2, f_3]$ span triangles of $\mathcal{C}$. The tame automorphism group acts on $\mathcal{C}$ by isometries, via pre-composition:

$$g \cdot [f_1, \dots, f_r] = [f_1 \circ g^{-1}, \dots, f_r \circ g^{-1}]$$

It is easy to see that $\mathcal{C}_n$ is connected.

Theorem 1. *$\mathcal{C}$ is contractible and Gromov-hyperbolic.*

We also exhibit a loxodromic weakly proper discontinuous element of $\mathrm{STame}(\mathbf{k}^3)$, in the sense of M. Bestvina and K. Fujiwara [1]. Recall that an isometry f of a metric space X is *loxodromic* if for some (hence any) $x \in X$ there exists $\lambda > 0$ such that for any $k \in \mathbb{Z}$ we have $|x, f^k \cdot x| \ge \lambda |k|$. Suppose that f belongs to a group G acting on X by isometries. We say that f is *weakly proper discontinuous* (WPD) if for some (hence any) $x \in X$ and any $C \ge 1$, for k sufficiently large there are only finitely many $j \in G$ satisfying $|x, j \cdot x| \le C$ and $|f^k \cdot x, j \circ f^k \cdot x| \le C$.

Let $n \ge 0$, and let $g, h, f \in \mathrm{Tame}(\mathbf{k}^3)$ be the automorphisms defined by

$$g^{-1}(x_1, x_2, x_3) = (x_2, x_1 + x_2 x_3, x_3),$$
$$h^{-1}(x_1, x_2, x_3) = (x_3, x_1, x_2),$$
$$f = g^n \circ h.$$

Theorem 2. *Let $n \ge 3$. Then f acts as a loxodromic element on $\mathcal{C}^1$. In particular, the complex $\mathcal{C}$ has infinite diameter. Moreover, if $n \ge 12$ is even and $G = \mathrm{Tame}(\mathbf{k}^3)$, then f acts as a* WPD *element on $\mathcal{C}^1$.*

By the work of F. Dahmani, V. Guirardel, and D. Osin [3, Thm 8.7], the existence of an action of a non-virtually cyclic group G on a Gromov-hyperbolic metric space, with at least one loxodromic WPD element, implies that G has a free normal subgroup, and in particular G is not simple. By the work of D. Osin [4, Thm

1.2], such a group is *acylindrically hyperbolic*: there exists a (different) Gromov-hyperbolic space on which the action of G is acylindrical, a notion introduced for general metric spaces by B. Bowditch [2].

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Compactifications of spaces of maximal representations and non-archimedean geometry

Marc Burger

(joint work with Alessandra Iozzi, Anne Parreau, Maria B. Pozzetti)

Maximal representations form certain components of the variety of $Sp(2n, \mathbb{R})$-representations of a compact surface group. We use length functions to compactify those components and present recent results concerning the structure of these boundaries and the properties of length functions forming them. The general picture is that this boundary decomposes into a closed subset consisting of length functions vanishing on subsurfaces or associated to $\mathbb{R}$-tree actions with small stabilisers and an open complement on which the mapping class group acts properly discontinuously. The approach is based on the study of an analogue of maximal representations over real closed, non-archimedean fields.

Boundary amenability of $\mathbf{Out}(F_N)$

Camille Horbez

(joint work with Mladen Bestvina, Vincent Guirardel)

A countable group Γ is *boundary amenable* (or *exact*) if it admits an action on a compact Hausdorff space X which is *topologically amenable*, i.e. such that there exists a sequence of continuous maps $\nu_n : X \to \text{Prob}(\Gamma)$ satisfying

$$\sup_{x\in X} ||\nu_n(\gamma.x) - \gamma.\nu_n(x)||_1 \to 0$$

as n goes to $+\infty$. Here the space $\text{Prob}(\Gamma)$ of all probability measures on Γ is equipped with the topology of pointwise convergence. This notion is a weakening of amenability, a countable group Γ being amenable if and only if the trivial Γ-action on a point is topologically amenable. Exactness of a countable group is also equivalent to the exactness of its reduced C^*-algebra [2, 15].

A key motivation behind the study of exactness of groups comes from the fact that every exact countable group satisfies the Novikov conjecture on higher signatures: this follows from results of Yu [19], Higson–Roe [10] and Higson [9].

All known examples of non-exact groups rely on Gromov's construction of groups containing copies of expanders [5]. On the other hand, exactness has already been established for hyperbolic groups (Adams [1]), relatively hyperbolic groups with exact parabolics (Ozawa [16]), linear groups (Guentner–Higson–Weinberger [6]), groups acting properly and cocompactly on a finite-dimensional CAT(0) cube complex (Campbell–Niblo [3]), groups acting properly on locally finite buildings (Lécureux [12]), mapping class groups of hyperbolic surfaces of finite type (Kida [11], Hamenstädt [8]). Our main result is the following.

Theorem 1. *Let G be either a free group, or more generally*

- *a torsion-free hyperbolic group (or hyperbolic relative to a finite collection of free abelian subgroups), or*
- *a right-angled Artin group.*

Then $Out(G)$ is exact (and therefore $Out(G)$ satisfies the Novikov conjecture).

In order to prove Theorem 1, we work in the more general setting of free products. Let $G_1, \dots, G_k$ be a finite collection of countable groups, let F_N be a free group of rank N, and let $G := G_1 * \cdots * G_k * F_N$. We let $\mathrm{Out}(G, \{G_i\})$ be the subgroup of $\mathrm{Out}(G)$ made of all outer automorphisms that preserve the conjugacy classes of each of the subgroups G_i, and $\mathrm{Out}(G, \{G_i\}^{(t)})$ be the subgroup made of all automorphisms acting by conjugation by an element $g_i \in G$ on each G_i. These groups fit in the following short exact sequence:

$$1 \to \mathrm{Out}(G, \{G_i\}^{(t)}) \to \mathrm{Out}(G, \{G_i\}) \to \prod_{i=1}^{k} \mathrm{Out}(G_i) \to 1.$$

Theorem 2. *Let $G_1, \dots, G_k$ be a finite collection of countable groups, let F_N be a free group of rank N, and let $G := G_1 * \cdots * G_k * F_N$.*
If G_i is exact for all $i \in \{1, \dots, k\}$, then $Out(G, \{G_i\}^{(t)})$ is exact.
If in addition $Out(G_i)$ is exact for all $i \in \{1, \dots, k\}$, then $Out(G, \{G_i\})$ is exact.

Theorem 1 follows from Theorem 2 in the following way. The case of $\mathrm{Out}(F_N)$ is the case where $k = 0$. In the other cases, Theorem 2 reduces the proof of Theorem 1 to the case where G is one-ended. By [18, 13], the outer automorphism group of a one-ended torsion-free hyperbolic group is built out of mapping class groups and free abelian subgroups (this is proved using JSJ theory), from which exactness follows. The case where G is hyperbolic relative to free abelian subgroups is understood in the same way using [7]. In the case where G is a right-angled Artin group, we argue by induction on the number of vertices in the underlying graph, using previous work of Charney and Vogtmann [4].

We now present the strategy of our proof of Theorem 2 in the case where $G = F_N$. We mention that this case is not really easier than the general case: in the course of the proof, we are naturally led studying subgroups of $\mathrm{Out}(F_N)$ that preserve factors arising in some decomposition of F_N as a free product.

