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Mini-Workshop: Poisson and Poisson-type algebras

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Abstract. The first historical encounter with Poisson-type algebras is with Hamiltonian mechanics. With the abstraction of many notions in Physics, Hamiltonian systems were geometrized into manifolds that model the set of all possible configurations of the system, and the cotangent bundle of this manifold describes its phase space, which is endowed with a Poisson structure. Poisson brackets led to other algebraic structures, and the notion of Poisson-type algebra arose, including transposed Poisson algebras, Novikov-Poisson algebras, or commutative pre-Lie algebras, for example. These types of algebras have long gained popularity in the scientific world and are not only of their own interest to study, but are also an important tool for researching other mathematical and physical objects.

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Introduction by the Organizers

Poisson algebras emerged naturally in the framework of Hamiltonian mechanics and the field developed rapidly following the advent of mathematical physics. Nowadays Poisson algebras play a central role in a wide range of areas in mathematics and physics, such as Poisson manifolds, algebraic geometry, operads, quantization theory, quantum groups, classical and quantum mechanics. A variety of related algebraic structures, the so-called Poisson-type algebras, gained popularity in recent years: these include Novikov-Poisson algebras, commutative pre-Lie algebras or the recently introduced transposed Poisson algebras, to name but a few. The purpose of this meeting was to bring together experts in various fields revolving around Poisson algebras, to discuss new approaches to open problems in the area and to initiate new research work. Discussions and talks were focused into the following directions:

- **Poisson-type structures:** Several talks considered the different Poisson-type structures existing in the literature. Guo's talk reported on the study of operads encoding algebraic structures with replicated copies of operations satisfying various compatibility conditions among these copies and explained the relations of the compatibility conditions with Koszul duality and Manin products. Admissible operads of various types have been discussed in the talk of Dzhumadil'daev. Burde presented various results and open conjectures concerning the existence of post-Lie algebra structures on a pair of Lie algebras over a fixed vectors space, emphasising the cases where either of the two Lie algebras is abelian, nilpotent, solvable, simple, semisimple, reductive, complete or perfect. The talk of Zusmanovich, considered the problem of whether an extension of the contact bracket (a natural generalisation of Poisson bracket) on the tensor product from the bracket on the factors is possible. Transposed Poisson algebras have been recently introduced as a dual notion of Poisson algebras, by exchanging the roles of the two multiplications in the Leibniz rule defining a Poisson algebra. The mini-workshop featured several talks which discussed the rich structure of transposed Poisson algebras. Bai presented a bialgebra theory for transposed Poisson algebras. Khrypchenko discussed transposed Poisson structures on Lie incidence algebras and Fernandez Ouaridi's talk focused on the simple transposed Poisson algebras.
- **Poisson algebras and superalgebras:** Another important part of the mini-workshop consisted of talks related to the study of certain specific classes of Poisson (super)algebras. Sierra reported on the study of the Poisson ideal structure of the symmetric algebras of the Virasoro algebra and the Witt algebra of algebraic vector fields on $\mathbb{C}^*$ and various other related Lie algebras. The talk of Yakimova highlighted the use of the symmetrisation map for obtaining various new explicit formulas for the generators of the Feigin-Frenkel center. Launois discussed algorithmic methods to study Poisson derivations of a semiclassical limit of a family of quantum second Weyl algebras. Agore presented certain universal objects for Poisson algebras and highlighted several applications of these constructions to the description of the automorphism group of a given Poisson algebra and to the classification of gradings by an abelian group. The talk of Siciliano gave an overview of the known results about solvable (truncated) symmetric Poisson algebras and their derived lengths as well as some open questions on these topics. The Gerstenhaber bracket on the Hochschild cohomology of a certain subalgebra of the Weyl algebra and its connection to the Virasoro Lie algebra have been highlighted by Lopes. Usefi's talk focused on the characterization of Lie superalgebras whose enveloping algebras satisfy a non-matrix polynomial identity.

Mini-Workshop: Poisson and Poisson-type algebras

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Abstracts

Universal constructions for Poisson algebras. Applications

Ana Agore
(joint work with Gigel Militaru)

We introduce and study some universal objects for Poisson algebras and highlight their main applications having as sourse of inspiration the previous work of Sweedler [15], Manin [11] and Tambara [13] for Hopf algebra (co)actions on associative algebras. From a categorical point of view, the existence of universal objects with a certain property, for a given category $\mathcal{C}$ can shed some light on the structure of the category $\mathcal{C}$ itself. In particular, the existence and description of universal objects (groups or "group like objects" such as Lie groups, algebraic groups, Hopf algebras, groupoids or quantum groupoids, etc.) which act or coact on a fixed object $\mathcal{O}$ in a certain category $\mathcal{C}$ has often various applications in many areas of mathematics. An elementary but illuminating example is the following: let $\mathcal{O}$ be a given object in a certain category $\mathcal{C}$ and consider the category $\mathrm{ActGr}_{\mathcal{O}}$ of all groups that *act* on $\mathcal{O}$, i.e. the objects in $\mathrm{ActGr}_{\mathcal{O}}$ are pairs (G, φ) consisting of a discrete group G and a morphism of groups $\varphi\colon G \to \mathrm{Aut}_{\mathcal{C}}(\mathcal{O})$, where $\mathrm{Aut}_{\mathcal{C}}(\mathcal{O})$ denotes the automorphisms group of the object $\mathcal{O}$ in $\mathcal{C}$. Then the category $\mathrm{ActGr}_{\mathcal{O}}$ has a final object, namely $\big(\mathrm{Aut}_{\mathcal{C}}(\mathcal{O}), \mathrm{Id}\big)$. Now, if we replace the discrete groups that act on the fixed object $\mathcal{O}$ in $\mathcal{C}$, by some other "groups like objects" from a certain more sophisticated category $\mathcal{D}$ (for instance, Lie groups, algebraic groups, Hopf algebras, etc.) which (co)act on $\mathcal{O}$ and if moreover we ask the (co)action to preserve the algebraic, differential or topological structures which might exist on $\mathcal{O}$, then things become very complicated. Indeed, the first obstacle we encounter is the fact that $\mathrm{Aut}_{\mathcal{C}}(\mathcal{O})$ might not be an object *inside* the category $\mathcal{D}$ anymore. However, even in this complicated situation, it is possible for the above result to remain valid but, however, the construction of the final object will be far more complicated. Furthermore, it is to expect that, if it exists, this final object will contain important information on the entire automorphisms group of the object $\mathcal{O}$. To the best of our knowledge, the first result in this direction was proved by Sweelder [15, Theorem 7.0.4] in the case where $\mathcal{C}$ is the category of associative algebras and $\mathcal{D}$ is the category of bialgebras: if A is a fixed associative algebra then the category of all bialgebras H that *act* on A (i.e. A is an H-module algebra) has a final object $\mathrm{M}(A)$, called by Sweedler the *universal measuring bialgebra* of A. The dual situation of *coactions* of bialgebras on a fixed algebra A, was first considered in the case when $\mathcal{C}$ is the category of graded algebras by Manin [11] for reasons related to non-commutative geometry, and in the general case by Tambara [13]. If A is an associative algebra, necessarily finite dimensional this time around, then the category of all bialgebras that coact on A (i.e. A is an H-comodule algebra) has an initial object $a(A)$. Furthermore, the usual automorphisms group $\mathrm{Aut}_{\mathrm{Alg}}(A)$ of A is indeed recovered as the group of all invertible group-like elements of the finite dual $a(A)^{\circ}$ [12, Theorem 2.1] and

$a(A)^{\circ}$ is just Sweedler's final object in the category of all bialgebras that act on A [13, Remark 1.3]. The two results above remains valid if we take the category of Hopf algebras instead of bialgebras: in particular, the Hopf envelope of $a(A)$, denoted by $\underline{\mathrm{aut}}(A)$, is called in non-commutative geometry the *non-commutative symmetry group* of A [14] and its description is a very complicated matter. The existence and description of these universal (co)acting bialgebras/Hopf algebras has been considered recently in [1] in the context of Ω-algebras. The duality between Sweedler's and Manin-Tambara's objects has been extended to this general setting and necessary and sufficient conditions for the existence of the universal coacting bialgebras/Hopf algebras, which roughly explains the need for assuming finite-dimensionality in Manin-Tambara's constructions, are given. Furthermore, universal coacting objects for Poisson algebras have also been considered in [2] but from a different perspective, leading to entirely different constructions. We only point out that in [2], the universal coacting object considered is actually a Poisson Hopf algebra. For more background on the importance and the applications of universal bialgebras/Hopf algebras in various areas of mathematics, we refer to [3, 5, 6, 7, 9, 10].

The key object of our work, namely the *universal algebra* of two Poisson algebras P and Q, is a pair $\big(\mathcal{P}(P,\, Q),\, \eta\big)$ consisting of a commutative algebra $A := \mathcal{P}(P,\, Q)$ and a Poisson algebra homomorphism $\eta\colon Q \to P \otimes \mathcal{P}(P,\, Q)$ satisfying a certain universal property. If P is finite-dimensional, then the universal algebra $\mathcal{P}(P,\, Q)$ of P and Q exists and we provide an explicit construction of it. This result has two important consequences: for a fixed Poisson P-module U there exists a canonical functor $U \otimes -\colon {}_A\mathcal{M} \to {}_Q\mathcal{PM}$ from the category of usual A-modules (i.e. representations of the associative algebra A) to the category of Poisson Q-modules (i.e. Poisson representations of Q) and moreover, if U is finite-dimensional this functor has a left adjoint. Secondly, if V is an A-module, then there exists a canonical functor $- \otimes V\colon {}_P\mathcal{PM} \to {}_Q\mathcal{PM}$ connecting the categories of Poisson modules over P and Q and, furthermore, if V is finite-dimensional then the aforementioned functor has a left adjoint. These results provide answers, at the level of Poisson algebras, to the following general problem:

If $\mathcal{O}_1$ and $\mathcal{O}_2$ are two mathematical objects (not necessary in the same category), is it possible to construct "canonical functors" between the representation categories $\mathrm{Rep}(\mathcal{O}_1)$ *and* $\mathrm{Rep}(\mathcal{O}_2)$ *of the two objects?*

Three more applications of our constructions are considered. For a Poisson algebra P of dimension n, we denote $\mathcal{P}(P) := \mathcal{P}(P,\, P)$ and we construct $\mathcal{P}(P)$ as the quotient of the polynomial algebra $k[X_{ij} \,|\, i, j = 1, \cdots, n]$ through an ideal generated by $2n^3$ non-homogeneous polynomials of degree ≤ 2. $\mathcal{P}(P)$ has a canonical bialgebra structure and, moreover, $\mathcal{P}(P)$ is the *initial object* of the category $\mathrm{CoactBialg}_P$ of all commutative bialgebras coacting on P and, for this reason, we call it the *universal coacting bialgebra* of P. As in the case of Lie [4] or associative algebras [12], the universal bialgebra $\mathcal{P}(P)$ has two important applications, which provide the theoretical answer for Poisson algebras, of the following open questions:

(1) *Describe explicitly the automorphisms group of a given Poisson algebra P;*
(2) *Describe and classify all G-gradings on P for a given abelian group G.*

More precisely, we prove that there exists an isomorphism of groups between the group of all Poisson automorphisms of P and the group of all invertible group-like elements of the finite dual $\mathcal{P}(P)^{\circ}$. The second application is given is the following: for an abelian group G, all G-gradings on a finite dimensional Poisson algebra P are described and classified in terms of bialgebra homomorphisms $\mathcal{P}(P) \to k[G]$. By taking Takeuchi's commutative Hopf envelope of $\mathcal{P}(P)$, we obtain that the category $\mathrm{CoactHopf}_P$ of all commutative Hopf algebras coacting on P has an initial object $\mathcal{H}(P)$. It is reasonable to hope that $\mathcal{H}(P)$ will play the role of a non-commutative symmetry group of the Poisson algebra P. This expectation is based on the fact that the concept of Poisson H-comodule algebra which we are dealing with, is the algebraic counterpart of the action of an algebraic groups on an affine Poisson variety [8, Example 2.20].

