Mathematisches Forschungsinstitut Oberwolfach

Report No. 23/2026

DOI: 10.4171/OWR/2026/23

Algebraische Zahlentheorie

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31 May – 5 June 2026

Abstract. This workshop focused on recent developments in algebraic number theory. Algebraic number theory is a central area of research in mathematics, and it has a long history. The workshop featured talks on topics intersecting several different areas, especially algebraic and (p-adic) analytic geometry and representation theory. The talks presented new results and techniques spanning various aspects of Euler systems, eigenvarieties, local-global compatibility, automorphic forms, and related topics that play prominent roles in algebraic number theory.

Mathematics Subject Classification (2020): 11-XX, 14-XX.

Introduction by the Organizers

The workshop *Algebraische Zahlentheorie*, organized by Fabrizio Andreatta (Milano), Ellen Eischen (Eugene), Eugen Hellmann (Münster), and Mahesh Kakde (Bangalore) was well-attended with over 40 participants with broad geographic representation from all continents. The workshop was held entirely in person. There were eighteen lectures. Outside of the lectures, there were ample opportunities for participants to discuss research with each other and build new connections. The workshop covered several interconnected topics within algebraic number theory, including Iwasawa theory, automorphic forms, Euler systems, Shimura varieties, eigenvarieties, and local-global compatibility.

Algebraic number theory is a central area of mathematics that has grown to intersect several areas of mathematics. The lectures reflected this aspect and introduced recent developments, particularly on topics intersecting algebraic geometry and representation theory. Broadly speaking, the results introduced in the lectures are tied to advances for Iwasawa theory (concerning p-adic approaches), automorphic forms and representations (of fundamental importance in the Langlands program), and geometry of Shimura varieties and eigenvarieties. While these three areas naturally overlap and talks do not necessarily fall neatly into just one of these three areas, we list them here in the area they most strongly represent:

Iwasawa theory

- Guido Kings: p-adic interpolation of critical L-values of Hecke characters
- David Loeffler: Ultraprimes and non-ordinary Selmer groups
- Shilin Lai: Euler systems and the relative Satake isomorphism
- Jaclyn Lang: Eisenstein congruence at prime-square level

Automorphic forms and representations

- George Boxer: Local-Global compatibility in low weight
- Juan Esteban Rodriguez Camargo: A Riemann–Hilbert functor in p-adic analytic geometry
- Jessica Fintzen: Reduction to depth-zero for $\bar{\mathbb{Z}}[1/p]$-representations of p-adic groups
- Luis García: Algebraic specialisations of the elliptic gamma function
- Alexander Horawa: Arithmetic of Fourier coefficients on G_2
- Jelena Ivančić: Infinitesimal characters in the completed cohomology for GL_n over CM fields
- James Newton: Integral Eichler–Shimura decompositions for ordinary p-adic automorphic forms
- Peter Scholze: $\mathrm{Aut}(\mathbb{A}^1_{\mathrm{dR}})$

Geometry of Shimura varieties and eigenvarieties

- John Bergdall: On the distribution of modular points on deformation rings
- Shaunak Deo: Greenberg's question for Siegel modular forms
- Linus Hamann: Intersection Cohomology of p-adic Shimura varieties
- Vaughan McDonald: Eigenvarieties over CM fields and Galois representations
- Lue Pan: Plectic Galois action on the cohomology of Hilbert modular varieties
- Jan Vonk: Heights on non-split Cartan modular curves $X^+_{\mathrm{ns}}(p)$

Workshop: Algebraische Zahlentheorie

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Abstracts

p-adic interpolation of critical L-values of Hecke characters

Guido Kings
(joint work with Johannes Sprang)