The $\mathrm{Out}(F_N)$-action on the compactification $\overline{CV}_N$ of Culler–Vogtmann's outer space is not topologically amenable since there are trees in the boundary whose stabilizer is non-amenable (and this is a general obstruction to amenability of the action). Examples of such trees arise as trees dual to some non-filling measured laminations on a punctured surface: the stabilizer of the tree then contains the mapping class group of the complementary subsurface to the support of the lamination.

Instead, following a strategy used by Kida for mapping class groups [11], we partition $\overline{CV}_N$ into two (non-compact) subspaces: the arational trees $\mathcal{AT}$ on the one side, and the non-arational trees on the other. A tree $T \in \partial CV_N$ is called *arational* if for every proper free factor $A \subsetneq F_N$, the A-action on its minimal subtree $T_A \subseteq T$ is free and simplicial. The key step in our proof is the following proposition.

Proposition 1. *The Out*(F_N)*-action on* $\mathcal{AT}$ *is topologically amenable.*

On the other hand, by a theorem of Reynolds [17], one can associate to any non-arational tree a canonical finite set of conjugacy classes of proper free factors of F_N. The following proposition of Ozawa, applied with $X = \overline{CV}_N$ and $\mathcal{D}$ the collection of all conjugacy classes of proper free factors of F_N, reduces the proof of the exactness of $\mathrm{Out}(F_N)$ to the proof of exactness of $\mathrm{Out}(F_N, A)$ (where $A \subsetneq F_N$ is a proper free factor). This allows for an inductive argument on some notion of complexity (for which $\mathrm{Out}(F_N, A)$ is simpler than $\mathrm{Out}(F_N)$).

Proposition 2 (Ozawa [16])**.** *Let* Γ *be a countable group, let* X *be a compact* Γ*-space, and let* $\mathcal{D}$ *be a countable* Γ*-set. Assume that*

(1) there exist Borel maps $\nu_n : X \to Prob(\mathcal{D})$ *such that for all* $\gamma \in \Gamma$ *and all* $x \in X$*, one has* $||\nu_n(\gamma.x) - \gamma.\nu_n(x)||_1 \to 0$ *as* n *goes to* $+\infty$*,*

(2) for all $d \in \mathcal{D}$*, the stabilizer* $Stab_\Gamma(d)$ *is exact.*

Then Γ *is exact.*

We finish with a few words about our proof of Proposition 1. The classical proof that the F_N-action on $\partial_\infty F_N$ is topologically amenable relies on the following geometric feature: two rays converging to a given point in $\partial_\infty F_N$ eventually coincide. Lemma 1 below (which was inspired by a preprint of Los–Lustig [14]) is our analogue of this phenomenon in the $\mathrm{Out}(F_N)$-context, and is a crucial step in our proof. Given an arational tree T, we define an equivalence relation $\sim$ on the set of all morphisms $f : S \to T$, where S is a tree in outer space: informally, two morphisms are equivalent if they take the same turns in T.

Lemma 1. *Let* $T \in \mathcal{AT}$*, let* S, S' *be two trees in outer space, and let* $f : S \to T$ *and* $f' : S' \to T$ *be two optimal morphisms such that* $f \sim f'$*.*
Then there exists $\epsilon > 0$ *such that if* U *is a tree in unprojectivized outer space of covolume at most* ϵ *and* f *factors through* U*, then* f' *also factors through* U*.*

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Lattice envelopes

Roman Sauer

(joint work with Uri Bader, Alex Furman)

Let Γ be a countable group. We are concerned with the study of its lattice envelopes, i.e. the locally compact groups containing Γ as a lattice. We aim at structural results that impose no restrictions on the ambient locally compact group and only abstract group-theoretic conditions on Γ.

We say that Γ satisfies (†) if it

(1) is not virtually isomorphic to a product of two infinite groups, and
(2) does not possess infinite amenable commensurated subgroups, and

(3) satisfies: For a normal subgroup N and a commensurated subgroup M with $N \cap M = \{1\}$ there exists a finite index subgroup $M' < M$ such that N and M' commute.

Linear groups with semi-simple Zariski closure satisfy conditions 2 and 3. Groups with some positive ℓ^2-Betti number satisfy condition 2. All the (†) conditions are satisfied by all linear groups with simple Zariski closure, by all groups with positive first ℓ^2-Betti number, and by all non-elementary acylindrically hyperbolic groups and convergence groups.

To state the main result let us introduce the following notion of S-arithmetic lattice embeddings up to tree extension: Let K be a number field. Let $\mathbf{H}$ be a connected, absolutely simple adjoint K-group, and let S be a set of places of K that contains every infinite place for which $\mathbf{H}$ is isotropic and at least one finite place for which $\mathbf{H}$ is isotropic. Let $\mathcal{O}_S \subset K$ denote the S-integers. The (diagonal) inclusion of $\mathbf{H}(\mathcal{O}_S)$ into $\prod_{\nu\in S}\mathbf{H}(K_\nu)$ is the prototype example of an S-arithmetic lattice.

Let H be a group obtained from $\prod_{\nu\in S}\mathbf{H}(K_\nu)^+$ by possibly replacing each factor $\mathbf{H}(K_\nu)^+$ with K_ν-rank 1 by an intermediate closed subgroup $\mathbf{H}(K_\nu)^+ < D < \mathrm{Aut}(T)$ where T is the Bruhat-Tits tree of $\mathbf{H}(K_\nu)$.

The lattice embedding of $\mathbf{H}(\mathcal{O}_S) \cap H$ into H is called an *S-arithmetic lattice embedding up to tree extension.* Consider the example of $\mathrm{SL}_2(\mathbb{Z}[1/p])$ embedded diagonally as a lattice into $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{Q}_p)$. The latter is a closed cocompact subgroup of $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{Aut}(T_{p+1})$, where T_{p+1} is the Bruhat-Tits tree of $\mathrm{SL}_2(\mathbb{Q}_p)$, i.e. a $(p+1)$-regular tree. So $\mathrm{SL}_2(\mathbb{Z}[1/p]) < \mathrm{SL}_2(\mathbb{R}) \times \mathrm{Aut}(T_{p+1})$ is an S-arithmetic lattice embedding up to tree extension.

We now state the main result which was announced in [1]: Let Γ be a group satisfying (†). Then every embedding of Γ as a lattice into a compactly generated locally compact group G is, up to passage to finite index subgroups and dividing out a normal compact subgroup of G, isomorphic to one of the following cases:

(1) an irreducible lattice in a center-free, semi-simple Lie group without compact factors;
(2) an S-arithmetic lattice embedding up to tree extension, where S is a finite set of places;
(3) a lattice in a totally disconnected group with trivial amenable radical.

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Embeddings of mapping class groups and friends into products of quasi-trees

Ursula Hamenstädt

Property (T) for a group Γ was introduced by Kazdan as a property for its representations. It is equivalent to the following more geometric property: Every action of Γ by affine isometries on a Hilbert space has a global fixed point. This implies the so-called *property (FA)* of Serre: Every action of Γ on a tree has a global fixed point.

There are groups without property (T) which have property (FA), and there are groups with property (FA) for which property (T) is unknown. An example of such groups are *mapping class groups* of surfaces of finite type. Property (FA) for such groups was established by Culler and Vogtmann [2]. On the other hand, there are groups with property (T) which admit isometric actions on *quasi-trees*, i.e. metric spaces quasi-isometric to trees, without global fixed point.

A beautiful characterization of groups with property (T) is due to Chatterji, Drutu and Haglund [1]. Namely, they show that a group Γ has property (T) if and only if every continuous action of Γ by isometries on a median space has bounded orbits. CAT(0)-cube complexes are prototypical examples of median spaces. Hagen [3] associates to a CAT(0)-complex X a quasi-tree, the so-called *contact graph* of X, whose vertices are hyperplanes and where two such hyperplanes are connected by an edge of length one if and only if they contact. The contact graph has infinite diameter if there exists a rank one isometry ϕ of X which maps some hyperplane H to a strongly separated hyperplane $\phi(H)$.