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A bialgebra theory for transposed Poisson algebras

Chengming Bai
(joint work with Guilai Liu)

Transposed Poisson algebras are the dual notion of Poisson algebras by exchanging the roles of two binary operations in the Leibniz rule defining the Poisson algebras. The approach for Poisson bialgebras characterized by Manin triples with respect to the invariant bilinear forms on both the commutative associative algebras and the Lie algebras is not available for giving a bialgebra theory for transposed Poisson algebras. Alternatively, we consider Manin triples with respect to the commutative 2-cocycles on the Lie algebras instead. Explicitly, we first introduce the notion of anti-pre-Lie bialgebras as the equivalent structure of Manin triples of Lie algebras with respect to the commutative 2-cocycles since anti-pre-Lie algebras are regarded as the underlying algebraic structures of Lie algebras with nondegenerate commutative 2-cocycles. Then we introduce the notion of anti-pre-Lie-Poisson bialgebras, characterized by Manin triples of transposed Poisson algebras with respect to the bilinear forms which are invariant on the commutative associative algebras and commutative 2-cocycles on the Lie algebras, giving a bialgebra theory for transposed Poisson algebras. They are commutative and cocommutative infinitesimal bialgebras and anti-pre-Lie bialgebras satisfying certain compatible conditions. Finally the coboundary cases and the related structures such as analogues of the classical Yang-Baxter equation and $\mathcal{O}$-operators are studied.

Pre-Lie and Post-Lie Algebra Structures

Dietrich Burde
(joint work with Karel Dekimpe)

Post-Lie algebras and post-Lie algebra structures are an important generalization of left-symmetric algebras, also called pre-Lie algebras, and left-symmetric algebra structures on Lie algebras, which arise in many areas of algebra and geometry [1], such as left-invariant affine structures on Lie groups, affine crystallographic groups, simply transitive affine actions on Lie groups, convex homogeneous cones, faithful linear representations of Lie algebras, operad theory and several other areas.

In this talk we present several results and open conjectures concerning the existence of post-Lie algebra structures on a pair of Lie algebras $(\mathfrak{g}, \mathfrak{n})$ over a fixed vector space V. In particular we are interested in cases, where either $\mathfrak{g}$ or $\mathfrak{n}$ has one of the following properties: it is abelian, nilpotent, solvable, simple, semisimple, reductive, complete or perfect. We may assume here that the algebras, if possible, do not belong to several classes simultaneously, to avoid an unnecessary overlap.

Over the last years we have obtained already several results on the existence of post-Lie algebra structures, see [2, 3, 4, 5]. The methods use the theory of Rota-Baxter operators, decomposition theory, cohomology theory and several other tools.

In a recent paper [6], we proved the following rigidity result.

Theorem. *Let* $(\mathfrak{g}, \mathfrak{n})$ *be a pair of Lie algebras, where* $\mathfrak{g}$ *is semisimple and* $\mathfrak{n}$ *is arbitrary. Suppose that* $(\mathfrak{g}, \mathfrak{n})$ *admits a post-Lie algebra structure. Then* $\mathfrak{n}$ *is isomorphic to* $\mathfrak{g}$.

We will give a sketch of the proof. It uses several non-trivial results about decompositions of reductive Lie groups and Lie algebras by Onishchik. The result shows that the condition of $\mathfrak{g}$ being semisimple is very strong. A similar result for $\mathfrak{n}$ being semisimple does not hold.

Proposition. *Let* $\mathfrak{n}$ *be a semisimple Lie algebra. Then there exists a solvable non-nilpotent Lie algebra* $\mathfrak{g}$*, such that* $(\mathfrak{g}, \mathfrak{n})$ *is a pair of Lie algebras admitting a post-Lie algebra structure.*

Currently we are working on the generalization of the results for the semisimple case to the case of perfect Lie algebras. Here a Lie algebra L is called *perfect*, if $[L, L] = L$. A typical example of a perfect Lie algebra, which is not semisimple, is the Lie algebra $\mathfrak{sl}_n(\mathbb{C}) \ltimes V(n)$, where $V(n)$ is the natural irreducible representation of $\mathfrak{sl}_n(\mathbb{C})$. We have the following conjecture.

Conjecture. *Let* $(\mathfrak{g}, \mathfrak{n})$ *be a pair of Lie algebras, where* $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{C}) \ltimes V(n)$ *and* $\mathfrak{n}$ *is nilpotent. Then there is no post-Lie algebra structure on* $(\mathfrak{g}, \mathfrak{n})$.

This would be the first step to a more general conjecture, which is as follows.

Conjecture. *Let* $(\mathfrak{g}, \mathfrak{n})$ *be a pair of Lie algebras, where* $\mathfrak{g}$ *is perfect non-semisimple, and* $\mathfrak{n}$ *is nilpotent. Then there is no post-Lie algebra structure on* $(\mathfrak{g}, \mathfrak{n})$.

We have proved the second conjecture for the case, where $\mathfrak{g}$ is semisimple. However, the case of perfect Lie algebras is much more complicated.

A further question is about the case where $\mathfrak{g}$ is perfect and $\mathfrak{n}$ is simple or semisimple. Using a classification of perfect Lie algebras of dimension $n \leq 8$ over $\mathbb{C}$, we proved the following result.

Proposition. *Let* $(\mathfrak{g}, \mathfrak{n})$ *be a pair of Lie algebras, where* $\mathfrak{g}$ *is perfect non-semisimple, and* $\mathfrak{n} = \mathfrak{sl}_3(\mathbb{C})$*. Then there is no post-Lie algebra structure on* $(\mathfrak{g}, \mathfrak{n})$.

We conjecture that the same conclusion holds for any semisimple Lie algebra $\mathfrak{n}$, and not only for $\mathfrak{sl}_3(\mathbb{C})$.

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Lie-Jordan elements and q-admissible operads

Askar Dzhumadil'daev

Notations: K be a field of characteristic $p \geq 0$; $\mathcal{M}ag = K[x_1, x_2, \dots]$ free Magma, i.e., an algebra of non-commutative non-associative polynomials with generators $x_1, x_2, \dots$; $\mathcal{M}ag(n)$ is multi-linear part of free magma of degree $n+1$; $\mathbb{T}_k$ and $\mathbb{T}_k^{np}$ are sets of planar and non-planar binary trees with $k+1$ leaves, $\mathbb{T} = \cup_{k\geq 0}\mathbb{T}_k$ and $\mathbb{T}^{np} = \cup_{k\geq 0}\mathbb{T}_k^{np}$; $\mathcal{T}, \mathcal{T}^{np}, \mathcal{T}_k, \mathcal{T}_k^{np}$ are linear spans of $\mathbb{T}, \mathbb{T}^{np}, \mathbb{T}_k, \mathbb{T}_k^{np}$; $\mathbb{T}_k^{q}$ set of planar binary trees whose i-th inner vertex is colored by $q_i \in K$, $1 \leq i \leq k$, where $\mathbf{q} = (q_1, q_2, \dots,)$; similar notations for $\mathbb{T}_k^{\mathbf{q}}, \mathbb{T}_k^{np,\mathbf{q}}, \mathbb{T}^{\mathbf{q}}, \mathbb{T}^{np,\mathbf{q}}$, etc. Then $\tau_q(\omega)(a,b) = \omega(a,b) + q\omega(b,a)$ corresponds to q-commutator of algebra (A, ω), We endow $\mathcal{T}$ by structure of algebra under bucket product $st = s \vee t$.

We introduce equivalency relation on non-planar trees: two such trees are equivalent if one can be obtained from the second one by permuting of branches. Take as representative of a non-planar tree a tree such that for any inner vertex its left sub-branch is no more than right-sub-branch. We identify an equivalency class with a representative and we can assume that $\mathbb{T}_n^{np} \in \mathbb{T}_n$.

Space of operations on free magma has a base generated by planar binary rooted trees. To construct elements of free magma one should label its leaves by elements of magma and inner vertices by multiplication of magma and correspond to each inner vertex products of its sons. Then an element obtained at a root is a product of leaf elements. For a tree, t denotes by $|t|$ its number of inner vertices. We obtain commutative monoid denoted G_1, under composition

$$\tau_q\tau_{q'} = (1 + q\, q')\tau_{\frac{q+q'}{1+qq'}}$$

It has unit τ_0 and the group of invertible elements is isomorphic to $\{q \in K | q^2 \neq 1\}$.

There are two kinds of extensions of coloring maps for any n. First way, all inner vertices are changed by q-commutator and $\tau_q : \mathbb{T}_n \to \mathbb{T}_n^{(q)}$ is defined in natural way. The second way, we numerate somehow inner vertices and each i-th inner vertex has its own color, say q_i, and in this case we have to consider $\tau_{\mathbf{q}} : \mathbb{T}_n \to \mathbb{T}_n^{\mathbf{q}}$, where $\mathbf{q} = (q_1, q_2, \dots)$ is a sequence of colors. It is clear that the first case is a particular case of the second one: take $\mathbf{q} = (q, q, \dots)$. Let

$$G_n = G_1 \times \cdots \times G_1 \cong K^n$$

be commutative monoid generated by coloring maps $\tau_{\mathbf{q}}$, where $\mathbf{q} = (q_1, q_2, \dots)$.