Let $L/\mathbb{Q}$ be a number field, $\mathfrak{f} \subset \mathscr{O}_L$ an ideal in the ring of integers and $I(\mathfrak{f})$ be the fractional ideals coprime to $\mathfrak{f}$. Choose an embedding $\iota_\infty : \overline{\mathbb{Q}} \to \mathbb{C}$. For an algebraic Hecke character

$$\chi : I(\mathfrak{f}) \to \overline{\mathbb{Q}}^\times$$

consider the associated L-function $L_\mathfrak{f}(\chi, s) = \sum_{(\mathfrak{a},\mathfrak{f})=1} \frac{\iota_\infty \circ \chi(\mathfrak{a})}{N\mathfrak{a}^s}$, where the sum runs over the integral ideals of $\mathscr{O}_L$ coprime to $\mathfrak{f}$. One calls χ critical, if $L_\mathfrak{f}(\chi, 0)$ is critical, i.e., the Γ-factors in the functional equation have no poles in $s = 0$ and $s = 1$. Critical characters can only occur for number fields which are either totally real or contain a CM field. As the algebraicity and p-adic interpolation is known in the totally real case by Siegel-Klingen and Deligne-Ribet, we concentrate on the case where L contains a CM field K.

The following theorem was shown in special cases by Shimura [14], Katz [8], Colmez [4], Bergeron-Charollois-Garcia [2] and others and by Kings-Sprang [10] in general.

Theorem 1 ([10] Theorem 4.10)**.** *Let $\mathfrak{c} \subset \mathscr{O}_L$ be an auxiliary ideal coprime to $\mathfrak{f}$. Then there exists a number field k such that*

$$\frac{(\chi(\mathfrak{c})N\mathfrak{c} - 1)L_\mathfrak{f}(\chi, 0)}{\Omega^\chi} \in \mathscr{O}_k \left[\frac{1}{\mathfrak{f}N\mathfrak{c}d_L}\right]$$

where d_L is the discriminant of L and Ω^χ is a product of periods of an abelian variety $\mathcal{A}$ with CM by $\mathscr{O}_L$.

The proof relies on the construction of an equivariant cohomology class $EK \in H^{d-1}(\mathcal{A} \setminus \mathcal{A}[\mathfrak{c}], \mathscr{O}_\mathfrak{f}^\times, \widehat{\mathscr{P}} \otimes \Omega^d_\mathcal{A})$, where $\mathscr{O}_\mathfrak{f}^\times$ are the one-units modulo $\mathfrak{f}$ and $\widehat{\mathscr{P}}$ is the completion of the Poincaré-bundle along $\mathscr{A} \times \{e^\vee\} \subset \mathscr{A} \times \mathscr{A}^\vee$.

We are interested in the p-adic interpolation of the algebraic numbers

$$\frac{(\chi(\mathfrak{c})N\mathfrak{c} - 1)L_\mathfrak{f}(\chi, 0)}{\Omega^\chi}.$$

When p is ordinary, p-adic L-functions were first constructed by Katz [8] in the case of a CM field i.e. $L = K$, by Colmez-Schneps [5] for certain extension of an imaginary quadratic field and in general by Kings-Sprang [10]. In the non-ordinary case only for $L = K$ imaginary quadratic, i.e. for elliptic curves, the p-adic interpolation was known by work of Katz [9], Boxall [3], Schneider-Teitelbaum [12] and Andreatta-Iovita [1].

Let $G(p^\infty\mathfrak{f}) = Gal(L(p^\infty\mathfrak{f})/L)$ be the Galois group of the ray class field of conductor $p^\infty\mathfrak{f}$ of L. Choosing an embedding $\iota_p : \overline{\mathbb{Q}} \to \mathbb{C}_p$, the algebraic Hecke character χ gives rise to a character $\chi : G(p^\infty\mathfrak{f}) \to \mathbb{C}_p^\times$. The Galois group $G(p^\infty\mathfrak{f})$

has a subgroup of finite index $Gal(L(p^\infty\mathfrak{f})/L(\mathfrak{f}))$, which by class field theory is isomorphic to $(\mathscr{O}_L\otimes\mathbb{Z}_p)^\times/E(\mathscr{O}_\mathfrak{f}^\times)$, where $E(\mathscr{O}_\mathfrak{f}^\times)$ is the closure of the one-units modulo $\mathfrak{f}$. Our main result is as follows:

Theorem 2 ([11] Theorem 9.1)**.** *There exists a locally analytic distribution $\mu_\mathfrak{f}$ on $G(p^\infty\mathfrak{f})$ such that for all critical algebraic Hecke characters χ whose conductor divides $p^\infty\mathfrak{f}$ and ∞-type $-\alpha$ supported in the induced CM type Σ one has*

$$\frac{1}{\Omega_p^\chi}\int_{G(p^\infty\mathfrak{f})}\chi\mu_\mathfrak{f}=(\alpha-1)!(\textit{explicit loc. factors})\frac{L_\mathfrak{f}(\chi,0)}{\Omega^\chi}$$

where Ω_p^χ is a product of p-adic periods attached to the abelian variety $\mathcal{A}$ with CM by $\mathscr{O}_L$.

The proof of this theorem relies on an extension of the p-adic Fourier theory of Schneider-Teitelbaum [12], which is developed in [11] and the previous results in [10]. An extension of the theory of Schneider-Teitelbaum was independently obtained in [7].

Let T be a free $\mathbb{Z}_p$-module of finite rank considered as p-adic Lie group, and $\mathscr{C}^{an}(T,\mathbb{C}_p)$ the locally analytic functions on T and write

$$\widehat{T}(\mathbb{C}_p)=\{x\in Hom(T,\mathbb{C}_p^\times)\mid\chi\in\mathscr{C}^{an}(T,\mathbb{C}_p)\}$$

for the locally analytic characters (same as continuous in fact). Let $Lie(T)=T\otimes\mathbb{Q}$ and consider a subvector space

$$W\subset(Lie(T))^\vee\otimes_{\mathbb{Q}_p}\mathbb{C}_p=Hom(T,\mathbb{C}_p),$$

where $(Lie(T))^\vee$ denotes the dual let

$$\mathscr{C}_W^{an}(T,\mathbb{C}_p)=\{f\in\mathscr{C}^{an}(T,\mathbb{C}_p)\mid df\in W\otimes\mathscr{C}^{an}(T,\mathbb{C}_p)\}.$$

Finally define the W-analytic characters by $\widehat{T}_W(\mathbb{C}_p):=\widehat{T}(\mathbb{C}_p)\cap\mathscr{C}_W^{an}(T,\mathbb{C}_p)$ and let $\mathscr{D}_W(T,\mathbb{C}_p)$ be the strong dual of $\mathscr{C}_W^{an}(T,\mathbb{C}_p)$, which we call the W-analytic distributions. Then one has

Theorem 3 ([11],[7])**.** *The Fourier transform gives an isomorphism*

$$\mathscr{D}_W(T,\mathbb{C}_p)\to\mathscr{O}^{rig}(\widehat{T}_W/\mathbb{C}_p)\qquad\qquad\lambda\mapsto\{\chi\mapsto\lambda(\chi)\}$$

where $\widehat{T}_W$ is a rigid variety whose $\mathbb{C}_p$-valued points are $\widehat{T}_W(\mathbb{C}_p)$ and $\mathscr{O}^{rig}(\widehat{T}_W/\mathbb{C}_p)$ denotes the ring of rigid functions on $\widehat{T}_W$.

To construct the W-analytic distribution $\mu_\mathfrak{f}\in\mathscr{D}_W(T,\mathbb{C}_p)$ in Theorem 2 one needs to connect $\widehat{T}_W$ with the p-divisible group $G=\mathcal{A}[p^\infty]$ associated to the abelian variety $\mathcal{A}$. This uses the classification of Fargues-Scholze-Weinstein of p-divisible groups over $\mathscr{O}_{\mathbb{C}_p}$, who classify these in terms of pairs (W,T) as above (in fact this is dual to the original classification, see [11] Section 4 for more details). In the case of $G=\mathcal{A}[p^\infty]$ one gets the pair $(Lie(\mathcal{A})\otimes_{\mathbb{Q}_p}\mathbb{C}_p,T_p(G^\vee))$, where $G^\vee$ is the dual p-divisible group. With this, the following theorem is a consequence of the theory of Fargues-Scholze-Weinstein.