Let now S be a closed surface of genus $g \geq 2$. The curve graph $\mathcal{CG}(S)$ of S is the graph whose vertices are isotopy classes of simple closed curves and where two such vertices are connected by an edge of length one if they can be realized disjointly. The curve graph is a hyperbolic geodesic metric graph. The mapping class group $\mathrm{Mod}(S)$ of S acts on $\mathcal{CG}(S)$ as a group of simplicial automorphisms.

There are many other hyperbolic geodesic $\mathrm{Mod}(S)$-graphs defined in terms of the surface S. In the talk we introduced the following variation of the curve graph. Its vertices are simple closed curves as before. Two such curves c, d are connected by an edge of length one if the following holds true. Put c, d in minimal position. Then $S - (c \cup d)$ is a union of complementary regions which are polygons with an even number of sides or which contain a simple closed curve which is not contractible in S. Connect c and d by an edge if there is at least one component of $S - (c \cup d)$ which neither is a quadrangle nor a six-gon. We call this graph the *principal curve graph*. We discuss how to use CAT(0)-cube complexes to establish the following result [4].

Theorem 1. *The principal curve graph is a quasi-tree of infinite diameter. The action of the mapping class group is acylindrical.*

This leads to the following

Theorem 2. *The mapping class group of S admits an equivariant quasi-isometric embedding into a product of quasi-trees.*

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Affine buildings, folded galleries and algebraic varieties

Petra Schwer and Anne Thomas
(joint work with Elizabeth Milićević)

Affine Deligne–Lusztig varieties (ADLVs) are certain algebraic varieties associated to algebraic groups which have a Bruhat–Tits building. These buildings are CAT(0) spaces that are unions of maximal flats, called apartments, with each apartment a copy of the Coxeter complex for an associated Euclidean reflection group called the affine Weyl group. In [4], we use the geometry and combinatorics of the building to study nonemptiness and the dimension of certain ADLVs.

Part I: The algebraic picture. Let G be a split, connected, reductive group, such as SL_n, and fix a Borel subgroup B of G and a maximal torus T contained in B. For example, if $G = \mathrm{SL}_n$ then we can take B to be the upper-triangular matrices and T the diagonal matrices. Now let k be an algebraic closure of the finite field $\mathbb{F}_q$, and let F be the nonarchimedean field $F = k((t))$, with ring of integers $\mathcal{O} = k[[t]]$. Then there is a natural projection $G(\mathcal{O}) \to G(k)$, obtained by putting $t = 0$, and $G(F)$ has *Iwahori subgroup* I defined as the preimage of $B(k)$ under this projection. The *affine Weyl group* is $W = N_G(T(F))/T(\mathcal{O})$, and this is an infinite Coxeter group, which splits as a semidirect product of a translation group and a finite Coxeter group W_0. For instance, if $G = \mathrm{SL}_3$ then W has Coxeter complex the tessellation of the Euclidean plane by equilateral triangles.

We use two decompositions of $G(F)$, the *affine Bruhat decomposition* on the left and the *affine Birkhoff decomposition* on the right:

$$G(F) = \bigsqcup_{x \in W} IxI \quad \text{and} \quad G(F) = \bigsqcup_{y \in W} U^- yI.$$

Here, $U^- = U^-(F)$ is the negative unipotent subgroup of $G(F)$.

The Bruhat–Tits building Δ for $G(F)$ has chambers the left cosets of I in $G(F)$, and apartments copies of the Coxeter complex for W. The group $G(F)$ acts on Δ with I the stabilizer of the base chamber, and U^- and its W_0-conjugates the stabilizers of the "chambers at infinity" at the boundary of the standard apartment.

The field $k = \overline{\mathbb{F}}_q$ admits the *Frobenius automorphism* $\sigma : a \mapsto a^q$, and this automorphism extends to F and to $G(F)$. For any $x \in W$ and $b \in G(F)$, the associated *afffine Deligne–Lusztig variety* $X_x(b)$ is given by:

$$X_x(b) = \{gI \in G(F)/I \mid g^{-1}b\sigma(g) \in IxI\}.$$

These varieties were introduced by Rapoport in 2000 [6], although they arose implicitly beforehand in the Langlands Program. Their p-adic analogs, affine Deligne–Lusztig sets, are important for the study of Shimura varieties.

The main questions concerning ADLVs in the affine flag variety are when they are nonempty, and if nonempty, their dimension. These questions have turned out to be very difficult. Before our work, they were only completely answered in the case that b is basic, for example $b = 1$ (see [3] and its references). For algebraic reasons, it is enough to consider $(x, b) \in W \times W$.

We study these questions for b a translation in W. The starting point is a theorem of Görtz, Haines, Kottwitz and Reuman [2] saying that for $x \in W$ and b a translation in W, $X_x(b) \neq \emptyset$ if and only if $IxI \cap (U^-)^w b^w I \neq \emptyset$ for some $w \in W_0$, and if nonempty, $\dim(X_x(b))$ equals $\max_{w \in W_0} \dim\left(IxI \cap (U^-)^w b^w I\right)$ in $G(F)/I$.

Part II: The geometric picture. Recall that the chambers of the Bruhat–Tits building Δ are in bijection with the left cosets of I and that each apartment is a copy of the Coxeter complex of the associated affine Weyl group W. In rank two, the Bruhat–Tits building associated with $\mathrm{SL}_2\left(\mathbb{F}_2((t))\right)$ is a trivalent tree as shown in Figure 1. The apartments are copies of simplicial bi-infinite lines. The standard apartment A fixed by the torus T in G is the horizontal line in this figure.

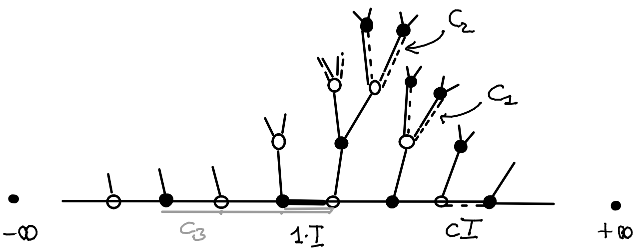


Figure 1. A rank two Bruhat-Tits building.

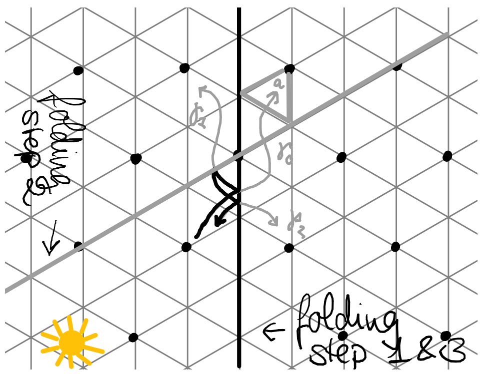
From the affine Bruhat decomposition we obtain that Δ is the union of all apartments containing the chamber labeled $1I$. Double cosets of the form IxI then correspond to pre-images of the chamber xI under the retraction $r : \Delta \to A$ centered at the chamber $1I$ in Δ. We may extend this retraction to galleries starting at I and may view a double coset IxI as the set of final chambers of all pre-images of a minimal gallery connecting I and xI, that is, a gallery of *type* x.