Let

$$M_n = \{\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n) | \alpha_i^2 = 1\}$$

and

$$M = \cup_{n\geq 1} M_n.$$

Say that $\boldsymbol{\alpha} \in M_n$ has (l_+, l_-)-*type* if numbers of components of $\boldsymbol{\alpha}$ are equal to $+1$ and -1 are l_+ and l_- respectively. Then $l_+ + l_- = n$. Let

$$M_n^{(l_+,l_-)} = \{\boldsymbol{\alpha} \in M_n \mid type(\boldsymbol{\alpha}) = (l_+, l_-)\}.$$

For $\boldsymbol{\alpha} \in M_n, t \in \mathbb{T}_n$ set

$$t_{\boldsymbol{\alpha}} = \tau_{\boldsymbol{\alpha}} t.$$

We call $t_{\boldsymbol{\alpha}}$ *Lie-Jordan element of type* $\boldsymbol{\alpha}$. Let $\mathcal{T}_n^{np,(l_+,l_-)}$ be subspace of $\mathcal{T}_n^{np}$ generated by trees $t_{\boldsymbol{\alpha}}$, where $\boldsymbol{\alpha} \in M_n^{(l_+,l_-)}$. For $X \in \mathcal{T}_n$ say that X is *Lie-Jordan element of* $\pm$*-type* (l_+, l_-), or shortly homogeneous LJ-element, if X is a linear combination of elements constructed by trees $t \in \mathcal{T}_n^{np,(l_+,l_-)}$.

Well known that space of $(n+1)$-ary operations on $\mathcal{M}ag$ can be generated by planar rooted trees with n inner vertices and $\mathbb{T}_n$ can be selected as a base.

Theorem. *Set of Lie-Jordan elements $t_{\boldsymbol{\alpha}}$, where $\boldsymbol{\alpha} \in M_n$ and $t \in \mathbb{T}_n^{np}$, forms base of the space of n-ary operations on $\mathcal{M}ag$. In particular, dim $\mathcal{M}ag(n) = 2^n(2n-1)!!$.*

Theorem. *The following conditions are equivalent*

- *X is Lie-Jordan element of type $\boldsymbol{\alpha} \in M_n$.*
- $\tau_{\boldsymbol{\alpha}} X = 2^n X$
- *$\tau_{\boldsymbol{\beta}} X = 0$, for any $\boldsymbol{\beta} \in M_n, \boldsymbol{\beta} \neq \boldsymbol{\alpha}$.*

In particular, the following conditions are equivalent

- *$X \in \mathcal{M}ag(n)$ is Lie element*
- *$\tau_{\boldsymbol{\alpha}} X = 2^n X$, where $\boldsymbol{\alpha} = (-1, -1, \ldots, -1) \in M_n$*
- *$\tau_{\boldsymbol{\beta}} X = 0$, for any $\boldsymbol{\beta} \neq (-1, -1, \ldots, -1).,\boldsymbol{\beta} \in M_n$*

and the following conditions are also equivalent

- *$X \in \mathcal{M}ag(n)$ is Jordan element*
- *$\tau_{\boldsymbol{\alpha}} X = 2^n X$, where $\boldsymbol{\alpha} = (1, 1, \ldots, 1) \in M_n$*
- *$\tau_{\boldsymbol{\beta}} X = 0$, for any $\boldsymbol{\beta} \neq (1, 1, \ldots, 1), \boldsymbol{\beta} \in M_n$*

Let $\mathcal{V} = \langle f_1, \ldots, f_k \rangle$ be an operad of algebras generated by polynomial identities $f_1 = 0, \ldots, f_k = 0$. Call an algebra $A = (A, \cdot)$ *q-admissible $\mathcal{V}$-algebra* and denote by $\mathcal{V}Adm^{(q)}$ class of such algebras, if A under q-commutator $A^{(q)} = (A, \cdot_q)$ became $\mathcal{V}$-algebra.

Theorem. *Let $q^2 \neq 1$. Then $\mathcal{V}Adm^{(q)}$ forms a variety of algebras, namely, variety generated by polynomial identities $f_1^{(-q)} = 0, \ldots, f_k^{(-q)} = 0$. As categories, varieties $\mathcal{V}$ and $\mathcal{V}Adm^{(q)}$ are isomorphic. Dimensions sequence of multi-linear parts of $d_{\mathcal{V},n} = dim\, \mathcal{V}(n)$ are not changed,*

$$d_{\mathcal{V},n} = d_{\mathcal{V}^{(q)},n}.$$

If $f_i, 1 \leq i \leq s$, are homogeneous Lie-Jordan polynomials, then

$$\mathcal{V}^{(q)} = \langle f_1, \ldots, f_s, f_{s+1}^{(-q)}, \ldots, f_k^{(-q)} \rangle.$$

If all $f_i, 1 \leq i \leq k$, are homogeneous Lie-Jordan polynomials, then

$$\mathcal{V}^{(q)} = \mathcal{V}.$$

An algebra with the following polynomial identities is called reverse-associative, anti-reverse-associative, and weak Leibniz, respectively:

$$a(bc) = (cb)a,$$

$$(a(bc) = -(cb)a,$$

$$[a, b]c = 2(a(bc) - b(ac)), \quad a[b, c] = 2((ab)c - (ac)b)$$

Applications of our results for these classes of algebras are given below.

Results on reverse-associative and anti-reverse-associative operads

Theorem. *Reverse-associative and anti-reverse-associative operads have the following properties.*

- **(a)** *Operads $\mathcal{R}evas$ and $\mathcal{A}revas$ are Koszul.*
- **(b)** *Any anti-reverse-associative algebra is associative-admissible and Lie-admissible.*
- **(c)** $\mathcal{R}evas^! = \mathcal{A}revas$.
- **(d)** $\mathcal{R}evas = \langle \{t_1, [t_2, t_3]\}, \quad [t_1, \{t_2, t_3\}] \rangle$.
- **(e)** $\mathcal{A}revas = \langle [t_1, [t_2, t_3]], \quad \{t_1, \{t_2, t_3\}\} \rangle$.
- **(f)** *Plus-colored trees generate a base of free reverse-associative operad. In particular,*

$$\mathcal{R}evas(n) = \mathcal{C}om^+(n) \oplus \mathcal{C}om^-(n), \qquad n > 1.$$

- **(g)** *Minus-colored trees generate a base of free anti-reverse-associative operad. In particular,*

$$Arevas(n) = Com^{\pm}(n) \oplus Com^{\mp}(n), \qquad n > 1,$$

- **(h)** $dim\ \mathcal{A}revas(n) = dim\ \mathcal{R}evas(n) = 2(2n-1)!!, \qquad n > 1,$

$$dim\ \mathcal{A}revas(1) = dim\ \mathcal{R}evas(1) = 1.$$

- **(i)** $\mathcal{A}revas = \mathcal{C}omNil_2 \star \mathcal{A}comNil_2$, *where*

$$\mathcal{C}omNil_2 = \langle t_1 t_2 - t_2 t_1, (t_1 t_2) t_3 \rangle,$$

$$\mathcal{A}comNil_2 = \langle t_1 t_2 + t_2 t_1, (t_1 t_2) t_3 \rangle.$$

- **(j)** *Multipliaction table in free reverse-associative algebra $F_{revas}(X)$ generated by elements $X = \{x_1, x_2, \ldots, x_n\}$ can be defined by*

$$x_i x_j = x_i \bullet x_j + x_i \circ x_j,$$

$$x_i u = x_i \bullet u_+ + x_i \circ u_-,$$

$$u x_j = u_+ \bullet x_j + u_- \circ x_j,$$

$$uv = u_+ \bullet v_+ + u_- \circ v_-,$$

where $u, v \in F_{revas}(X)^2$, $1 \leq i, j \leq n$.

(k) *Multipliaction table in free anti-reverse-associative algebra $F_{arevas}(X)$ generated by elements $X = \{x_1, x_2, \ldots, x_n\}$ can be constructed as in reverse-associative case,*

$$x_i x_j = x_i \bullet x_j + x_i \circ x_j,$$

$$x_i u = x_i \bullet u_- + x_i \circ u_+,$$

$$u x_j = u_- \bullet x_j + u_+ \circ x_j,$$

$$uv = u_- \bullet v_- + u_+ \circ v_+,$$

for any $u, v \in F_{arevas}(X)^2$, $1 \le i, j \le n$.

Results on associative-admissible operad

Recall that Non-Anti-Commutative Lie operad Lie$^\flat$ is generated by Jacobi identity $jac = 0$, reverse-associative identity

$$revas(t_1, t_2, t_3) = t_1(t_2t_3) - (t_3t_2)t_1 = 0,$$

and the identity

$$t_1(t_2t_3 + t_3t_2) = 0.$$

Left-Leibniz and right-Leibniz operads are defined by identities

$$llei = 0 \text{ and } rlei = 0$$

respectively, where

$$llei = (t_1t_2)t_3 - t_1(t_2t_3) + t_2(t_1t_3),$$

$$rlei = t_1(t_2t_3) - (t_1t_2)t_3 + (t_1t_3)t_2.$$

So, two-sided Leibniz operad $\mathcal{L}ei$ is defined by left- and right-Leibniz identities

$$\mathcal{L}ei = \langle llei, rlei \rangle.$$

Theorem. *Associative-admissible operad has the following properties.*

(a) $\mathcal{L}ie^\flat = \mathcal{L}ei$.
(b) *dim* $\mathcal{L}ie^\flat(n) = (n-1)!$, *if* $n \neq 2$ *and* $= 2$, *if* $n = 2$.
(c) *Operads* $\mathcal{A}sAdm$ *and* $\mathcal{L}ie^\flat$ *are Koszul.*
(d) $\mathcal{A}sAdm^! = \mathcal{L}ie^\flat$.
(e) $\mathcal{A}sAdm = \mathcal{A}sCom \star \mathcal{A}com$.
(f) *Dimensions of multi-linear parts of associative-admissible operad $d_n = \dim \mathcal{A}sAdm(n)$ can be found by the following recurrence relations*

$$d_n = \sum_{k=1}^{n-1} k! F_{k+2} B_{n-1,k}(d_1, d_2, \ldots, d_{n-k}), \quad n > 1,$$

$$d_1 = 1,$$

where F_n are Fibonacci numbers and $B_{n,k}(x_1, \ldots, x_{n-k+1})$ are Bell polynomials.