Theorem 4 ([11],[7])**.** *With $T = T_p(G^\vee)$ and $W = Lie(\mathcal{A}) \otimes_{\mathbb{Q}_p} \mathbb{C}_p$ one has an isomorphism*

$$\widehat{T}_W/\mathbb{C}_p \cong (G)_\eta^{rig}$$

of the rigid variety underlying the W-analytic characters with the rigid generic fibre of $G = \mathcal{A}[p^\infty]$.

To complete the proof of Theorem 2 one has to construct a function on $(G)_\eta^{rig}$. This is done by restricting the class $EK \in H^{d-1}(\mathcal{A} \setminus \mathcal{A}[\mathfrak{c}], \mathscr{O}_{\mathfrak{f}}^\times, \widehat{\mathscr{P}} \otimes \Omega^d_{\mathcal{A}})$ to $\mathcal{A}[p^\infty]$.

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Ultraprimes and non-ordinary Selmer groups

David Loeffler

(joint work with Pierre Colmez and Sarah Livia Zerbes)

1. Euler systems and Selmer groups

Many important problems in number theory, such as the Birch–Swinnerton-Dyer conjecture, the Beilinson–Bloch–Kato conjecture, and the Iwasawa main conjecture, involve studying *Selmer groups* of p-adic Galois representations. These are subgroups cut out by local conditions,

$$\{x \in H^1(\mathbb{Q}, V) : \mathrm{loc}_\ell(x) \in H^1_{\mathcal{F}}(\mathbb{Q}_\ell, V)\ \forall \ell\},$$

where ℓ varies over primes, and $H^1_{\mathcal{F}}(\mathbb{Q}_\ell, V) \subseteq H^1(\mathbb{Q}_\ell, V)$ is a family of subspaces (a "Selmer structure"). For $\ell \neq p$ there is a "standard" choice of local condition (the Bloch–Kato subspace), but at the p-adic place the story is more complicated.

The theory of *Euler systems* (developed by Kolyvagin, Rubin, and others) provides a very powerful tool for bounding Selmer groups when the local condition at p is the "strict" one, i.e. $H^1_{\mathcal{F}}(\mathbb{Q}_p, V) = \{0\}$. Using Poitou–Tate duality, one can up-grade this to bound Selmer groups where $H^1_{\mathcal{F}}(\mathbb{Q}_\ell, V)$ has dimension ≤ 1. However, in practice the "interesting" local conditions have dimension about $\frac{1}{2}\dim V$. This is no problem for the classical Euler systems of Kato, Kolyvagin, et al, where $\dim V = 1$ or 2; but studying Euler systems for larger Galois representations requires new ingredients.

Possibilities include:

- constructing several Euler systems (whose localisations at p are linearly independent and "fill up" the local condition space);
- constructing a *higher-rank* Euler system, i.e. a family of classes in exterior powers of cohomology groups;
- constructing Euler systems with some additional local property at p.

The first two approaches both suffer from the problem that it seems to be hard to construct the input (the cohomology classes forming the Euler system). So we shall focus on the third. The key difficulty in this strategy is to show that if the Euler system classes have some non-trivial local property, then the Kolyvagin "derivative" classes formed from them also have this property.

2. Ordinary and non-ordinary

If $V|_{G_{\mathbb{Q}_p}}$ is reducible, stabilizing a partial flag with appropriate block sizes, then we can use local conditions of "Greenberg type", where the local condition at p is given by the image of a $G_{\mathbb{Q}_p}$-subrepresentation $V^+ \subseteq V$. It is shown in [1] that the Kolyvagin derivative operator respects these Greenberg-type local conditions: if the Euler system lands locally in the cohomology of V^+, then the same holds for the Kolyvagin derivative classes. Moreover, there are naturally-arising examples of Euler systems satisfying this kind of Greenberg condition (such as the Beilinson–Flach elements). This this approach has been successfully used to prove many cases of the Iwasawa and Bloch–Kato conjectures for automorphic motives on various small rank groups, such as $\mathrm{GL}_2 \times \mathrm{GL}_2$ and GSp_4, assuming suitable ordinarity hypotheses. However, the ordinarity hypothesis is inconvenient in applications.