On the other hand, the affine Birkhoff decomposition yields a second type of retraction $\rho_- : \Delta \to A$ centered at a chamber at infinity $\partial C = -\infty$ in the boundary of A, which is stabilized by the unipotent subgroup U^-. Hence the double cosets $U^- yI$ correspond to sets of chambers that are pre-images of some yI under ρ_-.

So for a chamber to be simultaneously contained in IxI and $U^- yI$, the image of some chamber in IxI under ρ_- needs to be yI. In terms of galleries, we combine the above theorem from [2] with results from [5] to prove the following:

(1) For any $x \in W$ and b a translation in W, $X_x(b) \neq \emptyset$ if and only if there exists a choice of a chamber at infinity ∂C, and a gallery γ of type x which starts in I, ends in bI and is positively folded with respect to ∂C.
(2) If $X_x(b) \neq \emptyset$, then $\dim(X_x(b))$ is essentially given by the sum of the (maximal possible) number of positive folds of the gallery and the number of its positive crossings.

Thus in order to prove non-emptiness statements about (families of) ADLVs, and compute their dimension, we construct and manipulate positively folded galleries in such a way that one can read off the number of their positive folds and crossings.



We use tools from Coxeter combinatorics to explicitly construct and manipulate such galleries and thus directly show nonemptiness and compute dimensions of some ADLVs. An example of such a folded gallery is shown in the figure to the left. The black gallery is obtained from the grey one ending in a by repeatedly folding it along the thick black and grey lines. The resulting gallery is positively folded with respect to the chamber at infinity that is indicated by the sun.

Certain conjugation and extension tricks are used alongside the Gaussent–Littelmann root operators [1] to extend the explicit constructions to infinite families of galleries representing non-empty ADLVs and their dimensions. A sample result from [4], which establishes a sharpened version of a conjecture from [2], is:

Theorem (Milićević–S–T)**.** *Let $b = t^\mu$ be a translation and let $x = t^\lambda w \in W$. Assume that bI is in the convex hull of the chambers xI and $1I$, and two technical conditions on μ and λ hold. Then*

$$X_x(1) \neq \emptyset \implies X_x(b) \neq \emptyset$$

and if $w = w_0$, the longest element of W_0, then $X_x(1) \neq \emptyset$ and $X_x(b) \neq \emptyset$. Moreover, if both varieties are nonempty, then

$$\dim X_x(b) = \dim X_x(1) - \langle \rho, \mu^+ \rangle.$$

All our methods are independent of type and rank, and independent of the characteristic of the underlying field. Hence our results are also valid for affine Deligne Lusztig sets which are defined in characteristic zero.

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First order rigidity of high-rank arithmetic groups

Alex Lubotzky
(joint work with Nir Avni, Chen Meiri)

The family of high-rank arithmetic groups is a class of groups which is playing an important role in various areas of mathematics. It includes $\mathrm{SL}(n,\mathbb{Z})$, for $n > 2$, $\mathrm{SL}(n,\mathbb{Z}[1/p])$ for $n > 1$, their finite index subgroups and many more. A number of remarkable results about them have been proven including Mostow rigidity, Margulis Super rigidity and the Quasi-isometric rigidity.

We will talk about a new type of rigidity: *first order rigidity*. Namely if D is such a non-uniform characteristic zero arithmetic group and E a finitely generated group which is elementary equivalent to it (i.e., the same first order theory in the sense of model theory) then E is isomorphic to D.

This stands in contrast with Zlil Sela's remarkable work which implies that the free groups, surface groups and hyperbolic groups (many of whose are low-rank arithmetic groups) have many non isomorphic finitely generated groups which are elementary equivalent to them.

A profinitely rigid Kleinian group

Alan Reid
(joint work with Martin R. Bridson, Ben McReynolds, Ryan Spitler)

Recently, there has been considerable interest in the question of when the set of finite quotients of a finitely generated residually finite group determines the group up to isomorphism (see [1], [2], [3], [6], [7] and [8] for some recent work). In more sophisticated terminology, one wants to develop a complete understanding of the circumstances in which finitely generated residually finite groups have isomorphic *profinite completions*. Recall that if Γ is a finitely generated group, then the profinite completion of Γ is defined as

$$\widehat{\Gamma} = \varprojlim \Gamma/N$$

where the inverse limit is taken over the normal subgroups of finite index $N \lhd \Gamma$ ordered by reverse inclusion.

Motivated by this, say that a residually finite group Γ is *profinitely rigid*, if whenever $\widehat{\Delta} \cong \widehat{\Gamma}$, then $\Delta \cong \Gamma$.

However, as yet, little by way of positive results are known in any setting outside of the cases where the group Γ satisfies a law (for example see [5] for the cases of certain nilpotent groups of class 2). In the context of our work, the groups of most interest to us are lattices in semi-simple Lie groups (which contain free subgroups and so satisfy no laws), and in this setting even the cases of free groups of rank at least 2, hyperbolic surface groups or Kleinian groups of finite co-volume remain open.

This talk provided a preliminary report of a result that provides the first examples of finite co-volume Kleinian groups that are profinitely rigid. Namely we prove the following theorem (where $\omega^2 + \omega + 1 = 0$).

Theorem 1. *The arithmetic Kleinian groups* $\mathrm{PGL}(2, \mathbb{Z}[\omega])$ *and* $\mathrm{PSL}(2, \mathbb{Z}[\omega])$ *are profinitely rigid.*

The case of $\mathrm{PGL}(2, \mathbb{Z}[\omega])$ follows from that of $\mathrm{PSL}(2, \mathbb{Z}[\omega])$, and so we limit ourselves to briefly indicating the strategy of the proof of Theorem 1 for $\mathrm{PSL}(2, \mathbb{Z}[\omega])$. We then discuss why the group $\mathrm{PSL}(2, \mathbb{Z}[\omega])$ is almost uniquely placed for this line of argument to work. In what follows we set $\Gamma = \mathrm{PSL}(2, \mathbb{Z}[\omega])$.

There are three key steps in the proof which we summarize below.

Theorem 2 (Representation Rigidity)**.** *Let* $\iota : \Gamma \to \mathrm{PSL}(2, \mathbb{C})$ *denote the identity homomorphism, and* $c = \bar{\iota}$ *the complex conjugate representation. Then if* $\rho : \Gamma \to \mathrm{PSL}(2, \mathbb{C})$ *is a representation with infinite image,* $\rho = \iota$ *or* c*.*

Using Theorem 2 we are able to get some control on $\mathrm{PSL}(2, \mathbb{C})$ representations of a finitely generated residually finite group with profinite completion isomorphic to $\widehat{\Gamma}$, and to that end we prove:

Theorem 3. *Let* Δ *be a finitely generated residually finite group with* $\widehat{\Delta} \cong \widehat{\Gamma}$*. Then* Δ *admits an epimorphism to a group* $L < \Gamma$ *which is Zariski dense in* $\mathrm{PSL}(2, \mathbb{C})$*.*

Finally, we make use of Theorem 3, in tandem with an understanding of the topology and deformations of orbifolds $\mathbb{H}^3/G$ for subgroups $G < \Gamma$. Briefly, in the notation of Theorem 3, the case of L having infinite index can be ruled out using Teichmüller theory to construct explicit finite quotients of L and hence Δ that cannot be finite quotients of Γ. To rule out the finite index case we make use of the observation made via Magma that any subgroup of index ≤ 12 in Γ has first Betti number ≤ 1. This together with the construction of L, and 3-manifold topology shows that L contains the fundamental group of a 1-punctured torus bundle over the circle of index 12, and we can then invoke [4] to yield the desired conclusion that $\Delta \cong \Gamma$. □

We finish with some comments on why Γ is so well suited to this line of argument.