Theorem. *Let p be prime. Dimensions $d_n = dim\ \mathcal{A}s\mathcal{A}dm(n)$ of multi-linear part of associative-admissible operad satisfy the following congruences*

$$d_{p-1} \equiv \begin{cases} 1(mod\, p), & if\ p \neq 3, \\ -1 & if\ p = 3, \end{cases}$$

$$d_p \equiv \begin{cases} 1(mod\, p), & if\ p \neq 2, \\ 0 & if\ p = 2, \end{cases}$$

$$d_{p+1} \equiv 2(mod\, p),$$

$$d_{p+2} \equiv 10(mod\, p).$$

Results on weak Leibniz operad

Let us define left-weak Leibniz and right-weak-Leibniz polynomials by

$$lwlei = [t_1, t_2]t_3 - 2t_1(t_2t_3) + 2t_2(t_1t_3),$$

$$rwlei = t_1[t_2, t_3] - 2(t_1t_2)t_3 + 2(t_1t_3)t_2.$$

Let $\mathcal{L}wlei = \langle lwlei \rangle$ and $\mathcal{R}wlei = \langle rwlei \rangle$ are Left-weak-Leibniz and Right-weak-Leibniz operads, So, two-sided weak-Leibniz operad $\mathcal{W}lei$ is defined by

$$\mathcal{W}lei = \langle lwlei, rwlei \rangle.$$

Let I be some finite set of integers, $\varepsilon_i \in K$, for any $i \in I$. Let $L(I, \varepsilon)$ be infinite-dimensional algebra with base $\{e_i | i \in \mathbf{Z}\}$ and multiplication

$$e_i \cdot e_j = (i-j)e_{i+j} + \sum_{k \in I} \varepsilon_k e_{i+j+k}.$$

Theorem.

- $\mathrm{Wlei}^! = \mathrm{Wlei}$.
- *Weak Leibniz operad is not Koszul.*
- *Any weak Leibniz algebra is associative-admissible. Any weak Leibniz algebra is two-sided Alia, if $p \neq 3$. In particular, any weak Leibniz algebra is Lie-admissible, if $p \neq 3$.*
- *There exist weak Leibniz algebras that are not Leibniz.*
- *The algebra $L(I, \varepsilon)$ is a simple weak Leibniz algebra for any I and $\varepsilon_i \in K, i \in I$.*

Results on Associative-admissible and Lie-admissible operad

Let $\mathcal{A}sLie\mathcal{A}dm$ be operad representing associative-admissible and Lie-admissible algebras, i.e. algebras with the following identities

$$\{t_1,\{t_2,t_3\}\} = \{\{t_1,t_2\},t_3\},$$
$$[[t_1,t_2],t_3] + [[t_2,t_3],t_1] + [[t_1,t_2],t_3] = 0.$$

Let $\mathcal{A}s\mathcal{C}om$ be operad for associative and commutative algebras,

$$t_1(t_2t_3) = (t_1t_2)t_3, \quad t_1t_2 = t_2t_1.$$

Let

$$aslia' = rwlei - lwlei,$$

$aslia(t_1,t_2,t_3) = aslia'(t_1,t_2,t_3) + 2\langle t_3,t_1,t_2\rangle = [t_1,[t_2,t_3]] - 2(\langle t_2,t_1,t_3\rangle - \langle t_3,t_1,t_2\rangle)$.
In other words,

$$aslia' = aslia'(t_1,t_2,t_3) = [t_1,[t_2,t_3]] + 2(arevas(t_2,t_3,t_1) - arevas(t_3,t_2,t_1)) =$$
$$[t_1,[t_2,t_3]] + 2(t_2(t_3t_1) - t_3(t_2t_1) - (t_1t_2)t_3 + (t_1t_3)t_2).$$

Theorem. *The operad $\mathcal{A}sLie\mathcal{A}dm$ has the following properties.*

- *The operad $\mathcal{A}sLie\mathcal{A}dm$ is Koszul.*
- $\mathcal{A}sLie\mathcal{A}dm = \langle aslia\rangle$, *if* $p \neq 3$.
- $\mathcal{A}sLie\mathcal{A}dm = \mathcal{A}s\mathcal{C}om \star \mathcal{L}ie$.
- $\mathcal{A}sLie\mathcal{A}dm^! = \langle revas, lwlei \text{ or } rwlei\rangle$.
- $d_n^! = dim\ \mathcal{A}sLie\mathcal{A}dm^!(n) = (n-1)! + 1$.
- *Poincare series* $f^!_{\mathcal{A}sLie\mathcal{A}dm}(x) = \sum_{i\geq 1} d_i^! \frac{x^i}{i!} = -1 + e^x - x - \ln(1-x)$
- $d_n = dim\ \mathcal{A}sLie\mathcal{A}dm(n) = \sum_{k=1}^{n-1}(-1)^k\lambda_k B_{n-1,k}(d_1,d_2,\ldots,d_{n-k})$, *where*

$$\lambda_k = \sum_{s=1}^{k}(-1)^s s! B_{k,s}(1!+1, 2!+1, \ldots, i!+1, \ldots, (k-s+1)!+1).$$

Simple transposed Poisson algebras and Jordan superalgebras

Amir Fernández Ouaridi

Transposed Poisson algebras (TPAs, for short) were introduced as a dual class of the Poisson algebras in the sense that the roles of the two multiplications in the Leibniz rule are swapped [1]. Precisely, we have the identity

$$2x \circ \{y,z\} = \{x\circ y, z\} + \{y, x\circ z\}.$$

This identity can be realized as the left multiplication of the associative commutative algebra is a $\frac{1}{2}$-derivation of the Lie algebra. These derivations of Lie algebras are well-studied (for example, see [4]). The interest on this class has increased rapidly in the last four years (see [2] and the references therein). Some known facts about TPAs include the closure undertaking tensor products, the Koszul self-duality as an operad or the correspondence with weak Leibniz algebras by depolarization. TPAs coincide with commutative Gelfand-Dorfman algebras [1, 6]. In this talk, we will discuss about simple transposed Poisson algebra. For a further read on the topic of simple TPAs, we refer to the paper of the author [3].

Recall that an ideal of a Poisson-type algebra is a proper subspace such that it is simultaneously an ideal of both multiplications. Kantor [5] introduced an invertible way to construct a Jordan superalgebra from a Poisson algebra (the Kantor double), this construction preserves the simplicity in both directions, so a classification of the complex simple finite-dimensional Poisson algebras was obtained from the classification of simple Jordan superalgebras.

Our first approach to the problem of classifying simple TPAs took us to the study of the Kantor double of a TPA. It turns out that, as in the Poisson case, the Kantor double of a TPA is a Jordan superalgebra. Hence, TPAs are Jordan brackets. This motivates the following open question.

Question. *Characterize the subclass of Jordan brackets that arise from TPAs. Are these Jordan algebras special?*

The construction of simple TPA from simple Jordan superalgebras is partially possible. Indeed, we proved that a TPA is simple with perfect associative part if and only if its Kantor double is simple. Although we can not construct all the simple TPAs, a straightforward check of the inverse Kantor double of a complex simple finite-dimensional Jordan superalgebras shows that none of them produce non-trivial TPAs. In other words, there are no complex simple finite-dimensional TPAs with perfect associative part.

This result was improved, thanks to the next key result: over an arbitrary field and for any dimension, a TPA is simple if and only if its Lie bracket is simple. It is worth to point out that this result is also valid for the super case. The main idea to prove this fact is the introduction of the notion of a transposed quasi-ideal (see [3] for details).

As a consequence of the cited result, any complex simple finite-dimensional TPA is trivial. This is thanks to a result of Filippov, who showed that every simple complex finite-dimensional Lie algebra has trivial $\frac{1}{2}$-derivations [4]. However, there are simple finite-dimensional TPAs over fields of characteristic $p > 2$. This motivates the following question.

Question. *Classify the finite-dimensional TPAs.*

Another consequence is that any TPA with simple associative part has either abelian Lie part or simple Lie part. An example of a TPA with both multiplications being simple was presented by A. Dzhumadildayev on the field of formal series during the mini-workshop. This motives the following question:

Question. *Is there any finite-dimensional simple TPA with both multiplications being simple?*

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Compatible structures of operads by polarization, Koszul duality, and Manin products

Li Guo

(joint work with Xing Gao, Huhu Zhang)

Traditionally, a compatible structure is usually referring to a linearly compatible structure, where a vector space is equipped with two identical copies of operations in a given algebraic structure (Lie algebra, for example) so that the sums of these two copies of operations still yield the same algebraic structure. Together with several other algebraic compatible structures, they have been widely studied in mathematics and mathematical physics.

The first instance of linearly compatible structures appeared in the pioneering work [9] of Magri on bi-Hamiltonian systems, in which a Poisson algebra has two linearly compatible Poisson (Lie) brackets. Such a structure was abstracted to the notion of a bi-Hamiltonian algebra and was studied in the context of operads and Koszul duality [3]. Compatible Lie algebras have been studied in connection with integrable systems, classical Yang-Baxter equation, loop algebras and elliptic theta functions [5, 6, 7, 10]. In [2], quantum bi-Hamiltonian systems were built on linearly compatible associative algebras [11, 12].

Other algebraic structures with multiple copies of operations related by various compatibility conditions have appeared in recent studies in broad areas.

For example, a multiple pre-Lie algebra emerged in the remarkable work of Bruned, Hairer and Zambotti [1, 4] on algebraic renormalization of regularity structures. Matching Rota-Baxter algebras appeared in the algebraic study of Voterra integral equations [8, 15]. These structures can be broadly grouped into linearly compatible, matching, and totally compatible structures.

General studied of such structures using operads have been carried out with various restrictions [13, 14]. This talk presents some recent progress aiming at giving a unified approach to these various structures that can be applied to an arbitrary operad. We first introduce the notion of polarization in operads, leading to the notion of linearly compatible operads. Refining the polarization by the process of taking foliation, we obtain a general notion of matching type operads including those that have appeared. When we make all matching compatibilities of a given operad equal, we obtain the totally compatible operad of this operad.

For unary/binary quadratic operads, we prove that the linear compatibility and the total compatibility are in Koszul dual to each other, and there is a Koszul self-duality among the matching compatibilities. For binary quadratic operads, these three compatible operads can also be obtained by Manin products. For a finitely generated binary quadratic Koszul operad, we prove that the three types of compatible operads are also Koszul.

Back to the Poisson algebra origin of this study, natural questions arise, such as

(1) Study the transpose bi-Hamiltonian algebra?
(2) What should be the algebraic structure when the Poisson bracket in a Poisson algebra is replaced by any of the compatible Lie algebras, with the bi-Hamiltonian system or a bi-Hamiltonian algebra as a special case?
(3) The same question can be asked for the transposed Poisson algebra.

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Transposed Poisson structures on Lie incidence algebras

Mykola Khrypchenko
(joint work with Ivan Kaygorodov)

A *transposed Poisson algebra* [1] is a triple $(\mathfrak{L}, \cdot, [\cdot,\cdot])$ consisting of a vector space $\mathfrak{L}$ with two bilinear operations $\cdot$ and $[\cdot,\cdot]$, such that

(1) $(\mathfrak{L}, \cdot)$ is a commutative associative algebra;
(2) $(\mathfrak{L}, [\cdot,\cdot])$ is a Lie algebra;
(3) the *"transposed" Leibniz law* holds: $2z \cdot [x, y] = [z \cdot x, y] + [x, z \cdot y]$ for all $x, y, z \in \mathfrak{L}$.