Pottharst [2] has proposed a more general approach to Selmer groups in which subrepresentations $V^+ \subseteq V$ are replaced by sub-(φ, Γ)-modules $D^+ \subseteq D = \mathbb{D}_{\mathrm{rig}}(V)$. This widens the scope of the theory considerably, since D can have many "extra" submodules which do not come from subrepresentations (e.g. if $V|_{G_{\mathbb{Q}_p}}$ is crystalline, D will always be *trianguline*, i.e. admit a full flag of sub-(φ, Γ)-modules).

However, it is not so clear if the passage from Euler systems to Kolyvagin systems interacts well with Pottharst's theory, because (φ, Γ)-modules are fundamentally "$\mathbb{Q}_p$-linear" objects, with no natural integral structure, whereas Kolyvagin classes are families of $\bmod p^n$ cohomology classes that do not naturally lift to characteristic 0.

3. Ultraprimes

To bridge this gap, we used an approach due to Sweeting [3] (building on ideas of Scholze in the setting of Taylor–Wiles patching): rather than using Kolyvagin classes, each of which is only defined mod p^n for some finite n, one can use a non-principal ultrafilter on $\mathbb{N}$ to glue these together into "ultra-Kolyvagin classes" which *do* naturally live in characteristic 0.

In this setting, the natural definition of a "local condition" is given by a subspace

$$\mathcal{F} \subseteq H^1(K_\infty, V),$$

where K_∞ is the unramified $\mathbb{Z}_p$-extension of $\mathbb{Q}_p$. We show that:

- If $\mathcal{F}$ is *closed* in the Banach-space topology of $H^1(K_\infty, V)$, then the passage from Euler system classes to ultra-Kolyvagin classes respects the local condition $\mathcal{F}$.
- The Pottharst-style local conditions given by sub-(φ, Γ)-modules $D^+ \subseteq D$ are closed. This follows from a more general result that the H^1 of (φ, Γ)-modules over the Robba ring of $\widehat{K}_\infty$ carries a *functorial* Banach-space topology, and this topology is compatible with the natural topology on cohomology of Galois representations via the isomorphisms
$$H^1(\mathbb{D}_{\mathrm{rig}}(V|_{G_{K_\infty}})) = H^1(K_\infty, V).$$

Combining these results allows us to extend many of the results towards the Iwasawa main conjecture, previously proved for ordinary motives, to non-ordinary cases. For instance, we obtain one divisibility in Pottharst's main conjecture for a Rankin–Selberg convolution of (possibly non-ordinary) modular forms $f \times g$.

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Infinitesimal characters in the completed cohomology for $\mathbf{GL}_n$ over CM fields

Jelena Ivančić

(joint work with Vaughan McDonald)

Let p be a prime number, F a CM field, and L a finite extension over $\mathbf{Q}_p$ which is large enough so that there are $[F:\mathbf{Q}]$ different embeddings of F into L. Let us write $F_p := F \otimes_{\mathbf{Q}} \mathbf{Q}_p$, write $Z(\mathfrak{g}) := Z(U(\mathrm{Res}_{F/\mathbf{Q}}\mathrm{Lie}(\mathrm{GL}_n))_{\mathbf{Q}_p})$ for the center of the universal envelopping algebra and denote by $Z(\mathfrak{g})_L := Z(\mathfrak{g}) \otimes_{\mathbf{Q}_p} L$. Finally, we fix an isomorphism $\iota : \mathbf{Q}_p \simeq \mathbf{C}$.

