Mathematisches Forschungsinstitut Oberwolfach

Report No. 42/2020

DOI: 10.4171/OWR/2020/42

Mini-Workshop: Relativistic Fluids at the Intersection of Mathematics and Physics (online meeting)

Organized by
Shabnam Beheshti, London
Marcelo M. Disconzi, Nashville
Vu Hoang, San Antonio

13 December – 19 December 2020

Abstract. Relativistic Hydrodynamics is the description of fluid motion in regimes where relativistic effects are important. This is the case for fluids moving at high velocities or interacting with very strong gravitational fields, such as in the physics of black hole accretion disks or neutron star mergers but also in the microscopic dynamics of high-energy heavy-ion collisions. Although the first formulation of hydrodynamic equations dates back to the beginning stages of relativity theory, many mathematical problems remain wide open. In particular, the development of the theory of relativistic *viscous* fluids was slow and mathematical progress only made recently. The purpose of this Mini-Workshop was to bring together a diverse group of researchers, including specialists in nonlinear PDEs and physicists, to jump-start the mathematical development of this field. This allowed for a vital exchange of ideas between mathematics and physics communities.

Mathematics Subject Classification (2010): 35-06, 76-06, 83-06.

Introduction by the Organizers

The mini-workshop *Relativistic Fluids at the Intersection of Mathematics and Physics*, organized by Shabnam Beheshti (London), Marcelo M. Disconzi (Nashville) and Vu Hoang (San Antonio) took place 13 December - 16 December and was attended by 19 participants from Europe, Australia and the United States. The group was a diverse blend of researchers from the Mathematical Physics and Theoretical Physics communities, the majority of which is working on different areas of relativistic physics. Due to travel restrictions and the global pandemic situation, the mini-workshop took place in an online form. In order to emphasize interaction among its participants, the speakers agreed to giving 45 minute talks, so that discussion could take place after the talks.

The talks on the first day considered various models for relativistic plasmas, with Oliver Rinne opening the meeting by speaking about the Einstein-Vlasov system. This was followed by Michael Kiessling introducing the audience to the problem of deriving the kinetic description of plasmas from a microscopic model of particle motion. The final talk of the day was given by Michael Strickland, who gave an introduction to the physics of the quark-gluon plasma.

The second day of the meeting started with a talk by Juan Valiente Kroon, who discussed the stability of perturbations of cosmological solutions via conformal methods. This was followed by an hour of free remarks and discussions. The second day was concluded with A. Shadi Tahvildar-Zadeh's talk where he addressed a broad range of questions concerning the propagation of singularities in theories involving gravitational and other fields.

The focus on the third day was fluid models, with talks by Todd Oliynyk, Luciano Rezzolla and Jorge Noronha. Todd Oliynyk presented the state-of-the-art theory of relativistic liquid bodies, Luciano Rezzolla introduced the audience to cutting-edge applications of relativistic fluids to astrophysical problems, especially involving strong gravitational fields. Finally, Jorge Noronha discussed a novel theory of first-order viscous relativistic hydrodynamics.

On its fourth day, the meeting continued with Annegret Burtscher, Matthias Hanauske and Jeremie Joudioux. Annegret Burtscher addressed the question of singularity formation of a spherically symmetric Einstein-Euler system, whereas Matthias Hanauske gave an overview about the properties of hypermassive stars containing different phases of neutron and quark matter. Jérémie Joudioux gave an overview and open problem talk introducing the challenging problem of stability of steady states in relativistic kinetic theory.

On the final day of the workshop, talks were given by Susanne Reffert, who discussed large charge expansion in relativistic effective field theory and Michael Hott, who gave an overview talk about the derivation of mean field equations from an underlying many-particle quantum theory. In between the two talks, free discussions took place.

Many of the talks were followed by lively and stimulating discussion. A detailed description of all such discussions is beyond the scope of this report. But in order to provide an illustration of the depth and quality of the discussions, we highlight here three questions that generated significant interaction:

- How broad is the class of matter models for which the conformal method for the stability of cosmological solutions can be applied (Juan Valiente Kroon)?
- Is there a fully relativistic, well-posed theory of fluid bodies containing several phases of matter that includes effects of surface tension between the different phases (Luciano Rezzolla and Matthias Hanauske)?

- Why are there regimes of the quark-gluon plasma where one would expect hydrodynamics to fail, due to a low number of particles, still very well described by the relativistic fluid equations (Michael Strickland)?

The workshop was thus successful in providing a platform for exchanging ideas between the theoretical physics and mathematics communities and to inspire future collaborations.

The organizers invited all participants to respond to a brief anonymous survey about the workshop, consisting of the following questions: Q1: Please rate the overall quality of the workshop. Q2: Please rate your satisfaction with the level of the math-physics interaction in the workshop. Q3: If we organize a similar workshop in the future, would you be interest in attending it? In addition, the respondents could enter general comments about the workshop. In total, ten participants answered the survey, and their responses are summarized below.

Q1: Please rate the overall quality of the workshop.

Possible answer	Number of responses
Excellent	3
Very good	7
Good	0
Fair	0
Poor	0

Q2: Please rate your satisfaction with the level of the math-physics interaction in the workshop.

Possible answer	Number of responses
Excellent	4
Very good	2
Good	3
Fair	1
Poor	0

Q3: If we organize a similar workshop in the future, would you be interest in attending it?

Possible answer	Number of responses
Yes	10
No	0

In additions, respondents provided the following comments:

> I may be good to have one or more moderated panel discussions in addition to the talks
>
> Online workshops are not the optimal solution but I was surprised of how engaging this one was.

I liked the broad range of topics. In view of this maybe some of the talks should have been a little more introductory in order to be able to follow topics that are quite remote from one's own. Another idea would be dedicated discussion sessions on specific topics.

In-person would obviously work much better for such a workshop but under the circumstances it was fine.

Encouraging more socializing, even over Zoom would be great.

A more concrete common theme would help to align talks and discussions more (including introductory talks to bridge the gap between math and physics); sometimes motivations in physics talks did not come across enough for me (probably because I haven't heard these things before)

Mini-Workshop: Relativistic Fluids at the Intersection of Mathematics and Physics (online meeting)

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Abstracts

Numerical evolution of the axisymmetric Einstein-Vlasov system

Oliver Rinne
(joint work with Ellery Ames and Håkan Andréasson)

Numerical simulations of collisionless matter are challenging due to the number of phase-space dimensions. In order to alleviate this problem, we perform an axisymmetric reduction of the Einstein-Vlasov equations based on the (2+1)+1 formalism, including rotation [1]. The numerical implementation is based on a constrained (mixed hyperbolic-elliptic) evolution scheme. The collisionless matter is treated using a particle-in-cell method. We tune smooth one-parameter families of initial data to the threshold of black hole formation and observe type I critical behaviour, in particular power-law scaling of the lifetime of the near-critical solution. The qualitative behaviour close to the critical point is found to depend on the sign of the binding energy: in the case of positive binding energy, marginally supercritical evolutions perform a series of damped oscillations before forming a black hole, whereas in the case of negative binding energy they collapse immediately after the initial expansion phase. Further problems we intend to investigate using our code are the stability (under time evolution) of stationary rotating axisymmetric solutions [2] and a re-assessment of cosmic censorship in gravitational collapse [3].