A *transposed Poisson algebra structure* on a Lie algebra $(\mathfrak{L}, [\cdot,\cdot])$ is a (commutative associative) multiplication $\cdot$ on $\mathfrak{L}$ such that $(\mathfrak{L}, \cdot, [\cdot,\cdot])$ is a transposed Poisson algebra.

A transposed Poisson structure $\cdot$ on $(\mathfrak{L}, [\cdot,\cdot])$ is said to be *of Poisson type* if it is at the same time a usual Poisson structure on $\mathfrak{L}$. It was proved in [1] that this happens if and only if

$$x \cdot [y, z] = [x \cdot y, z] = 0$$

for all $x, y, z \in \mathfrak{L}$. Another class of transposed Poisson structures that can be defined on any Lie algebra $(\mathfrak{L}, [\cdot,\cdot])$ is as follows. Fix $c \in Z([\mathfrak{L}, \mathfrak{L}])$ and consider the following *mutation* of the product $[\cdot,\cdot]$:

$$a \cdot_c b = [[a, c], b] = [a, [c, b]].$$

Then $\cdot_c$ is a transposed Poisson structure on $\mathfrak{L}$ called *mutational*.

Given two binary operations $\cdot_1$ and $\cdot_2$ on a vector space V, their *sum* $*$ is defined by

$$a * b = a \cdot_1 b + a \cdot_2 b.$$

We say that $\cdot_1$ and $\cdot_2$ are *orthogonal*, if

$$V \cdot_1 V \subseteq \mathrm{Ann}(V, \cdot_2) \text{ and } V \cdot_2 V \subseteq \mathrm{Ann}(V, \cdot_1).$$

In this case $*$ defined above is called the *orthogonal* sum of $\cdot_1$ and $\cdot_2$.

Clearly, the sum $*$ of two transposed Poisson structures $\cdot_1$ and $\cdot_2$ on $(\mathfrak{L}, [\cdot,\cdot])$ is commutative and satisfies the transposed Leibniz law. If $\cdot_1$ and $\cdot_2$ are orthogonal, then $*$ is also associative, so we get the following.

Proposition. *The orthogonal sum of two transposed Poisson structures on a Lie algebra $\mathfrak{L}$ is a transposed Poisson structure on $\mathfrak{L}$.*

Observe that any mutational transposed Poisson structure on a Lie algebra $\mathfrak{L}$ is orthogonal to any transposed Poisson structure of Poisson type on $\mathfrak{L}$.

Corollary. *The (orthogonal) sum of a mutational transposed Poisson structure on $\mathfrak{L}$ and a transposed Poisson structure of Poisson type on $\mathfrak{L}$ is a transposed Poisson structure on $\mathfrak{L}$.*

Let X be a finite poset and K a field. Recall that the *incidence algebra* $I(X,K)$ of X over K (see [3]) is the associative K-algebra with basis $\{e_{xy} \mid x \le y\}$ and multiplication is given by

$$e_{xy}e_{uv} = \begin{cases} e_{xv}, & y = u, \\ 0, & y \ne u, \end{cases}$$

for all $x \le y$ and $u \le v$ in X. Given $f \in I(X,K)$, we write $f = \sum_{x \le y} f(x,y)e_{xy}$, where $f(x,y) \in K$. Let us denote $e_x := e_{xx}$, and for arbitrary $Y \subseteq X$ put $e_Y := \sum_{y \in Y} e_y$. Then e_Y is an idempotent and $e_Y e_Z = e_{Y \cap Z}$, in particular, $e_x e_y = 0$ for $x \ne y$. Notice that $\delta := e_X$ is the identity element of $I(X,K)$.

We consider $I(X,K)$ as a Lie algebra under the commutator product $[f,g] = fg - gf$. If X is connected, then one can easily prove that

$$Z(I(X,K)) = \langle \delta \rangle \text{ and } [I(X,K), I(X,K)] = \langle e_{xy} \mid x < y \rangle.$$

Moreover,

$$Z([I(X,K), I(X,K)]) = \langle e_{xy} \mid \mathrm{Min}(X) \ni x < y \in \mathrm{Max}(X) \rangle.$$

Diagonal elements of $I(X,K)$ are $f \in I(X,K)$ with $f(x,y) = 0$ for $x \ne y$. They form a commutative subalgebra $D(X,K)$ of $I(X,K)$ with basis $\{e_x \mid x \in X\}$. As a vector space,

$$I(X,K) = D(X,K) \oplus [I(X,K), I(X,K)].$$

Thus, each $f \in I(X,K)$ is uniquely written as $f = f_D + f_J$ with $f_D \in D(X,K)$ and $f_J \in [I(X,K), I(X,K)]$. Observe that $Z(I(X,K)) \subseteq D(X,K)$.

In this talk, we describe transposed Poisson structures on $(I(X,K), [\cdot,\cdot])$, where X is a finite connected poset and K is a field of characteristic zero. It is obvious that any transposed Poisson structure of Poisson type on $I(X,K)$ is of the form

$$e_x \cdot e_y = \mu(x,y)\delta$$

for some $\mu : X^2 \to K$ with $\mu(x,y) = \mu(y,x)$, where the associativity of the product is equivalent to

$$\mu(x,y) \sum_{v \in X} \mu(z,v) = \mu(y,z) \sum_{v \in X} \mu(x,v).$$

Observe that we write only non-trivial products here.

Each $\nu \in Z([I(X,K), I(X,K)])$ defines the mutational structure whose non-trivial products are:

$$e_x \cdot e_y = e_y \cdot e_x = \begin{cases} \nu(x,y)e_{xy}, & \mathrm{Min}(X) \ni x < y \in \mathrm{Max}(X), \\ -\sum_{x < v \in \mathrm{Max}(X)} \nu(x,v)e_{xv}, & x = y \in \mathrm{Min}(X), \\ -\sum_{\mathrm{Min}(X) \ni u < x} \nu(u,x)e_{ux}, & x = y \in \mathrm{Max}(X). \end{cases}$$

The definition of the third structure requires some preparation. We say that a pair (x,y) of elements of X is *extreme*, if $x < y$ is a maximal chain in X and there is no cycle in X containing x and y. Denote $X_e^2 = \{(x,y) \in X^2 \mid (x,y) \text{ is extreme}\}$. We set $\mathrm{sgn}_{u_0}(x,y) := 1$, if there is a path starting at u_0 and ending at (x,y).

Otherwise there is a path starting at u_0 and ending at (y, x), in which case set $\mathrm{sgn}_{u_0}(x, y) := -1$. Given $(x, y) \in X_e^2$ and $u, v \in X$, we say that u and v are *on the same side* with respect to (x, y), if there is a path from u to v that does not have (x, y) and (y, x) as edges. Otherwise u and v are said to be *on the opposite sides* with respect to (x, y). Fix $u_0 \in X$. For any $(x, y) \in X_e^2$ denote

$$V_{xy} = \{v \in X \mid u_0 \text{ and } v \text{ are on the opposite sides with respect to } (x, y)\}.$$

An arbitrary $\lambda : X_e^2 \to K$ determines the *λ-structure* on $I(X, K)$ as follows:

$$e_x \cdot e_{xy} = e_{xy} \cdot e_x = -e_{xy} \cdot e_y = -e_y \cdot e_{xy} = \lambda(x, y)e_{xy}, (x, y) \in X_e^2,$$

$$e_x \cdot e_y = e_y \cdot e_x = \begin{cases} \mathrm{sgn}_{u_0}(x, y)\lambda(x, y)e_{V_{xy}}, & (x, y) \in X_e^2, \\ -\sum_{(x,v)\in X_e^2} \mathrm{sgn}_{u_0}(x, v)\lambda(x, v)e_{V_{xv}}, & x = y \in \mathrm{Min}(X), \\ -\sum_{(u,x)\in X_e^2} \mathrm{sgn}_{u_0}(u, x)\lambda(u, x)e_{V_{ux}}, & x = y \in \mathrm{Max}(X). \end{cases}$$

Lemma. *Any λ-structure $\cdot$ is a transposed Poisson structure on $I(X, K)$ orthogonal to any structure of Poisson type.*

Lemma *The sum of any mutational structure and any λ-structure is a transposed Poisson structure on $I(X, K)$.*

The following theorem is the main result of our talk [2].

Theorem. *A binary operation $\cdot$ on $I(X, K)$ is a transposed Poisson algebra structure on $I(X, K)$ if and only if $\cdot$ is the sum of a structure of Poisson type, a mutational structure and a λ-structure.*

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Poisson derivations of a semiclassical limit of a family of quantum second Weyl algebras

Stéphane Launois

(joint work with Isaac Oppong)

In [1], we studied deformations $A_{\alpha,\beta}$ of the second Weyl algebra and computed their derivations. In this talk, we identify the semiclassical limits $\mathcal{A}_{\alpha,\beta}$ of these deformations and compute their Poisson derivations. Our results show that the first Hochschild cohomology group $\mathrm{HH}^1(A_{\alpha,\beta})$ is isomorphic to the first Poisson cohomology group $\mathrm{HP}^1(\mathcal{A}_{\alpha,\beta})$.

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Gerstenhaber algebra structure on Hochschild cohomology

Samuel A. Lopes

(joint work with Andrea Solotar)

The Hochschild cohomology $\mathsf{HH}^\bullet(\mathsf{A})$ of an associative algebra A encodes many nontrivial properties and features of the algebra, including crucial information about its deformations. In [2], Gerstenhaber constructed two operations on $\mathsf{HH}^\bullet(\mathsf{A})$: the cup product and a (graded) Lie bracket. Together, they form what is now called a Gerstenhaber algebra structure on $\mathsf{HH}^\bullet(\mathsf{A})$. In general, a Gerstenhaber algebra is just a graded Poisson algebra of degree -1. Another example is the exterior algebra $\Lambda^\bullet\mathfrak{g}$ of a Lie algebra $\mathfrak{g}$ or, more generally, $\Lambda^\bullet_R L$, for a Lie–Rinehart algebra (R, L).

The Lie bracket on $\mathsf{HH}^\bullet(\mathsf{A})$ is easily defined on the bar resolution, but in general it is quite difficult to compute from a minimal resolution of A. Nevertheless, the graded Lie algebra structure of $\mathsf{HH}^\bullet(\mathsf{A})$ can be quite interesting; in particular, $\mathsf{HH}^\bullet(\mathsf{A})$ is a Lie module for the Lie algebra $\mathsf{HH}^1(\mathsf{A})$ of outer derivations of A.