For example one can run the above proof idea for $\mathrm{SL}(3,\mathbb{Z})$ and one gets an analogous group L as in Theorem 3. However, the structure of Zariski dense infinite index (i.e. "thin") subgroups of $\mathrm{SL}(3,\mathbb{Z})$ remains rather mysterious. Theorem 2 places extremely strong control on $\mathrm{PSL}(2,\mathbb{C})$ representations of Γ, and indeed, Γ is unique amongst Bianchi groups in that it has "such a small character variety". Put more precisely, Γ is the unique Bianchi group that has Property FA. Amongst other things, this puts very strong restrictions on finite fields $\mathbb{F}$ for which possible quotients $\mathrm{PSL}(2,\mathbb{F})$ can occur.

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Introduction to geometric approximate group theory

Tobias Hartnick

(joint work with Michael Björklund, Matthew Cordes, Vera Tonić)

Definition. An *approximate group* is a pair $(\Lambda, \Lambda^\infty)$ where Λ^∞ is a group and $\Lambda \subset \Lambda^\infty$ is a subset satisying

(AG1) $e \in \Lambda$ and $\Lambda^{-1} = \Lambda$;
(AG2) there exists a *finite* subset $F \subset \Lambda^\infty$ such that $\Lambda \cdot \Lambda = F \cdot \Lambda$;
(AG3) Λ generates Λ^∞.

If Λ^∞ is a subgroup of a group Γ, then Λ is called an *approximate subgroup* of Γ.

Finite approximate subgroups play a mayor role in additive combinatorics. Approximate subgroups of $\mathbb{R}^n$ which are Delone sets can be constructed using cut-and-project methods and are models for mathematical quasi-crystals. Yves Meyer has provided a classification of Delone approximate subgroups in $\mathbb{R}^n$ in the 1970s [5], and recently Breuillard, Green and Tao [2] have established a structure theorem concerning finite approximate subgroups. So far there is no general structure theory for *infinite, non-abelian* approximate groups.

We suggest to use methods from geometric and measurable group theory to study approximate subgroups of finitely generated groups. If $(\Lambda, \Lambda^\infty)$ is an approximate group which is *finitely-generated* in the sense that Λ^∞ admits a finite generating set S, then the word metric d_S on Λ^∞ associated with S restricts to a metric on Λ, and the quasi-isometry (QI) type $[\Lambda]$ of $(\Lambda, d_S|_\Lambda)$ does not depend on the choice of S. This allows us to think of finitely-generated approximate groups as geometric objects (namely QI types) in the spirit of geometric group theory.

As for groups we can define the notion of a *(quasi-isometric) quasi-action* of an approximate group $(\Lambda, \Lambda^\infty)$ on a proper metric space X as a map ρ from Λ^∞ into the set of quasi-isometries of X such that $\rho(\Lambda^k)$ is uniform for all $k \geq 1$ and such that there exist constants $C_1, C_2, \ldots$ such that

$$d(\rho(g)\rho(h)x, \rho(gh)x) < C_k \quad (g, h \in \Lambda^k, x \in X).$$

Such a quasi-action is then called *geometric* provided the quasi-orbit $\rho(\Lambda).x$ is relatively dense in X for some (hence any) $x \in X$ and the induced map $\Lambda \times X \to X \times X$, $(g, x) \mapsto (x, \rho(g).x)$ is proper. We have the following theorem in the spirit of the classical Schwarz–Milnor lemma.

Theorem 1 ([1, 3])**.** *Let $(\Lambda, \Lambda^\infty)$ be an approximate group and let X be a coarsely-connected proper metric space.*

- *(i) If $(\Lambda, \Lambda^\infty)$ quasi-acts geometrically on X, then Λ^∞ is finitely-generated and Λ is coarsely equivalent to X. If X and Λ are moreover large-scale geodesic, then Λ is quasi-isometric to X.*
- *(ii) If Λ is quasi-isometric to X, then $(\Lambda, \Lambda^\infty)$ quasi-acts geometrically on X.*

While every finitely-generated approximate group admits a geometric quasi-action, not every such approximate group admits a geometric *isometric* action. The question whether every approximate group $(\Lambda, \Lambda^\infty)$ which is quasi-isometric to a specific proper metric space X acts geometrically on X is known as the *QI rigidity problem* for X. For example, it has a positive solution for higher rank symmetric spaces X of the non-compact type [1], but the rank one case is open.

To describe geometric isometric actions of approximate groups, let us call an approximate subgroup of a locally compact second countable (lcsc) group G an *approximate uniform lattice* if it is Delone, i.e. uniformly discrete and relatively dense with respect to some (hence any) proper left-invariant continuous metric on G. If $(\Lambda, \Lambda^\infty)$ is an approximate group and $f : \Lambda^\infty \to G$ is a homomorphism, then the spaces $\ker(f) \cap \Lambda^k$ are QI for $k \geq 2$, and we denote their common QI class by $[\ker f]$. Then we have:

Theorem 2 ([3])**.** *Let $(\Lambda, \Lambda^\infty)$ be an approximate group and let X be a proper metric space. Then a homomorphism $\rho : \Lambda \to \mathrm{Is}(X)$ induces a geometric action of $(\Lambda, \Lambda^\infty)$ on X if and only if $[\ker f]$ is trivial and $\rho(\Lambda)$ is a uniform approximate lattice in $\mathrm{Is}(X)$.*

If Λ is a uniform approximate lattice in a lcsc group G, then we call G an *envelope* for Λ. In this situation, G is compactly-generated if and only if Λ^∞ is finitely-generated and amenable if and only if $[\Lambda]$ is metrically amenable (see [1]).

Theorem 3 ([1])**.** *Every compactly-generated envelope and every amenable envelope of a uniform approximate lattice is unimodular.*

The world of finitely-generated approximate groups contains examples which behave rather differently from finitely-generated groups.

Example 1. There exist uniform approximate lattices in connected nilpotent Lie groups which are not quasi-isometric to any finitely-generated group. (This follows from results in [4] in view of the observation from [1] that a 1-connected nilpotent Lie group is an envelope if its Lie algebra is defined over the algebraic closure of $\mathbb{Q}$.)

Example 2 ([3])**.** If $\Lambda \subset \mathrm{PSL}_2(\mathbb{R})$ is a uniform approximate lattice and Λ^∞ is torsion-free, then $(\Lambda, \Lambda^\infty)$ is called an *approximate surface group*, and its Teichmüller space is defined as

$$\mathcal{T}(\Lambda, \Lambda^\infty) = \{\rho \in \mathrm{Hom}(\Lambda^\infty, \mathrm{PSL}_2(\mathbb{R})) \,|\, \rho \text{ injective}, \ \rho(\Lambda) \subset \mathrm{PSL}_2(\mathbb{R}) \text{ Delone}\}/\text{conj}.$$

There exist both *flexible* approximate surface groups with $\dim \mathcal{T}(\Lambda, \Lambda^\infty) > 0$ and *rigid* approximate surface groups with trivial Teichmüller space.

If an approximate group $(\Lambda, \Lambda^\infty)$ quasi-isometrically quasi-acts on a Gromov-hyperbolic space X, then Λ^∞ acts on the Gromov boundary of X. If the assumption that X be Gromov-hyperbolic is dropped, then one can still define an action of Λ^∞ on the Morse-boundary $\partial_M X$ of X, and the set of accumulation points of quasi-orbits of Λ defines a Λ^∞-invariant subset $\mathcal{L}(\Lambda) \subset \partial_M X$ called its *limit set*. We say that the quasi-action is *hyperbolically convex-cocompact* if it is proper with compact non-empty limit set and the hull of the limit set in X is at bounded distance from some (hence any) quasi-orbit of Λ.