References

[1] E. Ames, H. Andréasson and O. Rinne, *Dynamics of gravitational collapse in the axisymmetric Einstein-Vlasov system*, arXiv:2010.15771 (2020).
[2] E. Ames, H. Andréasson and A. Logg, *On axisymmetric and stationary solutions of the self-gravitating Vlasov system*, Class. Quantum Grav. **33** (2016), 155008.
[3] S. L. Shapiro and S. A. Teukolsky, *Formation of naked singularities: The violation of cosmic censorship*, Phys. Rev. Lett. **66** (1991), 994–997.

Microscopic foundations of relativistic kinetic plasma theory

Michael K.-H. Kiessling

It is argued that the relativistic Vlasov–Maxwell equations of the kinetic theory of plasma approximately describe a relativistic system of N charged point particles interacting with the electromagnetic Maxwell fields in a Bopp–Landé–Thomas–Podolsky (BLTP) vacuum, provided the microscopic dynamics lasts long enough. The purpose of this talk is not to supply an entirely rigorous vindication, but to lay down a conceptual road map for the microscopic foundations of the kinetic theory of special-relativistic plasma, and to emphasize that a rigorous derivation seems feasible. Rather than working with a BBGKY-type hierarchy of n-point marginal probability measures, the approach proposed in this paper works with the distributional PDE of the actual empirical 1-point measure, which involves

the actual empirical 2-point measure in a convolution term. The approximation of the empirical 1-point measure by a continuum density, and of the empirical 2-point measure by a (tensor) product of this continuum density with itself, yields a finite-N Vlasov-like set of kinetic equations which includes radiation-reaction and nontrivial finite-N corrections to the Vlasov–Maxwell–BLTP model. The finite-N corrections formally vanish in a mathematical scaling limit $N \to \infty$ in which charges $\propto 1/\sqrt{N}$. The radiation-reaction term vanishes in this limit, too. The subsequent formal limit sending Bopp's parameter $\varkappa \to \infty$ yields the Vlasov–Maxwell model.

Acknowledgement

The results reported in this talk have been obtained in various collaborations, with A. Shadi Tahvildar-Zadeh, Yves Elskens, and Holly Carley. Thanks are extended to all these; also to the organizers of this exciting workshop: Shabnam Beheshti, Marcelo Disconzi, Vu Hoang; and to the helpful staff at MFO.

References

[CK] Carley, H. K., and Kiessling, M. K.-H., *A study of the radiation-reaction on a point charge which moves along a constant applied electric field in an electromagnetic Bopp–Landé–Thomas–Podolsky vacuum*, in preparation (2021).

[EK] Elskens, Y., and Kiessling, M. K-H., *Microscopic foundations of kinetic plasma theory: The relativistic Vlasov–Maxwell equations and their radiation-reaction-corrected generalization*, J. Stat. Phys. **180**:749–772 (2020).

[K] Kiessling, M.K.-H., *Force on a point charge source of the classical electromagnetic field*, Phys. Rev. D **100**, 065012(19) (2019); *Errata*, Phys. Rev. D **101**:109901(E) (2020).

[KTZ] Kiessling, M.K.-H., and Tahvildar-Zadeh, A. S., *Bopp-Landé-Thomas-Podolsky electrodynamics as initial value problem*, in preparation (2021).

Relativistic hydrodynamics in the quark-gluon plasma

Michael Strickland

In the first part of my talk, I will review the application of relativistic hydrodynamics to the physics of the quark-gluon plasma created in ultrarelativistic heavy-ion collisions. I will present some of the phenomenological successes of hydrodynamical modeling and then turn to the challenges that must be faced going forward. In the second part of my talk, I will address the question of how is it possible for dissipative hydrodynamics to be applicable on such a short time scale. In the highest energy heavy-ion collisions, it is estimated that this time scale is on the order of $10^{-}24$ seconds after the nuclei pass through one another. I will discuss our current understanding of this astonishing finding in the context of non-equilibrium dynamical attractors in relativistic kinetic theory and dissipative hydrodynamical models.

Conformal methods for the Einstein-Euler and Einstein-Vlasov systems

Juan A. Valiente Kroon

In this talk I discuss how methods based on Friedrich's conformal Einstein field equations can be used to obtain stability results for the de Sitter spacetime in the case that the matter content is given by: (i) a perfect fluid with the equation of state of radiation [1]; (ii) a Vlasov model consisting of massless particles [2]; (iii) a collection of self-gravitating dust balls in an expanding Universe with positive Cosmological constant [3]. In all three cases the energy-momentum tensor is tracefree and, thus, the matter equations have good conformal transformation properties. The conformal approach advocated in this talk has the advantage of rendering global existence and stability results which require only of general properties of symmetric hyperbolic systems —in particular, Cauchy stability. A key limitation at the moment is that this approach is restricted to matter models with a tracefree energy-momentum tensor (so that the energy-momentum equation is conformally invariant). However, the analysis of the Einstein-dust system on which the analysis of self-gravitating dust balls is based gives the tantalising hope that these methods can be extended to more general classes of matter.

Details on the use of conformal methods in General Relativity can be found in the monograph [4].

References

[1] C. Lübbe & J.A. Valiente Kroon, *A conformal approach for the analysis of the non-linear stability of pure radiation cosmologies*, Ann. Phys. **328**, 1 (2013).

[2] J. Joudioux, M. Thaller & J.A. Valiente Kroon, *The conformal Einstein field equations with massless Vlasov matter*, In `arXiv arXiv:1906.10777[gr-qc]`. To appear in Ann. Inst. Fourier.

[3] S. Beheshti, M. Normann & J.A. Valiente Kroon, *Self-gravitating dust balls in an expanding universe*. In preparation.

[4] J.A. Valiente Kroon, *Conformal Methods in General Relativity* . Monographs in Mathematical Physics. Cambridge University Press (2016).

Classical and Quantum Laws of Motion for Singularities of Spacetime

Shadi Tahvildar-Zadeh

(joint work with Michael Kiessling, Matthias Lienert, Annegret Burtscher, et al.)