We will compute the Gerstenhaber bracket on $\mathsf{HH}^\bullet(\mathsf{A})$ in case $\mathsf{A} = \mathsf{A}_h$ is the subalgebra of the Weyl algebra $\mathsf{A}_1 = \mathbb{F}\{x, y\}/\langle [y, x] = 1\rangle$ generated by x and $h(x)y$, for an arbitrary polynomial $h(x)$ over a field $\mathbb{F}$ of characteristic 0. In this case, $\mathsf{HH}^1(\mathsf{A}_h)$ is related to the Virasoro Lie algebra and we will show that the Lie module $\mathsf{HH}^\bullet(\mathsf{A}_h)$ is related to the intermediate series modules for the Virasoro Lie algebra. This is the result of joint work with Andrea Solotar [4].

Some questions: A Gerstenhaber algebra is a Batalin–Vilkovisky algebra (BV algebra, for short) if the Lie bracket coming from the Gerstenhaber algebra is induced by a degree -1 operator Δ with $\Delta^2 = 0$. Thus,

$$[a, b] = (-1)^{|a|}\Delta(ab) - (-1)^{|a|}\Delta(a)b - a\Delta(b) + a\Delta(1)b.$$

BV structures appeared in mathematical physics in connection with the quantization of gauge theories but it is interesting in general to determine when a Gerstenhaber algebra is a BV algebra.

(1) In particular, when is $\mathsf{HH}^\bullet(\mathsf{A})$ a BV algebra?
(2) The former question has a positive answer in case A is a twisted Calabi–Yau algebra with a semisimple Nakayama automorphism [3]. The algebras A_h were shown in [4] to be twisted Calabi–Yau, although the Nakayama automorphism is not in general semisimple. Are there BV structures in $\mathsf{HH}^\bullet(\mathsf{A}_h)$ when the Nakayama automorphism of A_h is not semisimple?

(3) One can look at Poisson analogues of the above setting via semiclassical limit and try to answer similar questions (see also [1]).

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Solvability of symmetric Poisson algebras

Salvatore Siciliano

Let P be a Poisson algebra over a field $\mathbb{F}$. We recall that P is said to satisfy a nontrivial Poisson identity (or that P is a Poisson PI algebra) if there exists a nonzero element in the free Poisson algebra of countable rank which vanishes under any substitution in P (see e.g. [2]). A basic theory of Poisson PI algebras was carried out by Farkas [2, 3], and further developments on this theory were next considered by several authors. In particular, in [6], Mishchenko, Petrogradsky, and Regev developed the theory of so called codimension growth in characteristic zero, and proved that the tensor product of Poisson PI algebras is a Poisson PI algebra.

Now, let L be a Lie algebra over $\mathbb{F}$ and $\{U_n \mid n \geq 0\}$ the canonical filtration of its universal enveloping algebra $U(L)$. Set $U_{-1} = 0$ and consider the symmetric algebra $S(L) = \mathrm{gr}(U(L)) = \oplus_{n=0}^{\infty} U_n/U_{n-1}$, which we identify with the polynomial ring $\mathbb{F}[x_1, x_2, \ldots]$, where $x_1, x_2, \ldots$ is an $\mathbb{F}$-basis of L (cf [1, §2.3]). By linearity and the Leibniz rule, the Lie bracket $[\cdot,\cdot]$ of L can be uniquely extended to a Poisson bracket $\{\cdot,\cdot\}$ of $S(L)$ so that this algebra becomes a Poisson algebra, called the *symmetric Poisson algebra* of L. Moreover, when $\mathbb{F}$ has characteristic $p > 0$, the Poisson bracket of $S(L)$ naturally induces the structure of a Poisson algebra on the factor algebra $\mathbf{s}(L) = S(L)/I$, where I is the ideal generated by the elements x^p with $x \in L$. We will refer to $\mathbf{s}(L)$ as the *truncated symmetric Poisson algebra* of L.

Poisson identities of symmetric Poisson algebras of Lie algebras were first studied by Kostant [4], Shestakov [8], and Farkas [2, 3]. In particular, in [3] Farkas proved that, in characteristic zero, $S(L)$ satisfies a nontrivial Poisson identity if and only if L contains an abelian subalgebra of finite codimension. Some years later, in [5], Giambruno and Petrogradsky extended Farkas' result to arbitrary characteristic and, moreover, established when the truncated symmetric Poisson algebra of a restricted Lie algebra satisfies a nontrivial multilinear Poisson identity.

More recently, in [7], Monteiro Alves and Petrogradsky investigated the Lie identities of $S(L)$ and $\mathbf{s}(L)$. In particular, they determined necessary and sufficient conditions on L such that $S(L)$ and $\mathbf{s}(L)$ are Lie nilpotent, studied the Lie nilpotence class of $\mathbf{s}(L)$ and, in characteristic $p \neq 2$, established when $S(L)$ and $\mathbf{s}(L)$ are solvable. On the other hand, the harder problem of the solvability of $S(L)$ and $\mathbf{s}(L)$ in characteristic 2 remained unsettled and a related conjecture formulated. Afterwards, in [11] a corrected version of that conjecture was proved, thereby completing the classification. Further developments of these topics have been also carried out in [9, 10].

The aim of this talk is to present an overview of the known results about solvable (truncated) symmetric Poisson algebras and their derived lengths. We first recall some theorems about the Lie structure of ordinary and restricted enveloping algebras, which originally motivated the present subject. Next, we summarize results on the existence of nontrivial Poisson identities in symmetric and truncated symmetric Poisson algebras. Finally, we consider Lie nilpotence and solvability of these Poisson algebras and discuss some results concerning the derived lengths and the Lie nilpotence classes.

Some open questions on these topics are the following:

Problem 1. *Let L be a Lie algebra over a field of characteristic $p > 0$ such that $\mathbf{s}(L)$ is Lie nilpotent. It is shown in* [7] *that the Lie nilpotence class and the strong Lie nilpotency class of $\mathbf{s}(L)$ are the same, provided $p \geq 5$. Is this true also in characteristics $p = 2, 3$?*

Problem 2. *Let L be a Lie algebra over a field of characteristic $p > 2$ such that $\mathbf{s}(L)$ is solvable. Do the derived length and the strong derived length of $\mathbf{s}(L)$ coincide?*

Note that the derived lengths of a truncated symmetric Poisson algebra can be actually different in characteristic 2 (see [9, Remark 4.4]).

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The Poisson spectrum of the symmetric algebra of the Virasoro algebra

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(joint work with Alexey Petukhov)

Let G be a connected algebraic group over $\mathbb{C}$ with Lie algebra $\mathfrak{g}$, and consider the coadjoint action of G on $\mathfrak{g}^*$. This is a beautiful classical topic, with profound connections to areas from geometric representation theory to combinatorics to physics. Algebraic geometry tells us that coadjoint orbits in $\mathfrak{g}^*$ correspond to G-invariant radical ideals in the symmetric algebra $S(\mathfrak{g})$.

As is well known, $S(\mathfrak{g})$ is a Poisson algebra under the Kostant-Kirillov bracket:

$$\{f, g\} = \sum_{i,j} \frac{\partial f}{\partial e_i}\frac{\partial f}{\partial e_j}[e_i, e_j]$$

where $\{e_i\}$ is a basis of $\mathfrak{g}$. A basic fact is that I is G-invariant if and only if I is Poisson.

Thus to compute the closure of the coadjoint orbit of $\chi \in \mathfrak{g}^*$, let $\mathfrak{m}_\chi$ be the kernel of the evaluation morphism

$$\mathrm{ev}_\chi : \mathrm{S}(\mathfrak{g}) \to \mathbb{C},$$

and let $P(\chi)$ be the *Poisson core* of $\mathfrak{m}_\chi$: the maximal Poisson ideal contained in $\mathfrak{m}_\chi$. By definition, an ideal of the form $P(\chi)$ is called *Poisson primitive*; by a slight abuse of notation, we refer to $P(\chi)$ as the *Poisson core of* χ. The closure of the coadjoint orbit of χ is defined by $P(\chi)$:

$$\overline{G\cdot\chi} = V(P(\chi)) := \{\nu \in \mathfrak{g}^* \mid ev_\nu(P(\chi)) = 0\}, \tag{1}$$

and so $\chi, \nu \in \mathfrak{g}^*$ are in the same G-orbit if and only if $P(\chi) = P(\nu)$. In the case of algebraic Lie algebras over $\mathbb{C}$ or $\mathbb{R}$, coadjoint orbits are symplectic leaves for the respective Poisson structure.

We investigate how this theory extends to the *Witt algebra* $W = \mathbb{C}[t, t^{-1}]\partial_t$ of algebraic vector fields on $\mathbb{C}^\times$, and to its central extension the *Virasoro algebra* $Vir = \mathbb{C}[t, t^{-1}]\partial_t \oplus \mathbb{C}z$, with Lie bracket given by

$$[f\partial_t, g\partial_t] = (fg' - f'g)\partial_t + Res_0(f'g'' - g'f'')z, \quad z \text{ is central.}$$

(We also consider some important Lie subalgebras of W.) These infinite-dimensional Lie algebras, of fundamental importance in representation theory and in physics, have no adjoint group [3], but one can still study the Poisson cores of maximal ideals, and more generally the Poisson ideal structure of $S(W)$ and $S(Vir)$. Motivated by (1), we will say that functions $\chi, \nu \in Vir^*$ or in W^* are in the same *pseudo-orbit* if $P(\chi) = P(\nu)$. These (coadjoint) pseudo-orbits can be considered as algebraic symplectic leaves in Vir^* or W^*.

Taking the discussion above as our guide, we focus on prime Poisson ideals and Poisson primitive ideals of $S(Vir)$ and $S(W)$. Important questions here, which for brevity we ask here only for Vir, include:

- Given $\chi \in Vir^*$, can we compute the Poisson core $P(\chi)$ and the pseudo-orbit of χ? When is $P(\chi)$ nontrivial?
- How can we understand prime Poisson ideals of $S(Vir)$? Can we parameterise them in a reasonable fashion, ideally in a way which gives us further information about the ideal? How does one distinguish Poisson primitive ideals from other prime Poisson ideals?
- It is known, see [4, Corollary 5.1], that $S(Vir)$ satisfies the ascending chain condition on prime Poisson ideals. The augmentation ideal of $S(Vir)$, that is, the ideal generated by $Vir \subset S(Vir)$, is clearly a maximal Poisson ideal. What are the others? Conversely, does any nontrivial prime Poisson ideal have finite height?
- Do prime Poisson ideals induce any reasonable algebraic geometry on the uncountable-dimensional vector space Vir^*?