In this talk, we report on joint work with McDonald [6] concerning (a part of) the local–global compatibility at p for GL_n/F predicted by the following:

Conjecture 1 ([2], Conjecture 3.2.1). *Let π be a L–algebraic cuspidal automorphic representation for GL_n/F. Then, there exists a finite set S of primes of F, which includes all primes that divide $p\cdot\infty$ and all primes where π ramifies, such that there exists a continuous Galois representation $\rho_\pi : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_n(\overline{\mathbf{Q}}_p)$ satisfying the following (among others) properties:*

1. *For all $v \notin S$, $\rho_\pi|_{\mathrm{Gal}(\overline{F}_v/F_v)}$ is unramified and the characteristic polynomial of a geometric Frobenius is determined by the Satake parameters of π_v (via the unramified local Langlands correspondence);*
2. *(Local–global compatibility at p) For all $v \mid p$, $\rho_\pi|_{\mathrm{Gal}(\overline{F}_v/F_v)}$ is de Rham and its Hodge–Tate weights are explicitly determined by the infinitesimal character of π_w, where $w \mid \infty$ is the place corresponding to v via ι.*

Let us refer to the predicted relationship between the Hodge–Tate weights and the infinitesimal characters as 'the weak local–global compatibility at p'. In [6], we show that the weak local–global compatibility holds for (locally analytic vectors of) the completed cohomology for GL_n/F (localized suitably). Let us now illustrate our result and its proof.

To begin, let $K \subset \mathrm{GL}_n(\mathbb{A}_F^\infty)$ be a compact open subgroup and denote by $X_K := \mathrm{GL}_n(F)\backslash(X \times \mathrm{GL}_n(\mathbb{A}_F^\infty)/K)$ the corresponding locally symmetric space, with X being the symmetric space for GL_n. Recall that the completed cohomology for a tame level K^p and L–coefficients is given by

$$\tilde{H}^i(K^p, L) := \varprojlim_n \varinjlim_{K_p} H^i(X_{K_pK^p}, \mathbf{Z}_p/(p^n)) \otimes_{\mathbf{Z}_p} L \ .$$

It is equipped with a natural $\mathrm{GL}_n(F_p) \times \mathbf{T}^S(K^p)$ action where $\mathbf{T}^S(K^p)$ denotes a corresponding big Hecke algebra (see [6, Section 2.1] for precise definitions). In particular, it is an admissible Banach space representation for $\mathrm{GL}_n(F_p)$.

Let us now translate Conjecture 1 (in particular, the weak local–global compatibility at p) to this setting. In order to study the infinitesimal action on the automorphic side, we pass to the locally analytic vectors for $\mathrm{GL}_n(F_p)$ in the completed cohomology, denoted by $\tilde{H}^i(K^p, L)^{\mathrm{la}}$, which then carries an action of $Z(\mathfrak{g})_L \times \mathbf{T}^S(K^p)$. For technical reasons, let us further pass to a localization $\tilde{H}^i(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$, where $\mathfrak{m} \subset \mathbf{T}^S(K^p)$ is a non–Eisenstein maximal ideal. Note that, since $\mathbf{T}^S(K^p)_{\mathbf{m}}$ is a complete Noetherian local ring with finite residue field, the action of $\mathbf{T}^S(K^p)_{\mathfrak{m}}$ on $\tilde{H}^i(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$ upgrades to an action of the Banach algebra $\mathbf{T}^S(K^p)^{\mathrm{rig}}_{\mathfrak{m}} := \mathcal{O}(\mathrm{Spf}(\mathbf{T}^S(K^p)_{\mathbf{m}})^{\mathrm{rig}})$.

On the Galois side, by [4][1] (which removes the nilpotent ideal from [9]), we have a continuous semisimple representation $\rho_{\mathfrak{m}} : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_n(\mathbf{T}^S(K^p)_{\mathfrak{m}})$ satisfying the property (1) of Conjecture 1.