Theorem 4 ([3])**.** *A quasi-isometric quasi-action on a proper geodesic metric space X is hyperbolically convex cocompact if and only if its quasi-orbits are unbounded stable subsets of X. In this case $[\Lambda]$ is Gromov-hyperbolic, all limit points are conical, and the quasi-orbits maps $\Lambda \to X$ extend to Λ^∞-equivariant homeomorphisms between the Gromov boundary of $[\Lambda]$ and the limit set $\mathcal{L}(\Lambda) \subset \partial_M X$.*

If (Ξ, Ξ^∞), $(\Lambda, \Lambda^\infty)$ are approximate groups, then a surjective homomorphism $\Xi^\infty \to \Lambda^\infty$. which maps Ξ onto Λ is called an extension of $(\Lambda, \Lambda^\infty)$. In order to further develop the structure theory of approximate groups, it would be of major interest to obtain a better understanding of such extensions. So far we have the following results:

Theorem 5 ([3])**.** *Let $(\Lambda, \Lambda^\infty)$ be an approximate group.*

(i) If $p : \Gamma \to \Lambda^\infty$ is a surjective group homomorphism which admits a section σ such that the cocycle $c_\sigma(g,h) := \sigma(gh)^{-1}\sigma(g)\sigma(h)$ satisfies $|c_\sigma(\Lambda \times \Lambda^3)| < \infty$, then $\Xi := p^{-1}(\Lambda)$ is an approximate subgroup of Γ, and if $\Xi^\infty := \langle \Xi \rangle < \Gamma$, then p restricts to an extension $(\Xi, \Xi^\infty) \to (\Lambda, \Lambda^\infty)$.

(ii) If $p : (\Xi, \Xi^\infty) \to (\Lambda, \Lambda^\infty)$ is an extension of countable approximate groups and $[\ker p]$ is the common coarse equivalence class of the spaces $\ker p \cap \Lambda^k$, $k \geq 2$, then

$$\mathrm{asdim}(\Xi) \leq \mathrm{asdim}(\Lambda) + \mathrm{asdim}([\ker p]).$$

Part (i) of the theorem indicates that there is a connection between approximate groups and bounded cohomology of groups, which is not fully understood yet.

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Elementary equivalence of hyperbolic groups

Gilbert Levitt
(joint work with Vincent Guirardel, Rizos Sklinos)

I discussed the classification of torsion-free hyperbolic groups up to elementary equivalence, due to Zlil Sela. After sketching the main steps, I defined floors and towers using local preretractions, and gave examples. The core of G is the group $c(G)$ (unique up to isomorphism) such that G is a tower over $c(G)$, and $c(G)$ is not a floor. Two groups are elementary equivalent if and only if their cores are isomorphic. I discussed the non-uniqueness of the core as a subgroup of G, and gave an example where $c(G)$ cannot be elementarily embedded into G.

H–wide fillings and Virtual Fibering of finite-volume hyperbolic 3–manifolds

Daniel Groves
(joint work with Jason Manning)

The following result is due to Agol [2] in the closed case and Wise [9] in the non-compact case.

Theorem 1. *Suppose that M is the fundamental group of a finite-volume hyperbolic 3–manifold. Then $\pi_1(M)$ is the fundamental group of a compact virtual special cube complex.*

The following is then an immediate consequence of the main result in [1].

Corollary. *Let M be as in Theorem 1. Then M has a finite cover which fibers over S^1.*

The main goal of this talk was to explain a new proof of Theorem 1 in the non-compact setting. We utilize a recent theorem of Cooper and Futer [5] who prove that $\pi_1(M)$ is the fundamental group of a compact nonpositively curved cube complex X. Thus, it remains to prove that X is virtually special.

In order to do this, one needs (see [8]) to prove that certain geometrically finite subgroups, and certain double cosets of geometrically finite subgroups are separable. That the subgroups are separable follows from [2, Theorem 9.4], so we must separate the double cosets.

In order to prove this double coset separability, we perform a long (orbifold) Dehn filling on M to produce a closed hyperbolic 3–orbifold, and we need to control the behavior of the geometrically finite subgroups and their double cosets under this filling. The main novelty of this talk was to describe a method for achieving this that improves upon the existing results from [3, 4]. In particular, this new technique allows control over the behavior of many more kinds of geometrically finite subgroups under filling.

The new condition on filling is called *H–wide*, for a geometrically finite subgroup H. The idea is to make sure that the slope σ along which we fill in a cusp subgroup P has the property that for any $\gamma \in \pi_1(M)$ either $\sigma \in H^\gamma \cap P$, or else σ is a long way from $H^\gamma \cap P$. There are similar criteria for the double cosets. It is straightforward to see that there are the right kinds of slope.

The results are proved using the methods of relatively hyperbolic Dehn filling as in [7, 6, 3, 4].

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On weakly separable commensurated subgroup

Pierre-Emmanuel Caprace

(joint work with Peter H. Kropholler, Colin D. Reid, Phillip Wesolek)

We present a new theorem that provides a common explanation to the following three known results.

Theorem 1 (S. Meskin [5]). *The Baumslag–Solitar group*

$$BS(m,n) = \langle a, b \mid ba^m b^{-1} = a^n \rangle$$

is residually finite if and only if $\#\{1, |m|, |n|\} \leq 2$.

Recall that a subgroup A of a group G is called **separable** if it is an intersection of finite index subgroups of G. Equivalently, for each element $g \in G - A$, there exists a homomorphism $\varphi \colon G \to Q$ onto a finite group Q such that $\varphi(g) \notin \varphi(A)$.

Theorem 2 (D. Wise [7, Lemmas 5.7 and 16.2])**.** *Let T_1, T_2 be locally finite leafless trees and $\Gamma \leq \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ be a group acting properly and cocompactly on the product $T_1 \times T_2$. If for all $i \in \{1, 2\}$ and all edges $e \in ET_i$, the stabilizer Γ_e is a separable subgroup of Γ, then Γ has a finite index subgroup that splits as a direct product $\Gamma_1 \times \Gamma_2$, where Γ_j is a discrete cocompact subgroup of $\mathrm{Aut}(T_j)$.*

Recall that a subgroup A of a group G is called **profinitely dense** if it is dense with respect to the profinite topology on G or, equivalently, if A maps surjectively onto each finite quotient of G.

Theorem 3 (Folklore)**.** *For all $m, n \geq 2$, the group $\mathrm{SL}_n(\mathbf{Z})$ is profinitely dense in $\mathrm{SL}_n(\mathbf{Z}[\frac{1}{m}])$.*

Proof. By the solution of the congruence subgroup problem [1], [6], the profinite completion of $\mathrm{SL}_n(\mathbf{Z}[\frac{1}{m}])$ is the product over all primes p not dividing m of the groups $\mathrm{SL}_n(\mathbf{Z}_p)$. The density of the diagonal embedding of $\mathrm{SL}_n(\mathbf{Z})$ into that product is a consequence of the Chinese Remainder Theorem (this is a simple manifestation of Strong Approximation). An alternative more direct argument consists in enumerating the distinct prime divisors of m, say $p_1, \ldots, p_d$, and observing that in the chain

$$\mathrm{SL}_n(\mathbf{Z}) < \mathrm{SL}_n(\mathbf{Z}[\frac{1}{p_1}]) < \cdots < \mathrm{SL}_n(\mathbf{Z}[\frac{1}{p_1 \cdots p_d}]) = \mathrm{SL}_n(\mathbf{Z}[\frac{1}{m}]),$$

each term is a maximal subgroup of infinite index in the next. Therefore each term is profinitely dense in the next term, and the conclusion follows. $\square$

In order to state our main result, we need to introduce additional terminology.