We answer all of these questions, almost completely working out the structure of the Poisson spectra of $S(Vir)$ and $S(W)$.

Let us begin by discussing the idea of algebraic geometry on Vir^*. A priori, this seems intractable as Vir^* is an uncountable-dimensional affine space; little interesting can be said about $S(\mathfrak{a})$ where $\mathfrak{a}$ is a countable-dimensional *abelian* Lie algebra. However, Vir and W are extremely noncommutative and so Poisson ideals in their symmetric algebras are very large: in particular, by a result of Iyudu and the second author [1, Theorem 1.3], if I is a nontrivial Poisson ideal of $S(W)$ (respectively, a non-centrally generated Poisson ideal of $S(Vir)$), then $S(W)/I$ (respectively, $S(Vir)/I$) has polynomial growth. This suggests that a Poisson primitive ideal, and more generally a prime Poisson ideal, might correspond to a finite-dimensional algebraic subvariety of Vir^*, which we could investigate using tools from affine algebraic geometry. We will see that this is indeed the case.

From the discussion above, it is important to characterise which functions $\chi \in Vir^*$ have nontrivial Poisson cores. Strikingly, we show that such χ must vanish on the central element z. Further, the induced function $\overline{\chi} \in W^*$ is given by evaluating *local* behaviour on a proper (that is, finite) subscheme of $\mathbb{C}^\times$. We have:

Theorem. *Let $\chi \in Vir^*$. The following are equivalent:*

(1) *The Poisson core of χ is nontrivial: that is, $P(\chi) \supsetneqq (z - \chi(z))$.*
(2) *$\chi(z) = 0$ and the induced function $\overline{\chi} \in W^*$ is a linear combination of functions of the form*
$$f\partial_t \mapsto \alpha_0 f(x) + \ldots + \alpha_n f^{(n)}(x)$$
where $x \in \mathbb{C}^\times$ and $\alpha_0, \ldots, \alpha_n \in \mathbb{C}$.
(3) *The isotropy subalgebra Vir^χ of χ has finite codimension in Vir.*

We call functions $\chi \in Vir^*$ satisfying the equivalent conditions of Theorem *local functions* as by condition (2) they are defined by local data.

Motivated by condition (3) of Theorem , we investigate subalgebras of Vir of finite codimension. We prove:

Theorem. *Let $\mathfrak{k} \subseteq Vir$ be a subalgebra of finite codimension. Then there is $f \in \mathbb{C}[t, t^{-1}] \setminus \{0\}$ so that $\mathfrak{k} \supseteq \mathbb{C}z + f\mathbb{C}[t, t^{-1}]\partial_t$. In particular, any finite codimension subalgebra of Vir contains z.*

As an immediate corollary of Theorem, we show:

Corollary. *If $0 \neq \zeta \in \mathbb{C}$, then $S(Vir)/(z - \zeta)$ is* Poisson simple*: it has no nontrivial Poisson ideals.*

We then study the pseudo-orbits of local functions on Vir, W, and related Lie algebras; we describe our results here for Vir only. If $\chi \in Vir^*$ is local, then by combining Theorem and [1, Theorem 1.3] $S(Vir)/P(\chi)$ has polynomial growth and we thus expect the pseudo-orbit of χ to be finite-dimensional. We show that pseudo-orbits of local functions in Vir^* are in fact orbits of a finite-dimensional solvable algebraic (Lie) group acting on an affine variety which maps injectively to Vir^*, and we describe these orbits explicitly. This allows us to completely determine the pseudo-orbit of an arbitrary local function in Vir^* and thus also determine the Poisson primitive ideals of $S(Vir)$. We also classify maximal Poisson ideals in $S(Vir)$: they are the augmentation ideal, the ideals $(z - \zeta)$ for $\zeta \in \mathbb{C}^\times$, and the defining ideals of all but one of the two-dimensional pseudo-orbits.

Through this analysis, we obtain a nice combinatorial description of pseudo-orbits in W^*: pseudo-orbits of local functions on W, and thus Poisson primitive ideals of $S(W)$, correspond to a choice of a partition λ and a point in an open subvariety of a finite-dimensional affine space $\mathbb{A}^k$, where k can be calculated from λ. We further expand this correspondence to obtain a parameterisation of all prime Poisson ideals of $S(W)$ and $S(Vir)$. We also study the related Lie algebra $W_{\geq -1} = \mathbb{C}[t]\partial_t$, and prove that Poisson primitive and prime Poisson ideals of $S(W_{\geq -1})$ are induced by restriction from $S(W)$.

Our understanding of prime Poisson ideals allows us to determine exactly which prime Poisson ideals of $S(Vir)$ obey the *Poisson Dixmier-Moeglin equivalence*, which generalises the characterisation of primitive ideals in enveloping algebras of finite-dimensional Lie algebras due to Dixmier and Moeglin. The central question is when a Poisson primitive ideal of $S(Vir)$ is *Poisson locally closed*: that is, locally closed in the Zariski topology on Poisson primitive ideals. (If $\dim \mathfrak{g} < \infty$ then a prime Poisson ideal of $S(\mathfrak{g})$ is Poisson primitive if and only if it is Poisson locally closed [2, Theorem 2].) We show that (z) is the only Poisson primitive ideal of $S(Vir)$ which is not Poisson locally closed. We further prove that $S(W)$ has no nonzero prime Poisson ideals of finite height.

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Non-matrix polynomial identities on enveloping algebras

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(joint work with David Riley, Jeff Bergen)

A variety of associative algebras over a field $\mathbb{F}$ is called non-matrix if it does not contain $M_2(\mathbb{F})$, the algebra of 2×2 matrices over $\mathbb{F}$. A polynomial identity (PI) is called non-matrix if $M_2(\mathbb{F})$ does not satisfy this identity. Latyshev in his attempt to solve the Specht problem proved that any non-matrix variety generated by a finitely generated algebra over a field of characteristic zero is finitely based [9]. The complete solution of the Specht problem in the case of characteristic zero is given by Kemer [7].

Although several counterexamples are found for the Specht problem in the positive characteristic [1], the development in this area has lead to some interesting results. Kemer has investigated the relation between PI-algebras and nil algebras. Amitsur [2] had already proved that the Jacobson radical of a relatively-free algebra of countable rank is nil. Restricting to non-matrix varieties, Kemer [6] proved that the Jacobson radical of a relatively-free algebra of a non-matrix variety over a field of positive characteristic is nil of bounded index. These varieties have been further studied in [5, 6] and recently generalized for alternative and Jordan algebras in [14].

Enveloping algebras satisfying polynomial identities were first considered by Latyshev [10] by proving that the universal enveloping algebra of a Lie algebra L over a field of characteristic zero satisfies a PI if and only if L is abelian. Latyshev's result was extended to positive characteristic by Bahturin [4]. Passman [11] and Petrogradsky [13] considered the analogous problem for restricted Lie algebras.

Let $A = A_0 \oplus A_1$ be a vector space decomposition of a non-associative algebra over a field $\mathbb{F}$ of characteristic not 2. We say that this is a $\mathbb{Z}_2$-grading of A if $A_iA_j \subseteq A_{i+j}$, for every $i, j \in \mathbb{Z}_2$ with the understanding that the addition $i + j$ is mod 2. The components A_0 and A_1 are called even and odd parts of A, respectively. Note that A_0 is a subalgebra of A. One can associate a Lie super-bracket to A by defining $(x, y) = xy - (-1)^{ij}yx$ for every $x \in A_i$ and $y \in A_j$. If A is associative, then for any $x \in A_i$, $y \in A_j$ and $z \in A$ the following identities hold:

(1) $(x, y) = -(-1)^{ij}(y, x)$,
(2) $(x,(y, z)) = ((x, y), z) + (-1)^{ij}(y,(x, z))$.

The above identities are the defining relations of Lie superalgebras. Furthermore, A can be viewed as a Lie algebra by the usual Lie bracket $[u, v] = uv - vu$. The bracket of a Lie superalgebra $L = L_0 \oplus L_1$ is denoted by $(,)$. We denote the enveloping algebra of L by $U(L)$. Lie superalgebras whose enveloping algebras satisfy a PI were characterized by Bahturin [3] and Petrogradsky [12]. In this talk we characterize Lie superalgebras whose enveloping algebras satisfy a non-matrix PI. Our first main result is as follows.

Theorem. *Let $L = L_0 \oplus L_1$ be a Lie superalgebra over a field of characteristic $p > 2$. The following conditions are equivalent:*

(1) *$U(L)$ satisfies a non-matrix PI.*
(2) *$U(L)$ satisfies a PI, L_0 is abelian, and there exists a subspace M of L_1 of codimension at most 1 such that $(L_0, L_1) \subseteq M$ and $(M, L_1) = 0$.*
(3) *The commutator ideal of $U(L)$ is nil of bounded index.*
(4) *$U(L)$ satisfies a PI of the form $([x, y]z)^{p^m} = 0$, for some m.*

The equivalence of (1) and (4) is well known to hold for all algebras: it follows easily from standard PI-theory. The deeper fact that (1) and (3) are equivalent follows from the structure theory of PI algebras. We emphasize that the term Lie solvable is used with respect to the usual Lie bracket $[\,,]$.

Theorem *Let $L = L_0 \oplus L_1$ be a Lie superalgebra over a field of characteristic not 2. Then $U(L)$ is Lie solvable if and only if (L, L) is finite-dimensional, L_0 is abelian, and there exists a subspace M of L_1 of codimension at most 1 such that $(L_0, L_1) \subseteq M$ and $(M, L_1) = 0$.*

Kemer [8] proved that an algebra R over a field of characteristic zero satisfies a non-matrix PI if and only if R is Lie solvable. The following is now easily deduced from Theorem.

Corollary. *Let $L = L_0 \oplus L_1$ be a Lie superalgebra over a field of characteristic zero. The following conditions are equivalent:*

(1) *$U(L)$ satisfies a non-matrix PI.*
(2) *$U(L)$ is Lie solvable.*
(3) *(L, L) is finite-dimensional, L_0 is abelian, and there exists a subspace M of L_1 of codimension at most 1 such that $(L_0, L_1) \subseteq M$ and $(M, L_1) = 0$.*

The adjoint representation of L is given by ad $x : L \to L$, ad $x(y) = (y, x)$, for all $x, y \in L$. The notion of restricted Lie superalgebras can be easily formulated as follows.