In order to relate the p–adic Hodge–Tate properties of $\rho_{\mathfrak{m}}$ in terms of the infinitesimal action on $\tilde{H}^i(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$, we follow the general recipe provided by [5]. To begin, extend the coefficients of $\rho_{\mathfrak{m}}$ to $\mathbf{T}^S(K^p)^{\mathrm{rig}}_{\mathfrak{m}}$ and for each $v \mid p$, let $\Theta^{\mathrm{Sen}}_v \in M_{n\times n}(\mathbf{T}^S(K^p)^{\mathrm{rig}}_{\mathfrak{m}}\hat{\otimes}_{\mathbf{Q}_p}\mathbf{C}_p)$ be the Sen operator of $\rho_{\mathfrak{m}}|_{\mathrm{Gal}(\overline{F}_v/F_v)}$. Then, using the Harish–Chandra isomorphism together with the Chevalley restriction isomorphism and evaluating the Sen operators (twisted appropriately to accommodate the fact that automorphic representations appearing in the completed cohomology are C–algebraic, rather than L–algebraic), the authors in [5] construct from $\rho_{\mathfrak{m}}$ a *generalized Hodge–Tate–Sen* character $\zeta^C_{\rho_{\mathfrak{m}}} = \otimes_{v|p}\zeta^C_{\rho_{\mathfrak{m}},v} : Z(\mathfrak{g})_L \to \mathbf{T}^S(K^p)^{\mathrm{rig}}_{\mathfrak{m}}$. Our main theorem then says the following.

Theorem 2 ([6])**.** *Let F be a CM field containing an imaginary quadratic field where p splits. Let $\mathfrak{m} \subset \mathbf{T}^S(K^p)$ be a non–Eisenstein, decomposed generic maximal ideal. Then, for any $i \geq 0$, the infinitesimal action of $Z(\mathfrak{g})_L$ on $\tilde{H}^i(K^p, L)^{la}_{\mathfrak{m}}$ is given by the generalized Hodge–Tate–Sen character $\zeta^C_{\rho_{\mathfrak{m}}}$ attached to $\rho_{\mathfrak{m}}$.*

Let us first make some remarks. Firstly, specializing the above statement to a Hecke eigenspace, one gets a relationship predicted by (2) in Conjecture 1 (twisted appropriately to accommodate for the difference between C–algebraicity and L–algebraicity). Secondly, in [5] the authors made a conjecture similar to the above statement for any reductive group G which they proved in some cases when G is associated with a Shimura variety, $i = d$ is the middle degree of the Shimura variety, and $\mathfrak{m}$ is a maximal ideal such that $\tilde{H}^i_{\mathfrak{m}} \neq 0$ only if $i = d$. Although our case does not satisfy these restrictions, we still make use of their strategy indirectly, which consists of two parts as follows:

(1) Show density of (some) locally algebraic vectors, i.e. show that the image of
$$\bigoplus_{V\in\mathrm{Irr}(\mathrm{Res}_{F/\mathbf{Q}}G)_L} V(\tau)\otimes_L \mathrm{Hom}_{K_p}(V(\tau), \tilde{H}^d(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}) \to \tilde{H}^d(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$$
is dense (for example, this follows if there is a surjection $\mathcal{C}^{\mathrm{la}}(K_p, L)^{\oplus m} \to \tilde{H}^d(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$). Here, $K_p \subset G(F_p)$ is a compact open subgroup, τ is a supercuspidal type, and $V(\tau) := V \otimes_L \tau$.

(2) Show the weak local–global compatibility at p for the points on the LHS of (1) (i.e. use/show the weak local–global compatibility at p for cohomological automorphic representations).