In this talk I report on recent developments towards a relativistic quantum-mechanical theory of motion for a fixed, finite number of electrons, photons, and their anti-particles. I will briefly explain the necessary conditions under which world-lines of charged particles can be identified with time-like singularities of spacetime and/or classical fields permeating the spacetime, and show examples of classical as well as quantum theories of motion for them when these conditions are satisfied. I will then show how one can define a quantum-mechanical wave function for a single photon, and use that to obtain a Lorenz-covariant system of multi-time wave equations for an interacting two-body system in one space dimension, comprised of one electron and one photon. I will demonstrate that the corresponding initial-boundary-value problem is well-posed, and that both electron and photon trajectories exist globally for typical initial particle positions. I will conclude by presenting preliminary results of numerical experiments that illustrate Compton scattering in this context. This talk is a summary of joint work with Michael Kiessling, Matthias Lienert, Annegret Burtscher, and others.

References

[1] A. Burtscher, M. Kiessling, & A. S. Tahvildar-Zadeh, "Weak second Bianchi identity for spacetimes with time-like singularities," (in preparation.)

[2] M. Kiessling, M. Lienert, & A. S. Tahvildar-Zadeh, "A Lorentz-Covariant Interacting Photon-Electron System in One Space Dimension," *Lett. Math. Phys.* **110** 3153–3195 (2020) [arXiv:1906.03632].

[3] M. Kiessling & A. S. Tahvildar-Zadeh, "The Dirac point electron in zero-gravity Kerr–Newman spacetime," *Journal of Mathematical Physics* **56** 042303 (2015) [arXiv:1410.0419].

[4] M. Kiessling & A. S. Tahvildar-Zadeh, "A novel quantum-mechanical interpretation of the Dirac equation," *Journal of Physics A* **49** 135301 (2016) [arXiv:1411.2296].

[5] M. Kiessling & A. S. Tahvildar-Zadeh, "On the Quantum Mechanics of a Single Photon," *Jour. Math Phys.* **59**, 112302 (2018).[arXiv:1801.00268]

[6] A. S. Tahvildar-Zadeh, "On a zero-gravity limit of Kerr–Newman spacetimes and their electromagnetic fields," *Journal of Mathematical Physics.* **56** 042501 (2015) [arXiv:1410.0416].

Dynamical relativistic liquid bodies: local-in-time existence and uniqueness

Todd Oliynyk

In this talk, I will discuss a new approach to establishing the well-posedness of the relativistic Euler equations for liquid bodies in vacuum. During the talk, I will focus on giving an overview of the main ideas and I will try to keep the talk as non-technical as possible. The complete well-posedness proof and all of the technical details can be found in the article [1]. It is also worth noting that related ideas for establishing a prioi estimates are given in [2].

The new approach is based on a wave formulation of the relativistic Euler equations that consists of a system of non-linear wave equations in divergence form together with a combination of acoustic and Dirichlet boundary conditions. The equations and boundary conditions of the wave formulation differs from the standard one by terms proportional to certain constraints, and one of the main technical problems to overcome is to show that these constraints propagate, which is necessary to ensure that solutions of the wave formulation determine solutions to the Euler equations with vacuum boundary conditions. During the talk, I will describe the derivation of the wave equation and boundary conditions, the origin of the constraints, and how one shows that the constraints propagate. Time permitting, I will also discuss how energy estimates can be obtained from this new formulation paying particular attention to the role of the acoustic boundary conditions.

References

[1] T.A. Oliynyk, *Dynamical relativistic liquid bodies*, preprint [arXiv:1907.08192].
[2] T.A. Oliynyk, *A priori estimates relativistic liquid bodies*, Bull. Sci. Math. **141** (2017), 105–222.

Relativistic hydrodynamics in strong gravitational fields

Luciano Rezzolla

Relativistic hydrodynamics is an incredibly successful approach to study a large variety of problems that range from relativistic collisions of heavy ions to the inspiral and merger of neutron stars. I have reviewed the basic mathematical and numerical frameworks for the solution of the equations of relativistic hydrodynamics in arbitrary spacetimes and discuss how the corresponding solutions can be used to answer fundamental questions in physics and astrophysics.

Given the broad range of expertise in the workshop's participants, the presentation was organised in two distinct parts. The first one provided a brief overview of the mathematical tools and approaches developed to solve the equation relativistic hydrodynamics and magnetohydrodynamics (MHD) in general-relativistic regimes. The second one, on the other hand, concentrated on specific applications of fully general-relativistic hydrodynamics – such as the merger of two neutron stars – or of general-relativistic MHD – such as the problem of disk accretion onto a rapidly rotating black hole.

In both examples it was highlighted the ability of these approaches not only to reproduce the results of the observations, but also the considerable predictive power of some of the simulations performed. Finally, the prospects and the areas of future development – both at the mathematical and at the numerical level – have been illustrated and discussed.

References

[1] Baiotti, L., & Rezzolla, L. 2017, RPPh, 80, 096901
[2] Event Horizon Telescope Collaboration, Astrophys. J. Lett. 875, L1 (2019);

Deconstructing General-Relativistic Viscous Fluid Dynamics

Jorge Noronha

With the dawn of the multi-messenger astronomy era marked by the detection of a binary neutron star merger, it became imperative to understand how extremely dense fluids behave under very strong gravitational fields. In this talk I will critically review the foundations of relativistic viscous fluid dynamics and its formulation in curved spacetime. I will present the first set of fluid dynamic equations [1] that satisfies all of the following properties: (a) the system when coupled to Einstein's equations is causal and strongly hyperbolic (the initial value problem is well-posed); (b) equilibrium states are stable; (c) all leading dissipative contributions are present, i.e., shear viscosity, bulk viscosity, and thermal conductivity;

(d) effects from non-zero baryon number are included; (e) entropy production is non-negative in the regime of validity of the theory. The properties above hold in the nonlinear regime without any simplifying symmetry assumptions and are mathematically rigorously established. This is achieved using a new formulation of relativistic fluid dynamics [2, 3] containing only the hydrodynamic variables and their first-order derivatives. The framework presented here provides the starting point for systematic investigations of general-relativistic viscous phenomena in neutron star mergers.

References

[1] F. S. Bemfica, M. M. Disconzi, and J. Noronha, *General-Relativistic Viscous Fluid Dynamics*, arXiv:2009.11388 [gr-qc], 2020.
[2] F. S. Bemfica, M. M. Disconzi, and J. Noronha, *Causality and existence of solutions of relativistic viscous fluid dynamics with gravity*, Phys. Rev. D **98** (2018), 104064.
[3] F. S. Bemfica, M. M. Disconzi, and J. Noronha, *Nonlinear Causality of General First-Order Relativistic Viscous Hydrodynamics*, Phys. Rev. D **100** (2019), 104020.