A subgroup A of a group G is called **virtually normal** if A has a finite index subgroup that is normal in G. The subgroup A is called **weakly separable** if it is an intersection of virtually normal subgroups. Equivalently, for each element $g \in G - A$, there exists a homomorphism $\varphi \colon G \to Q$ onto a group Q such that $\varphi(A)$ is finite and $\varphi(g) \notin \varphi(A)$. Any subgroup $A \leq G$ is contained in a smallest separable (resp. weakly separable) subgroup, denoted by $\overline{A}$ (resp. $\widetilde{A}$) and called the **profinite closure** (resp. **residual closure**) of A in G. Every separable subgroup is weakly separable, so that the inclusion $\widetilde{A} \leq \overline{A}$ always holds. That inclusion can moreover be strict: for example, if G is an infinite simple group, then $\widetilde{\{1\}} = \{1\}$ and $\overline{\{1\}} = G$.

The subgroup A is called **commensurated** if the index $[A : A \cap gAg^{-1}]$ is finite for all $g \in G$.

Main Theorem ([3])**.** *Let G be a group and $A \leq G$ be a commensurated subgroup. If G is generated by finitely many cosets of A, then the residual closure $\widetilde{A}$ is virtually normal. In particular the profinite closure $\overline{A} = \overline{\widetilde{A}}$ is virtually normal.*

We sketch a proof that $\overline{A}$ is virtually normal; it is based on the following fact.

Proposition ([4, Corollary 4.1])**.** *Let J be a compactly generated totally disconnected locally compact group. If the intersection of all open normal subgroups of J is trivial, then every identity neighbourhood contains a compact open normal subgroup.*

Proof of the Main Theorem for the profinite closure. Let $\varphi\colon G \to \widehat{G}$ be the canonical homormorphism of G to its profinite completion $\widehat{G}$. Let J be the subgroup of $\widehat{G}$ abstractly generated by $\varphi(G) \cup \overline{\varphi(A)}$. Since A is commensurated by G, it follows that $\overline{\varphi(A)}$ is commensurated by $\varphi(G)$, so that J carries a unique group topology that makes the inclusion $\overline{\varphi(A)} \to J$ continuous and open. Since $\overline{\varphi(A)}$ is profinite and open in J, it follows that J is totally disconnected and locally compact. Since G is generated by finitely many cosets of A, it follows that J is generated by finitely many cosets of $\overline{\varphi(A)}$, and is thus compactly generated. The inclusion $J \to \widehat{G}$ is continuous (because it is continuous at the identity, since $\overline{\varphi(A)}$ is open in J). Therefore the intersection of all open normal subgroups of J is trivial, because $\widehat{G}$ is profinite. The Proposition therefore affords an open normal subgroup of J contained in $\overline{\varphi(A)}$. The preimage of that subgroup under φ is a normal subgroup of G that is contained as a finite index subgroup in the profinite closure $\overline{A}$. □

An alternative direct proof of the Main Theorem, not relying on the Proposition, may be found in [3] (where it is moreover observed that the Proposition can also be derived from the Main Theorem).

Corollary 1. *Let G be a residually finite group and A be a commensurated soluble subgroup such that G is generated by finitely many cosets of A. For any non-elementary cobounded isometric action of G on a Gromov hyperbolic metric space X, we have $[E_X(G) : A \cap E_X(G)] < \infty$, where $E_X(G) = \{g \in G \mid \sup_{x \in X} d(x, gx) < \infty\}$.*

Proof. Since G is residually finite, the profinite closure of a soluble subgroup is itself soluble. Hence $\overline{A}$ is soluble. By the Main Theorem it contains a finite index subgroup R that is normal in G. Since R is soluble, its action on X is elementary, i.e. its limit set contains at most 2 points. If the limit set of R contains 1 or 2 points, then the same holds for G since R is normal, contradicting the hypothesis that the G-action is non-elementary. Thus the limit set of R is empty, so that R has a bounded orbit. Since R is normal and the G-action is cobounded, it follows that R is contained in $E_X(G)$. □

The 'only if' part of Theorem 1 follows from Corollary 1 by setting $A = \langle a \rangle$ and letting X be the Bass–Serre tree of the Baumslag–Solitar group viewed as an HNN-extension. The condition $|m| \neq 1 \neq |n|$ ensurates that the G-action on X is non-elementary, while the condition $|m| \neq |n|$ ensurates that $E_X(G)$ is trivial. The action of the non-residually finite Baumslag–Solitar groups on their

Bass–Serre tree shows that the hypothesis of residual finiteness of G cannot be discarded in Corollary 1.

The following result generalizes Theorem 2.

Corollary 2. *Let T_1, T_2 be locally finite leafless trees and $\Gamma \leq \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ be a group acting properly and cocompactly on the product $T_1 \times T_2$. If for some $i \in \{1, 2\}$ and some vertex or edge $y \in VT_i \cup ET_i$, the stabilizer Γ_y is a weakly separable subgroup of Γ, then Γ has a finite index subgroup that splits as a direct product $\Gamma_1 \times \Gamma_2$, where Γ_j is a discrete cocompact subgroup of $\mathrm{Aut}(T_j)$.*

Proof. Since T_i is a connected locally finite graph, it follows that $A = \Gamma_y$ is a commensurated subgroup of Γ. Moreover Γ is finitely generated since it acts properly and cocompactly on $T_1 \times T_2$. Therefore, if A is weakly separable, then it is virtually normal by the Main Theorem. A finite index subgroup of A that is normal in Γ must act trivially on T_i (because its fixed point set is a non-empty Γ-invariant subtree of T_i, which is leafless by hypothesis). This implies that the projection of Γ to $\mathrm{Aut}(T_i)$ has discrete image. The reducibility of Γ then follows from a basic observation of Burger and Mozes [2, Proposition 1.2]. □

A group G is called **just-infinite** if it is infinite and all its proper quotients are finite. This is in particular the case of $\mathrm{PSL}_n(\mathbf{Z}[\frac{1}{m}])$ for all $m, n \geq 2$ by Margulis' Normal Subgroup Theorem. Therefore, Theorem 3 is an illustration of the following general fact.

Corollary 3. *In a finitely generated just-infinite group, the profinite closure of an infinite commensurated subgroup is of finite index.*

Proof. Let G be a finitely generated just-infinite group and A be an infinite commensurated subgroup. Then $\overline{A}$ is infinite and virtually normal by the Main Theorem. It must thus be of finite index since G is just-finite. □

Further applications of the Main Theorem are described in [3].

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Transgression in bounded cohomology and Monod's conjecture

Andreas Ott

Starting with Gromov's work [5], bounded cohomology of discrete groups has proved to be a useful concept in geometry and group theory. However, it is notoriously difficult to compute. Extending Gromov's original definition, Burger and Monod [1, 2, 3] introduced the continuous bounded cohomology $H^\bullet_{\mathrm{cb}}(G;\mathbb{R})$ for any locally compact topological group G. It is defined like the continuous group cohomology $H^\bullet_{\mathrm{c}}(G;\mathbb{R})$, but with the additional requirement that cochains on G be bounded. The resulting cohomology rings are then related by a natural comparison map

$$H^n_{\mathrm{cb}}(G;\mathbb{R}) \to H^n_{\mathrm{c}}(G;\mathbb{R})$$

for $n \geq 0$. Despite being powerful enough for applications, most notably in rigidity theory, the continuous bounded cohomology of Lie groups should be easier to compute than the bounded cohomology of discrete groups. In fact, Dupont and Monod conjectured that it coincides with the continuous group cohomology of G, which is classically known.