Definition. *A Lie superalgebra $L = L_0 \oplus L_1$ over a field $\mathbb{F}$ of characteristic $p \geq 3$ is called* restricted, if there is a pth power map $L_0 \to L_0$, denoted as ${}^{[p]}$, satisfying

(a) $(\alpha x)^{[p]} = \alpha^p x^{[p]}$, for all $x \in L_0$ and $\alpha \in \mathbb{F}$,
(b) $(y, x^{[p]}) = (y,_p x)$, for all $x \in L_0$ and $y \in L$,
(c) $(x + y)^{[p]} = x^{[p]} + y^{[p]} + \sum_{i=1}^{p-1} s_i(x, y)$, for all $x, y \in L_0$ where is_i is the coefficient of λ^{i-1} in (ad $(\lambda x + y))^{p-1}(x)$.

In short, a restricted Lie superalgebra is a Lie superalgebra whose even subalgebra is a restricted Lie algebra and the odd part is a restricted module by the adjoint action of the even subalgebra. For example, every $\mathbb{Z}_2$-graded associative algebra inherits a restricted Lie superalgebra structure.

Let L be a restricted Lie superalgebra over a field $\mathbb{F}$ of characteristic $p \geq 3$. We denote the enveloping algebra of L by $u(L)$. Restricted Lie superalgebras whose enveloping algebras satisfy a polynomial identity have been characterized by Petrogradsky [12].

Theorem. *Let $L = L_0 \oplus L_1$ be a restricted Lie superalgebra over a perfect field and denote by M the subspace spanned by all $y \in L_1$ such that (y, y) is p-nilpotent. The following statements are equivalent:*

(1) *$u(L)$ satisfies a non-matrix PI.*
(2) *The commutator ideal of $u(L)$ is nil of bounded index.*
(3) *$u(L)$ satisfies a PI, (L_0, L_0) is p-nilpotent, $\dim L_1/M \leq 1$, (M, L_1) is p-nilpotent, and $(L_1, L_0) \subseteq M$.*

We show that (3) implies (2) over any field. However, given that $u(L)$ satisfies a non-matrix PI, the restriction on the field is necessary to be able to show that $\dim L_1/M \leq 1$. We show that over a non-perfect field there exists a restricted Lie superalgebra $L = L_0 \oplus L_1$ such that $\dim L_1 = 2$, $u(L)$ is Lie solvable and yet (y, y) is not p-nilpotent, for every $y \in L_1$. This is in complete contrast with Theorem .

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Symmetrisation and the Feigin–Frenkel centre

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Let $\mathfrak{g}$ be a complex reductive Lie algebra. The Feigin–Frenkel centre $\mathfrak{z}(\widehat{\mathfrak{g}}) \subset \mathcal{U}(t^{-1}\mathfrak{g}[t^{-1}])$ is a remarkable commutative subalgebra. Its structure is described by a theorem of Feigin and Frenkel (1992), if $\ell = \operatorname{rk}\mathfrak{g}$ and $\tau = -\partial_t$, then $\mathfrak{z}(\widehat{\mathfrak{g}}) = \mathbb{C}[\tau^k(S_i) \mid k \geqslant 0,\, 1 \leqslant i \leqslant \ell]$, where the generators $\tau^k(S_i)$ are algebraically independent.

The classical counterpart of $\mathfrak{z}(\widehat{\mathfrak{g}})$ is the Poisson-commutative subalgebra of $\mathfrak{g}[t]$-invariants in $\mathcal{S}(\mathfrak{g}[t,t^{-1}])/(\mathfrak{g}[t]) \cong \mathcal{S}(t^{-1}\mathfrak{g}[t^{-1}])$, which is a polynomial ring with infinitely many generators according to a direct generalisation of a Raïs–Tauvel theorem (1992). Unlike the finite-dimensional case, no natural isomorphism between $\mathcal{S}(t^{-1}\mathfrak{g}[t^{-1}])^{\mathfrak{g}[t]}$ and $\mathfrak{z}(\widehat{\mathfrak{g}})$ is known. Explicit formulas for the elements S_i appeared first in type A [1, 2] and were extended to all classical types in [3]. In [4], it is shown that for all classical Lie algebras, the symmetrisation map ϖ can produce generators of $\mathfrak{z}(\widehat{\mathfrak{g}})$. Note that ϖ is a homomorphism of $\mathfrak{g}[t^{-1}]$-modules and it behaves well with respect to taking various limits.

One of the tools in [4] is a certain map $\mathsf{m}\colon \mathcal{S}^k(\mathfrak{g}) \to \Lambda^2\mathfrak{g} \otimes \mathcal{S}^{k-3}(\mathfrak{g})$. Let $F[-1] \in \mathcal{S}^k(\mathfrak{g}t^{-1})$ be obtained from $F \in \mathcal{S}^k(\mathfrak{g})^{\mathfrak{g}}$ by the canonical isomorphism $\mathfrak{g}t^{-1} \cong \mathfrak{g}$. Then $\varpi(F[-1]) \in \mathfrak{z}(\widehat{\mathfrak{g}})$ if and only if $\mathsf{m}(F) = 0$. More generally, if $H \in \mathcal{S}^k(\mathfrak{g})^{\mathfrak{g}}$ is such that

$$\mathsf{m}^d(H) = \mathsf{m}(\mathsf{m}^{d-1}(H)) \in \mathcal{S}(\mathfrak{g}) \quad \text{for all} \quad 1 \le d < k/2,$$

then there is a way to produce an element of $\mathfrak{z}(\widehat{\mathfrak{g}})$ corresponding to H.

For each classical $\mathfrak{g}$, there is a generating set $\{H_i \mid 1 \leqslant i \leqslant \ell\} \subset \mathcal{S}(\mathfrak{g})^{\mathfrak{g}}$ such that $\mathsf{m}(H_k) \in \mathbb{C}H_j$ with $j < k$ for each k. In types A and C, we are using the coefficients Δ_k of the characteristic polynomial, for $\mathfrak{g} = \mathfrak{so}_n$, we work with coefficients Φ_{2k} of $\det(I_n - q(F_{ij}))^{-1}$.

In type A_{n-1}, $\mathsf{m}(\Delta_k) = \frac{(n-k+2)(n-k+1)}{k(k-1)}\Delta_{k-2}$; in type C_n, we have

$$\mathsf{m}(\Delta_{2k}) = \frac{(2n-2k+3)(2n-2k+2)}{2k(2k-1)}\Delta_{2k-2};$$

and finally for $\mathfrak{g} = \mathfrak{so}_n$, we have $\mathsf{m}(\Phi_{2k}) = \frac{(n+2k-3)(n+2k-2)}{2k(2k-1)}\Phi_{2k-2}$. This leads to the following sets of *Segal–Sugawara vectors* $\{S_i \mid 1 \le i \le \ell\}$ [4]:

$$\{S_{k-1} = \varpi(\Delta_k[-1]) + \sum_{1 \le r < (k-1)/2} \binom{n-k+2r}{2r} \varpi(\tau^{2r}\Delta_{k-2r}[-1]){\cdot}1 \mid 2 \le k \le n\}$$

in type A_{n-1};

$$\{S_k = \varpi(\Delta_{2k}[-1]) + \sum_{1 \le r < k} \binom{2n-2k+2r+1}{2r} \varpi(\tau^{2r}\Delta_{2k-2r}[-1]){\cdot}1 \mid 1 \le k \le n\}$$

in type C_n;

$$\{S_k = \varpi(\Phi_{2k}[-1]) + \sum_{1\le r<k} \binom{n+2k-2}{2r} \varpi(\tau^{2r}\Phi_{2k-2r}[-1])\cdot 1 \mid 1 \le k < \ell\}$$

for $\mathfrak{so}_n$ with $n = 2\ell - 1$ with the addition of $S_\ell = \varpi(\mathrm{Pf}[-1])$ for $\mathfrak{so}_n$ with $n = 2\ell$.

The advantage of our method is that it reduces questions about elements of $\mathfrak{z}(\widehat{\mathfrak{g}})$ to questions on the structure of $\mathcal{S}(\mathfrak{g})^{\mathfrak{g}}$ in a type-free way. For example, it is possible to deal with type G_2 by hand [4]. It is quite probable, that other exceptional types can be handled on a computer. Conjecturally, each exceptional Lie algebra possesses a set $\{H_k \mid 1 \le k \le \ell\}$ of generating symmetric invariants such that for each k there is i with $\mathsf{m}(H_k) \in \mathbb{C}H_i$.

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Contact brackets and other structures on the tensor product

Pasha Zusmanovich

The purpose of this report is once more to call attention to an elementary and, in some cases, very effective technique of computing various kinds of structures on tensor products. Such problems often can be reduced to the simultaneous evaluation of kernels of several tensor product maps, i.e., maps of the form $S \otimes T$, where S and T are linear operators on the respective spaces of linear maps. Using the fact that

$$Ker(S \otimes T) = Ker(S) \otimes * + * \otimes Ker(S), \tag{1}$$

the question reduces to evaluation of the intersection of several linear spaces having the form as on the right-hand side of (1), for various operators S and T. The intersection of two such spaces satisfies the distributivity, and so can be handled effectively, due to the following elementary linear algebraic lemma:

Lemma ([4, Lemma 1.1]). *Let U_1, U_2 be subspaces of a vector space U, and V_1, V_2 be subspaces of a vector space V. Then*

$$(U_1\otimes V + U\otimes V_1)\cap(U_2\otimes V + U\otimes V_2) = (U_1\cap U_2)\otimes V + U_1\otimes V_2 + U_2\otimes V_1 + U\otimes(V_1\cap V_2).$$

This technique was used for the first time in [4] to derive some formulas for the low degree cohomology of current Lie algebras, i.e., Lie algebras of the form $L\otimes A$, where L is a Lie algebra, and A is an associative commutative algebra. The paper [5] contains further results about such cohomology, as well as about Poisson and

Hom-Lie structures on current and related Lie algebras. The last our result in this direction is in [6], which answers a recent question from [2] about extension of contact bracket on the tensor product from the bracket on the factors.

Recall that the contact bracket on a commutative associative algebra A with unit is a bilinear map $[\cdot,\cdot] : A \times A \to A$ such that

$$[ab, c] = [a, c]b + [b, c]a + [c, 1]ab$$

for any $a, b, c \in A$. Contact brackets are an obvious generalization of Poisson brackets, the latter being contact brackets satisfying $[A, 1] = 0$. It was asked in [2] whether, given contact brackets on two algebras A and B, is it always possible to extend them to the tensor product $A \otimes B$? In [6], using some general formulas for the space of contact brackets on some particular classes of algebras, a procedure was devised for constructing examples showing that such extension is not always possible.

This linear algebraic method is sometimes very effective, but its applicability is severely limited by the fact that no statement similar to Lemma is true for intersection of three or more spaces. The proper contexts of Lemma might be criteria for distributivity of a set of subspaces of a vector space (for an exposition, see, for example, [3, Chap. 1, §7]) and, more speculatively, the "four subspaces problem" of Gelfand–Ponomarev, [1].

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