Singularity formation for perfect fluids with linear equation of state

Annegret Burtscher

The Penrose singularity theorem relates the presence of a black hole to the existence of a trapped surface [7]. A trapped surface is a closed spacelike two-surface with the property that its surface area is decreasing in the direction of both future-directed null normals. In what follows we provide a brief overview of the current mathematical research on showing that such trapped surfaces can form during evolution from initial data that do not already contain trapped surfaces. We focus here on spacetimes with matter, in particular, on perfect fluids and elaborate on some open questions in this direction. The vacuum case studied by Christodoulou [5] and subsequently by Klainerman, Luk and Rodnianski [6] has been surveyed by Bieri [2]. Some discussion related to the scalar-field model studied by Christodoulou and the collisionless gas model studied by Andréasson, Kunze and Rein as well as a further discussion of the perfect fluid results can be found in [3].

Singularity formation for matter models has been studied in the spherically symmetric situation. The principal result that an accumulation of matter leads to black holes via Penrose's singularity theorem is already due to Schoen and Yau [12] but it took several more years for the mathematical observation of dynamical trapped surface formation, starting with an extensive investigation of the scalar field model by Christodoulou in the 1990s. For Einstein–Euler equations describing compressible fluids with a linear equation of state this involves the analysis of a system of two balance laws for the fluid coupled to two integral equations describing the geometry. Using generalized Eddington–Finkelstein coordinates and a generalized Glimm scheme LeFloch and the author have established a local existence result for solutions of bounded variation, including an estimate of the existence time and growth of parameters involved. Initial data have been used that are large pertubations of static solutions, and it was shown that the set of untrapped initial data leading to trapped surfaces during the evolution is nonempty [4]. This result has been shown in a rather restricted setting, and several improvements would be desirable, related to:

(1) The local existence proof is based on the theory of conservation laws (and thus a compactness argument) and does not imply *uniqueness*, certainly a major shortcoming.
(2) The assumption of spherical *symmetry* should be dropped. This property has been imposed for all matter models studied so far, but could already not be used in the vacuum case [2, 5, 6] (due to Birkhoff's Theorem).
(3) The *regularity* obtained is too low to actually apply the singularity theorem. It should either be improved or the singularity theorems strengthened. Some result related to an improvement of the regularity by adjusting the coordinates used are due to the work of Reintjes and Temple [8, 9, 10].
(4) The trapping result has been shown for linear equations of state with small sound speeds. A generalization to other *equations of state* is highly desirable. This requires a thorough understanding and analysis of suitable initial data (see also next point).
(5) The initial data used are large perturbations of static solutions localized in a shell around the center. Andersson and the author [1] have further analyzed the asymptotic behavior of static solutions with the aim to extend the set of admissible *initial data*, in part achieved by the author in [3]. While global existence and uniqueness of solutions to the Tolman–Oppenheimer–Volkhoff equation for reasonable equations of state was shown by Rendall and Schmidt [11] (and spherical symmetry is expected by the fluid ball conjecture/theorem), further analysis of the qualitative and global behavior of static solutions for general equations of state is still needed.

Ideally, of course, one would like to be able to tell from properties of the initial data (and equation of state) alone whether trapped surfaces and black holes will ever form – or not. And those initial data should not just be some sort of very special large localized perturbations of static spherically symmetric solutions but indeed very arbitrary and physically relevant initial data.

References

[1] L. Andersson and A.Y. Burtscher, *On the asymptotic behavior of static perfect fluids*, Ann. Henri Poincaré **20** (2019), no. 3, 813–857.

[2] L. Bieri, *Black hole formation and stability: a mathematical investigation*, Bull. Amer. Math. Soc. (N.S.) **55** (2018), no. 1, 1–30.

[3] A.Y. Burtscher, *Initial data and black holes for matter models*, in "Hyperbolic Problems: Theory, Numerics, Applications", by A. Bressan, M. Lewicka, D. Wang, Y. Zheng (Eds.), AIMS Ser. Appl. Math., vol. 10, 336–345, Am. Inst. Math. Sci. (AIMS), Springfield, MO, 2020.

[4] A.Y. Burtscher and P.G. LeFloch, *The formation of trapped surfaces in spherically-symmetric Einstein–Euler spacetimes with bounded variation*, J. Math. Pures Appl. **102** (2014), no. 6, 1164–1217.

[5] D. Christodoulou, *The formation of black holes in general relativity*, EMS Monographs in Mathematics, European Mathematical Society (EMS), Zürich, 2009, x+589 pp.

[6] S. Klainerman, J. Luk, Jonathan and I. Rodnianski, *A fully anisotropic mechanism for formation of trapped surfaces in vacuum*, Invent. Math. **198** (2014), no. 1, 1–26.
[7] R. Penrose, *Gravitational collapse and space-time singularities*, Phys. Rev. Lett. **14** (1965), 57–59.
[8] M. Reintjes, *Spacetime is locally inertial at points of general relativistic shock wave interaction between shocks from different characteristic families*, Adv. Theor. Math. Phys. **21** (2017), no. 6, 1525–1612.
[9] M. Reintjes and B. Temple, *No regularity singularities exist at points of general relativistic shock wave interaction between shocks from different characteristic families*, Proc. A. **471** (2015), no. 2177, 20140834, 17 pp.
[10] M. Reintjes and B. Temple, *Shock wave interactions and the Riemann-flat condition: the geometry behind metric smoothing and the existence of locally inertial frames in general relativity*, Arch. Ration. Mech. Anal. **235** (2020), no. 3, 1873–1904.
[11] A.D. Rendall and B.G. Schmidt, *Existence and properties of spherically symmetric static fluid bodies with a given equation of state*, Classical Quantum Gravity **8** (1991), no. 5, 985–1000.
[12] R. Schoen and S.T. Yau, *The existence of a black hole due to condensation of matter*, Comm. Math. Phys. **90** (1983), no. 4, 575–579.

Properties of hypermassive hybrid stars from binary compact star mergers

Matthias Hanauske

With the first detection of gravitational waves from a binary system of neutron stars GW170817, a new window was opened to study the properties of relativistic fluids at and above the nuclear-saturation density. Reaching densities a few times that of nuclear matter and temperatures up to 100 MeV, such mergers also represent potential sites for a phase transition (PT) from confined hadronic matter to deconfined quark matter. If the remnant of a binary compact star merger does not immediately collapse into a black hole, a hypermassive/supramassive compact star is created. Hypermassive/supramassive hybrid stars (HMHS,SMHS) are extreme astrophysical objects and in contrast to their purely hadronic counterparts (hypermassive/supramassive neutron stars (HMNS,SMNS)), these highly differentially rotating objects contain deconfined strange quark matter in their slowly rotating inner region. HMHS and HMNS are both metastable configurations and can survive only shortly after the merger, before collapsing to rotating Kerr black holes, whereas SMHS and SMNS end up in a stable final configuration.