Conjecture (Dupont [4] and Monod [8, Problem A])**.** *Let G be a connected semisimple Lie group with finite center. Then the comparison map is an isomorphism.*

The part on surjectivity is due to Dupont, and the more recent part on injectivity is due to Monod. The conjecture is known to be true in degrees $n \leq 2$. Surjectivity has been proved for many Lie groups, while injectivity is presently known only for $\mathrm{SL}_2(\mathbb{R})$ in degrees $n = 3$ [3] and $n = 4$ [6], and for $\mathrm{SO}(1,n)$ in degree $n = 3$ [10]. We study injectivity of the comparison map for the group

$$G = \mathrm{SL}_2(\mathbb{R}).$$

Since the continuous cohomology of $\mathrm{SL}_2(\mathbb{R})$ vanishes for $n > 2$, the conjecture predicts that $H^n_{\mathrm{cb}}(G;\mathbb{R}) = 0$ for $n > 2$. Our approach builds on the analytic techniques introduced in [6].

By work of Burger and Monod [7], the continuous bounded cohomology of G is computed by the complex of G-invariant essentially bounded measurable cochains on the boundary S^1 of G:

$$H^n_{\mathrm{cb}}(G;\mathbb{R}) \cong H^n(L^\infty(\mathbb{T}^{\bullet+1},\mathbb{R})^G, \delta^\bullet),$$

where $\mathbb{T}^n = (S^1)^n$ is the torus, equipped with the diagonal action of G, and δ is the usual coboundary operator. Let us fix a maximal compact subgroup K of G. We denote by $S^\infty(\mathbb{T}^n,\mathbb{R})$ the space of classes of real valued functions on $\mathbb{T}^n$ that are bounded and smooth with bounded derivatives along the G-orbits in $\mathbb{T}^n$, where two functions are identified if they agree away from a G-invariant set of measure zero. The subspace of K-invariants will be denoted by $S^\infty(\mathbb{T}^n,\mathbb{R})^K$. Moreover, we denote by $S^\infty_\tau(\mathbb{T}^n,\mathbb{C})$ the space of classes of complex valued functions u on $\mathbb{T}^n$

that are bounded, smooth with bounded derivatives along the G-orbits in $\mathbb{T}^n$, and *tame* in the following sense:

$$\sup_{z\in\mathbb{T}^n,\ T\in\mathbb{R}} \left| \int_0^T (\operatorname{Re} u)(a_t.z)\, dt \right| < \infty,$$

where $A = \{a_t\}_{t\in\mathbb{R}}$ is the 1-parameter subgroup of G obtained from the Iwasawa decomposition $G = KAN$. As before, two functions are identified if they agree away from a G-invariant set of measure zero. The subspace of K-invariants will be denoted by $S^\infty_\tau(\mathbb{T}^n, \mathbb{C})^K$. Here we think of $\mathbb{C}$ as a K-module equipped with the standard action of $K \cong S^1$. Hence the K-invariants in $S^\infty_\tau(\mathbb{T}^n, \mathbb{C})$ are actually K-equivariant functions. The infinitesimal actions of A and N on the torus $\mathbb{T}^n$ determine differential operators L_A and L_N, acting on orbitwise smooth functions. Consider the complexes

$$\mathcal{C} = (L^\infty(\mathbb{T}^{\bullet+1}, \mathbb{R})^G, \delta^\bullet), \quad \mathcal{S} = (S^\infty(\mathbb{T}^{\bullet+1}, \mathbb{R})^K, \delta^\bullet), \quad \mathcal{T} = (S^\infty_\tau(\mathbb{T}^{\bullet+1}, \mathbb{C})^K, \delta^\bullet),$$

and define operators

$$L = L_A + i\, L_N, \quad Q = \operatorname{Im}(\operatorname{Id} - \overline{L}).$$

We may then introduce the following complex

$$0 \longrightarrow \mathcal{C} \longrightarrow \mathcal{S} \xrightarrow{L} \mathcal{T} \xrightarrow{Q} \mathcal{S} \longrightarrow 0,$$

which we call the *Cauchy-Frobenius complex.* Its usefulness stems from the following two propositions.

Proposition 1. *The Cauchy-Frobenius complex is acyclic.*

Proposition 2. *The complex $\mathcal{S}$ is acyclic.*

Hence passing to the long exact sequences in cohomology associated to the Cauchy-Frobenius complex, we infer the following theorem.

Theorem. *(i)* $H^2_{\mathrm{cb}}(G;\mathbb{R}) \cong H^1(S^\infty_\tau(\mathbb{T}^{\bullet+1}, \mathbb{C})^K, \delta^\bullet) \cong \mathbb{R}$.
(ii) $H^n_{\mathrm{cb}}(G;\mathbb{R}) \cong H^{n-1}(S^\infty_\tau(\mathbb{T}^{\bullet+1}, \mathbb{C})^K, \delta^\bullet)$ *for every* $n > 2$.

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Noetherian groups rings, amenability, dimension flatness and the weak Nullstellensatz

Peter Kropholler
(joint work with Ilaria Castellano, Dawid Kielak, Karl Lorensen)

We combine recent work of Bartholdi–Kielak with results of Ceccherini-Silberstein–Coornaert on cellular automata to establish a criterion for amenability of groups. We apply this to confirm a conjecture of Lück concerning the dimension-flatness over the group von Neumann algebra. A further application states that groups whose group rings have finite uniform dimension are amenable. We also apply the results to groups satisfying a weak form of Hilbert's Nullstellensatz. Concluding that such groups must have Noetherian group rings and hence must be amenable with the maximal condition on subgroups. There are connections with the literature on Tarski monsters and related groups.

Topology of generalized tridiagonal isospectral Hermitian matrices

Tadeusz Januszkiewicz

Let T be a graph on the vertices $\{0,\dots,n\}$ and $\lambda = (\lambda_0,\dots,\lambda_n)$ be a vector of real numbers with $\lambda_0 < \dots < \lambda_n$. $O_\lambda \subset H_{n+1}$ denotes the set of Hermitian $(n+1)\times(n+1)$-matrices with spectrum λ, L_T is the linear subspace of H_{n+1} given by $L_T := \{(z_{i,j})_{i,j} | z_{i,j} = 0$ if (i,j) is not an edge of $T\}$. We want to study

$$O_\lambda \cap L_T =: M_{\mathbb{C}}(T) \supset M_{\mathbb{R}}(T) := M_{\mathbb{C}}(T) \cap Sym.$$

One tool to study $M(T)$ is the action of the diagonal matrices on $M_{\mathbb{C}}(T)$. This is especially effective if T is a tree. We have very explicit results on the topology (π_1, H^*, H^*_T) of both $M_{\mathbb{C}}(T)$ and $M_{\mathbb{R}}(T)$ when T is a line graph (this is classical) or if T is the complete bipartite graph $K_{1,n}$. In particular $M_{\mathbb{R}}(T)$ is a closed aspherical manifold, carrying cubical metric which is locally CAT(0). This cubical structure can be constructed either combinatorially or by the study of the *moment map*: projection from $M_{\mathbb{R}}(T)$ to diagonal matrices.

Reporter: Robin Loose

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