The gravitational wave signatures of the production of quark matter, both during the inspiral [1], merger and postmerger phase of a compact star merger had been addressed in this talk. The evolution of the density and temperature distributions and the rotational properties inside the produced HMHS were visualised by using fully general-relativistic hydrodynamic simulations [2, 3, 4, 5]. Depending on the properties of the PT, a HMHS/SMHS can be created promptly after the merger or during the post-merger evolution [6]. During the collapse of a HMHS to a Kerr Black the color degrees of freedom of the pure quark core gets macroscopically confined by the formation of the event horizon [7, 8].

References

[1] G. Montana, L. Tolós, L., M. Hanauske, and L. Rezzolla, *Constraining twin stars with GW170817*, PRD **99(10)** (2019), 103009.

[2] M. Hanauske, K. Takami, L. Bovard, L. Rezzolla, J.A. Font, F. Galeazzi, and H. Stöcker, *Rotational properties of hypermassive neutron stars from binary mergers*, PRD **96(4)** (2017), 043004.

[3] M.G. Alford, L. Bovard, M. Hanauske, L. Rezzolla, and K. Schwenzer, *Viscous dissipation and heat conduction in binary neutron-star mergers*, PRL **120(4)** (2018), 041101.

[4] M. Hanauske, et.al., *Neutron star mergers: Probing the EoS of hot, dense matter by gravitational waves*, Particles **2(1)** (2020), 44–56.

[5] M. Hanauske, et.al., *Detecting the Hadron-Quark Phase Transition with Gravitational Waves*, Universe **5(6)** (2019), 156.

[6] L.R. Weih, M. Hanauske and L. Rezzolla, *Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Mergers*, PRL **124(17)** (2020), 171103.

[7] E. Most, L. Papenfort, V. Dexheimer, M. Hanauske, S. Schramm, H. Stöcker, and L. Rezzolla, *Signatures of quark-hadron phase transitions in general-relativistic neutron-star mergers*, PRL **122(6)** (2019), 061101.

[8] E. Most, L. Papenfort, V. Dexheimer, M. Hanauske, H. Stöcker, and L. Rezzolla, *On the deconfinement phase transition in neutron-star mergers*, The European Physical Journal A **56(2)** (2020), 1-11.

Stability of steady states in kinetic theory - Overview and selected results

Jérémie Joudioux

The purpose of this note is to give an overview of existing mathematical results of the stability of steady states for self-gravitating systems of collisionless particles. The bibliography is not exhaustive but provides references which we believe are important to illustrate the research in the field. Unless explicitly mentioned, all the results stated here are for massive particles. The interested reader must refer to the living review by Andréasson [3].

Popular models of self-gravitating matter interacting solely through gravity are given by the Vlasov-Poisson system in the classical case or the Einstein-Vlasov system in the relativistic case. In these models, the matter is described by a density function defined on phase space, coupled either to a Newtonian potential satisfying a Poisson equation or a Lorentzian metric satisfying Einstein's equations. The matter fields of these systems admit static compactly supported solutions that model astrophysical objects such as clouds of gas, or stars. Several important conjectures for the stability of these physical objects were formulated in the '60s. Antonov [7] stated in his seminal work that static spherically-symmetric solutions of the Vlasov-Poisson system were stable provided that the number of particles is a decreasing function of their total energy. This work was later revisited and extended by Ipser-Thorne to the relativistic case [17]. Antonov used the so-called Casimir-energy method to obtain his results. Simultaneously, Zel'dovich and Podurets [29] stated that relativistic spherically symmetric steady states of "high temperature" (modeled by a Maxwellian) are unstable.

A key method in the stability study is the Casimir-energy method, see [15, Section 2] for a comprehensive introduction. Consider a differential system given by a Hamiltonian, admitting a collection of constants of motions, the so-called Casimir functionals. If an equilibrium is a critical point of a combination of the Hamiltonian and a constant of motions, and if the second variation of this combination defines a positive quadratic form, then, under some assumptions, the equilibrium is orbitally (or Lyapunov) stable. If a Hamiltonian formulation for the Vlasov-Poisson system is a classical result, such a formulation was derived for the Einstein-Vlasov system in the '90s in [18]. Note that a Lagrangian formulation of the system was recently obtained in [2].

The dynamical stability of the vacuum state of the Vlasov-Poisson system was proven in [8]. Shortly after, [9] provided the proof of the classification of static spherically symmetric steady states of the Vlasov-Poisson system: the matter density can be expressed as a function of the total energy and the angular momentum of the particles. This statement is known as Jeans' theorem. The first use of the Casimir-energy method to prove that the stability of these steady states is done in [23]. The most complete proof of the nonlinear stability under the criteria introduced by Antonov [7] was obtained by Lemou-Méhats-Raphael [19]. They prove that steady states which satisfy Antonov's criterium and which do not depend on angular momentum are orbitally stable. Techniques of the proof involve the characterization of the stable steady states as a fixed point of a transformation decreasing the Hamiltonian, coercivity estimates for the second variation of energy, and a compactness argument. Spherically symmetric steady states which depend on angular momentum may be unstable under some conditions [10]. Axially symmetric stationary steady states have been studied numerically, see for instance [27, 1], and constructed [24], but the problem of their stability remains largely open.

Amongst the steady states, the vacuum states play a particular role. The stability of the vacuum solution to the Einstein-Vlasov system with vanishing cosmological constant under spherically symmetric perturbations was proven by Rein-Rendall [22]. The first result for generic perturbations is the stability of de Sitter space by Ringström [25]. The instability of anti-de Sitter space under spherically symmetric perturbation as a solution of the massless Einstein-Vlasov system is obtained in [21]. The nonlinear dynamical stability of Minkowski space as a vacuum state of the Einstein-Vlasov system has only been recently obtained [11, 20]. Both approaches rely on a vector-field method for the transport equation, see for instance [12]. Initial data are restricted to compactly supported matter density and compactly-supported perturbations of the Schwarzschild metric far from the horizon. There exist a priori arbitrarily small steady states of the Einstein-Vlasov system. The reason why those are excluded by the dynamical nonlinear stability result [11, 20] should be investigated. The source of pointwise decay of [11, 20] is a weighted Sobolev inequality and requires high regularity in velocities. This is in strong contrast with the small-data global existence result [16] for the Vlasov-Poisson system.

The construction of spherically symmetric steady states in the form of those of Jeans' theorem was initiated in [22]. Counterexamples to Jeans' theorem have been constructed [26]. Wolansky attempted to use the Casimir-energy method [28], but the proof was unfortunately not correct [4]. Axisymmetric stationary solutions to the Einstein-Vlasov are obtained in [6, 5], and are investigated numerically in [1]. [14] provides the first linear (in)stability proof of relativistic static spherically symmetric solutions. The criterium for instability is the central redshift of the compact object, in strong contrast to the criterium in the classical case. The authors obtained more specifically a classical exponential trichotomy for admissible perturbations of the steady states, and provide the first proof of a turning point principle for those steady states. A detailed numerical analysis of the stability of spherical steady states is presented in [13]. The extension of [14] beyond spherical symmetry and linearization is a deeply challenging and wide-open problem.

References

[1] E. Ames, H. Andréasson, and A. Logg. On axisymmetric and stationary solutions of the self-gravitating Vlasov system. *Class. Q. Grav.*, 33(15):155008, 26, 2016.

[2] L. Andersson and M. Korzyński. Variational principle for the Einstein-Vlasov equations. *arXiv e-prints*, October 2019.

[3] H. Andréasson. The Einstein-Vlasov system/kinetic theory. *Living Rev. Relativ.*, 14:2011–14, 55, 2011.

[4] H. Andréasson and M. Kunze. Comments on the paper 'Static solutions of the Vlasov-Einstein system' by G. Wolansky. *Arch. Ration. Mech. Anal.*, 235(1):783–791, 2020.

[5] H. Andréasson, M. Kunze, and G. Rein. Existence of axially symmetric static solutions of the Einstein-Vlasov system. *Comm. Math. Phys.*, 308(1):23–47, 2011.

[6] H. Andréasson, M. Kunze, and G. Rein. Rotating, stationary, axially symmetric spacetimes with collisionless matter. *Comm. Math. Phys.*, 329(2):787–808, 2014.

[7] V. A. Antonov. Remarks on the problems of stability in stellar dynamics. *Soviet Astronom. AJ*, 4:859–867, 1960.

[8] C. Bardos and P. Degond. Global existence for the Vlasov-Poisson equation in 3 space variables with small initial data. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 2(2):101–118, 1985.

[9] J. Batt, W. Faltenbacher, and E. Horst. Stationary spherically symmetric models in stellar dynamics. *Arch. Rational Mech. Anal.*, 93(2):159–183, 1986.

[10] J. Binney and S. Tremaine. *Galactic Dynamics: Second Edition*. 2008.

[11] D. Fajman, J.Joudioux, and J. Smulevici. The Stability of the Minkowski space for the Einstein-Vlasov system. *arXiv e-prints*, July 2017. To appear in Analysis & PDE.

[12] D. Fajman, J. Joudioux, and J. Smulevici. A vector field method for relativistic transport equations with applications. *Anal. PDE*, 10(7):1539–1612, 2017.

[13] S Günther, J. Korner, T. Lebeda, B. Potzl, G. Rein, C. Straub, and J. Weber. A numerical stability analysis for the Einstein-Vlasov system. *arXiv e-prints*, September 2020.

[14] M. Hadžic, Z. Lin, and G. Rein. Stability and instability of self-gravitating relativistic matter distributions. *arXiv e-prints*, October 2018.

[15] D. D. Holm, J. E. Marsden, T. Ratiu, and A. Weinstein. Nonlinear stability of fluid and plasma equilibria. *Phys. Rep.*, 123(1-2):116, 1985.

[16] A. D. Ionescu, B. Pausader, X. Wang, and Klaus Widmayer. On the asymptotic behavior of solutions to the Vlasov-Poisson system. *arXiv e-prints*, May 2020.

[17] J. R. Ipser and K. S. Thorne. Relativistic, spherically symmetric star clusters. I. Stability theory for radial perturbations. *Astrophys. J.*, 154:251–270, 1968.

[18] H. E. Kandrup and E. O'Neill. Hamiltonian structure of the Vlasov-Einstein system for generic collisionless systems and the problem of stability. *Phys. Rev. D (3)*, 49(10):5115–5125, 1994.
[19] M. Lemou, F. Méhats, and P. Raphaël. Orbital stability of spherical galactic models. *Invent. Math.*, 187(1):145–194, 2012.
[20] H. Lindblad and M. Taylor. Global stability of Minkowski space for the Einstein-Vlasov system in the harmonic gauge. *Arch. Ration. Mech. Anal.*, 235(1):517–633, 2020.
[21] G. Moschidis. The characteristic initial-boundary value problem for the Einstein–massless Vlasov system in spherical symmetry. *arXiv e-prints*, December 2018.
[22] G. Rein and A. D. Rendall. Global existence of solutions of the spherically symmetric Vlasov-Einstein system with small initial data. *Comm. Math. Phys.*, 150(3):561–583, 1992.
[23] G. Rein. Non-linear stability for the Vlasov-Poisson system—the energy-Casimir method. *Math. Methods Appl. Sci.*, 17(14):1129–1140, 1994.
[24] G. Rein. Stationary and static stellar dynamic models with axial symmetry. *Nonlinear Anal.*, 41(3-4, Ser. A: Theory Methods):313–344, 2000.
[25] H. Ringström. *On the topology and future stability of the universe*. Oxford Mathematical Monographs. Oxford University Press, Oxford, 2013.
[26] J. Schaeffer. A class of counterexamples to Jeans' theorem for the Vlasov-Einstein system. *Comm. Math. Phys.*, 204(2):313–327, 1999.
[27] S. L. Shapiro and S. A. Teukolsky. Equilibrium Stellar Systems with Spindle Singularities. *Astrophysical Journal*, 388:287, April 1992.
[28] G. Wolansky. Static solutions of the Vlasov-Einstein system. *Arch. Ration. Mech. Anal.*, 156(3):205–230, 2001.
[29] Ya. B. Zel'dovich and M. A. Podurets. The Evolution of a System of Gravitationally Interacting Point Masses. *Soviet Astronomy*, 9:742, April 1966.

The large charge expansion for relativistic and non-relativistic models

Susanne Reffert

It has become clear in recent years that working in sectors of large global charge of strongly coupled and otherwise inaccessible CFTs leads to important simplifications. It is indeed possible to formulate an effective action in which the large charge appears as a control parameter. In this talk, I will explain the basic notions of the large-charge expansion using the simple example of the 2+1-dimensional O(2) model at the Wilson-Fisher point. The same approach can also be applied to non-relativistic systems.

Derivation of Mean-Field equations from many-body QM

Michael Hott

In this talk, we will see some main ideas on how to derive dynamical effective theories starting with the many-body Schroedinger equation. We will study two approaches in particular: One coming from unitary transformations applied in the second-quantization formalism, found, e.g., in the celebrated works of Erdos, Schlein and Yau, and one coming from deviation estimates as introduced by Peter Pickl.

Starting with the N-body Hamiltonian

$$H_N := \sum_{i=1}^{N}(-\Delta_{x_i})^\sigma + \frac{\lambda}{N-1}\sum_{i<j}\frac{1}{|x_i - x_j|} \tag{1}$$

acting on the bosonic Hilbert space $L^2_s(\mathbb{R}^{3N})$, we are interested in an effective description for the Cauchy problem

$$\begin{cases} i\partial_t \Psi_{N,t} &= H_N \Psi_{N,t}\,, \\ \Psi_{N,t}\Big|_{t=0} &= \Psi_{N,0}\,. \end{cases} \tag{2}$$

For that, we assume a factorized initial state $\Psi_{N,0} = \phi_0^{\otimes N}$ that can be found in a weakly interacting Bose-Einstein condensate. The power $\sigma \in \{\frac{1}{2}; 1\}$ distinguishes the semi-relativistic ($\sigma = \frac{1}{2}$) and the non-relativistic ($\sigma = 1$) case. It can be proved that (2) converges to the Hartree equation

$$\begin{cases} i\partial_t \phi_t &= (-\Delta)^\sigma \phi_t \,+\, \lambda\frac{1}{|\cdot|} * |\phi_t|^2\phi_t\,, \\ \phi_t\big|_{t=0} &= \phi_0 \end{cases} \tag{3}$$

in the sense that

$$\lim_{N\to\infty} |\langle \Psi_{N,t}, A\Psi_{N,t}\rangle - \langle \phi_t^{\otimes k}, A\phi_t^{\otimes k}\rangle| \,=\, 0 \tag{4}$$

for a class of observables $A : L^2_s(\mathbb{R}^{3k}) \to L^2_s(\mathbb{R}^{3k})$. This convergence has been shown with a rate of convergence and for a class containing the kinetic energy operator $(-\Delta)^\sigma$. In the semi-relativistic case $\sigma = \frac{1}{2}$, convergence has been shown after choosing a regularization of H_N by replacing $\frac{1}{|\cdot|} \to \frac{1}{|\cdot|+\alpha_N}$ for a null sequence $(\alpha_N)_{N\in\mathbb{N}}$. This is due to the fact that H_N ceases to be bounded from below when $\lambda < 0$ is chosen such that kinetic energy and potential energy balance each other. In this case, there is no unique or physical self-adjoint extension of H_N, as pointed out during the conference, by Michael Kiessling.

More recent works study the evoluation beyond the Hartree approximation, including propagation of soundwaves with Bogoliubov dispersion. In the static, Fermionic case, the highest order correction that has been established, is the one coming from diagonalizing pair excitations. This leads to the Random Phase Approximation. Moreover, instead of considering the Mean-Field scaling for the potential, scalings where the interaction becomes more δ-like or comparable to the kinetic energy are studied.

References

[1] I. Anapolitanos, M. Hott, *A simple proof of convergence to the Hartree dynamics in Sobolev trace norms.* Journal of Mathematical Physics **57.12** (2016), 122108.

[2] I. Anapolitanos, M. Hott, D. Hundertmark, *Derivation of the Hartree equation for compound Bose gases in the mean field limit.* Reviews in Mathematical Physics **29.07** (2017), 1750022.

[3] Z. Ammari, F. Nier, *Mean field limit for bosons and infinite dimensional phase-space analysis.* Annales Henri Poincaré. Vol. **9**. No. **8** (2008), 1503–1574.

[4] V. Bach, S. Breteaux, T. Chen, J. Fröhlich, I. M. Sigal, *The time-dependent hartree-fock-bogoliubov equations for bosons.* arXiv preprint arXiv:1602.05171, 1–36 (2016).

[5] N. Benedikter, M. Porta, B. Schlein, *Effective evolution equations from quantum dynamics* (Vol. **7**). Berlin: Springer (2016).

[6] N. Benedikter, P. T. Nam, M. Porta, B. Schlein, R. Seiringer, *Optimal upper bound for the correlation energy of a Fermi gas in the mean-field regime.* Communications in Mathematical Physics, **374(3)** (2020), 2097-2150.

[7] N. Benedikter, P. T. Nam, M. Porta, B. Schlein, R. Seiringer, *Correlation Energy of a Weakly Interacting Fermi Gas.* arXiv preprint arXiv:2005.08933 (2020).

[8] L. Boßmann, S. Petrat, P. Pickl, A. Soffer, *Beyond Bogoliubov Dynamics.* arXiv preprint arXiv:1912.11004 (2019).

[9] T. Chen, C. Hainzl, N. Pavlović, R. Seiringer, *Unconditional Uniqueness for the Cubic Gross-Pitaevskii Hierarchy via Quantum de Finetti.* Communications on Pure and Applied Mathematics, **68(10)** (2015), 1845-1884.

[10] T. Chen, N. Pavlović, *The quintic NLS as the mean field limit of a Boson gas with three-body interactions.* J. Funct. Anal. **260(4)** (2011), 959–997.

[11] T. Chen, N. Pavlović, *Derivation of the cubic NLS and Gross-Pitaevskii hierarchy from manybody dynamics in d= 3 based on spacetime norms.* Annales Henri Poincaré. Vol. **15**, No. **3** (2014), pp. 543-588. Springer Basel.

[12] X. Chen, J. Holmer, *On the rigorous derivation of the 2D cubic nonlinear Schrödinger equation from 3D quantum many-body dynamics.* Archive for Rational Mechanics and Analysis (2013), **210(3)**, 909-954.

[13] X. Chen, J. Holmer, *Focusing quantum many-body dynamics: the rigorous derivation of the 1D focusing cubic nonlinear Schrödinger equation.* Archive for Rational Mechanics and Analysis, **221(2)** (2016), 631-676.

[14] X. Chen, J. Holmer, *The derivation of the* $\mathbb{T}^3$ *energy-critical NLS from quantum many-body dynamics.* Inventiones mathematicae, **217(2)** (2019), 433-547.

[15] J. J. W. Chong, M. G. Grillakis, M. Machedon, Z. Zhao, *Global estimates for the Hartree-Fock-Bogoliubov equations.* arXiv preprint arXiv:2008.01753 (2020).

[16] L. Erdös, B. Schlein, H.-T. Yau, *Derivation of the Gross-Pitaevskii hierarchy for the dynamics of Bose-Einstein condensate*, Comm. Pure Appl. Math. **59** (12) (2006), 1659-1741.

[17] L. Erdös, B. Schlein, H.-T. Yau, *Derivation of the cubic non-linear Schrödinger equation from quantum dynamics of many-body systems*, Invent. Math. **167** (2007), 515-614.

[18] L. Erdös, B. Schlein, H.-T. Yau, *Rigorous derivation of the Gross-Pitaevskii equation with a large interaction potential.* J. Amer. Math. Soc. **22** (4) (2009), 1099-1156 .

[19] L. Erdös, B. Schlein, H.-T. Yau, *Derivation of the Gross-Pitaevskii equation for the dynamics of Bose-Einstein condensates.* Ann. of Math. (2) **172** (1) (2010), 291–370.

[20] M. Grillakis, M. Machedon, *Pair excitations and the mean field approximation of interacting bosons, I.* Communications in Mathematical Physics **324.2** (2013): 601-636.

[21] M. Grillakis, M. Machedon, *Pair excitations and the mean field approximation of interacting Bosons, II.* Communications in Partial Differential Equations **42.1** (2017), 24-67.

[22] M. Hott, *Convergence rate towards the fractional Hartree-equation with singular potentials in higher Sobolev norms.* arXiv preprint arXiv:1805.01807 (2018).

[23] E. H. Lieb, H.-T. Yau, *The Chandrasekhar theory of stellar collapse as the limit of quantum mechanics.* Communications in Mathematical Physics **112(1)** (1987), 147–174.

[24] D. Mitrouskas, S. Petrat, P. Pickl, *Bogoliubov corrections and trace norm convergence for the Hartree dynamics.* Reviews in Mathematical Physics, **31(08)**, 1950024 (2019).

[25] A. Michelangeli, B. Schlein, *Dynamical collapse of boson stars.* Communications in Mathematical Physics **311.3** (2012), 645–687.

[26] P.T. Nam, M. Napiórkowski, *Bogoliubov correction to the mean-field dynamics of interacting bosons.*Advances in Theoretical and Mathematical Physics **21**, no. **3** (2017), 683–738.

[27] P.T. Nam, M. Napiórkowski, *A note on the validity of Bogoliubov correction to mean-field dynamics.* Journal de Mathématiques Pures et Appliquées, **108(5)** (2017), 662–688.

[28] P.T. Nam, M. Napiórkowski, *Norm approximation for many-body quantum dynamics: focusing case in low dimensions.* Advances in Mathematics, **350** (2019), 547–587.

[29] P.T. Nam, M. Napiórkowski, J. Ricaud, A. Triay, *Optimal rate of condensation for trapped bosons in the Gross–Pitaevskii regime.* arXiv preprint arXiv:2001.04364 (2020).

[30] P.T. Nam, R. Salzmann, *Derivation of 3D Energy-Critical Nonlinear Schrödinger Equation and Bogoliubov Excitations for Bose Gases.* Communications in Mathematical Physics **375 (1)** (2019), 1–77 .

[31] A. Knowles, P. Pickl, *Mean-Field Dynamics: Singular Potentials and Rate of Convergence.* Communications in Mathematical Physics. Volume **298**, Issue 1 (2010). 101–138.

[32] P. Pickl, *A simple derivation of mean field limits for quantum systems*, Lett. Math. Phys., **97 (2)** (2011), 151 – 164.

Reporter: Marvin R. Schulz

Participants

Dr. Shabnam Beheshti
School of Mathematical Sciences
Queen Mary University of London
Mile End Road
London E1 4NS
UNITED KINGDOM

Dr. Annegret Burtscher
Dept. of Mathematics
Radboud University Nijmegen
P.O. Box 9010
6500 GL Nijmegen
NETHERLANDS

Prof. Dr. Marcelo M. Disconzi
Department of Mathematics
Vanderbilt University
1326 Stevenson Center
Nashville TN 37240-0001
UNITED STATES

Dr. Matthias Hanauske
Institut für Theoretische Physik
Arbeitsgruppe Relativistische
Astrophysik
Max-von-Laue-Str. 1
60438 Frankfurt am Main
GERMANY

Dr. Vu Hoang
Department of Mathematics
The University of Texas at San Antonio
One UTSA Circle
San Antonio TX 78249-0664
UNITED STATES

Michael Hott
Department of Mathematics
The University of Texas at Austin
2515 Speedway, RLM 8.100
Austin TX 78712-1082
UNITED STATES

Dr. Jeremie Joudioux
MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14476 Golm
GERMANY

Prof. Dr. Michael Kiessling
Department of Mathematics
Rutgers, The State University of New
Jersey
110 Frelinghuysen Rd. (Hill Center)
Piscataway NJ 08854-8019
UNITED STATES

Prof. Dr. Jorge Noronha
Department of Physics
The University of Illinois
at Urbana-Champaign
1110 W Green St.
Urbana IL 61801-9013
UNITED STATES

Dr. Jacquelyn Noronha-Hostler
Department of Physics
The University of Illinois
at Urbana-Champaign
1110 W Green St.
Urbana IL 61801-9013
UNITED STATES

Prof. Dr. Todd Oliynyk
School of Mathematical Sciences
Monash University
Clayton Victoria 3800
AUSTRALIA

Prof. Dr. Susanne Reffert
Albert Einstein Institute for Theoretical Physics
Universität Bern
Sidlerstr. 5
3012 Bern
SWITZERLAND

Prof. Dr. Luciano Rezzolla
Institute for Theoretical Physics
Goethe-University
Max-von-Laue-Strasse 1
60438 Frankfurt am Main
GERMANY

Prof. Dr. Oliver Rinne
Hochschule für Technik und Wirtschaft Berlin
Fachbereich 4
Treskowallee 8
10318 Berlin
GERMANY

Marvin R. Schulz
Fakultät für Mathematik
Institut für Analysis
Karlsruher Institut für Technologie (KIT)
Englerstrasse 2
76131 Karlsruhe
GERMANY

Prof. Dr. Michael Strickland
Dept. of Physics
Kent State University
Smith Hall 206B
Kent, OH 44242-0001
UNITED STATES

Prof. Dr. A. Shadi Tahvildar-Zadeh
Department of Mathematics
Rutgers University
Hill Center, Busch Campus
110 Frelinghuysen Road
Piscataway, NJ 08854-8019
UNITED STATES

Dr. Juan A. Valiente Kroon
School of Mathematical Sciences
Queen Mary
University of London
Mile End Road
London E1 4NS
UNITED KINGDOM