However, the locally algebraic vectors in $\tilde{H}^i(K^p, L)^{\mathrm{la}}_{\mathfrak{m}}$ are not expected to be dense for $G = \mathrm{GL}_n/F$, so we cannot apply the above argument directly. Instead, our strategy is to first apply the strategy of [5] to the unitary group $\tilde{G} = U(n, n)/F^+$, where $F^+ \subset F$ is a maximal totally real subfield, which has

[1] The authors assume that p splits completely in F – we will not need this assumption in our theorem because we have an extra assumption that $\mathfrak{m}$ is decomposed generic.

a Levi subgroup isomorphic to $G := \mathrm{Res}_{F/F^+}\mathrm{GL}_n$, and then deduce it for G.[2] Let us sketch our argument in four steps. Let $\tilde{K}^p \subset \tilde{G}(\mathbb{A}_{F^+}^{p;\infty})$ be an appropriate tame level with $K_p = \tilde{K}^p \cap G(\mathbb{A}_{F^+}^{p;\infty})$ and let $\tilde{\mathfrak{m}}$ be the pullback of $\mathfrak{m}$ via the Satake map. The first step is to relate $\tilde{H}^i(K^p, L)_{\mathfrak{m}}^{G(F_p^+)-\mathrm{la}}$ and $\tilde{H}^i_\partial(\tilde{K}^p, L)_{\tilde{\mathfrak{m}}}^{\tilde{G}(F_p^+)-\mathrm{la}}$ using [1] (here, we also show that the corresponding infinitesimal actions are related by an appropriate unnormalized Harish–Chandra map $\mathrm{HC}^{\overline{P}}$). Secondly, we show that the infinitesimal action on $\tilde{H}^i_\partial(\tilde{K}^p, L)_{\tilde{\mathfrak{m}}}^{\tilde{G}(F_p^+)-\mathrm{la}}$ is given by the appropriate generalized Hodge–Tate–Sen character (which exists by [8] which generalizes [5]). Here we are able to follow the strategy of [5] by using the vanishing results of [3] (as generalized by [7]). In the third step, we show that the generalized Hodge–Tate–Sen characters for $\tilde{G}$ and G fit into a commutative diagram using the map $\mathrm{HC}^{\overline{P}}$ appearing in step 1 and the Satake map between the Hecke algebras. Finally, to account for the fact that $\mathrm{HC}^{\overline{P}}$ is not surjective, in the fourth step we twist $\mathfrak{m}$ by a suitable character χ. Relating the commutative diagrams for $\mathfrak{m}$ and $\mathfrak{m}(\chi)$, we deduce that the whole algebra $Z(\mathfrak{g})_L$ acts on $\tilde{H}^i(K^p, L)_{\mathfrak{m}}^{G(F_p^+)-\mathrm{la}}$ by the generalized Hodge–Tate–Sen character $\zeta^C_{\rho_\mathfrak{m}}$, as wanted.

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[2]This way, we get the weak local–global compatibility at p for cohomological automorphic representations as a corollary of Theorem 1, rather than using it in the proof.

Plectic Galois action on the cohomology of Hilbert modular varieties

Lue Pan
(joint work with Yuanyang Jiang)

Fix a prime number p. Let F be a totally real field extension of $\mathbb{Q}$ of degree d. For a neat open compact subgroup $K \subseteq GL_2(\mathbb{A}_f \otimes_{\mathbb{Q}} F)$, we denote by Sh_K the Hilbert modular variety of level K over $\mathbb{Q}$. Consider the interior cohomology of Sh_K

$$H^i_!(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p}) := \mathrm{Im}\left(H^i_c(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p}) \to H^i(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p})\right).$$

The absolute Galois group $G_{\mathbb{Q}}$ acts on $H^i_!(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p})$. A remarkable result of Nekovar [1] says that this action is *semisimple*. In fact, Nekovar allowed the coefficient systems to be arbitrary automorphic local systems and proved the semisimplicity result for the intersection cohomology.

In this talk, we explain a new proof of Nekovar's result for the non-CM part of the cohomology. Let $\mathbb{T}$ denote the abstract Hecke algebra generated by spherical Hecke operators. For a Hecke eigensystem $\lambda : \mathbb{T} \to \overline{\mathbb{Q}_p}$ that shows up in $H^d(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p})$ and corresponds to a cuspidal Hilbert modular form, it was known that one can attach a two-dimensional irreducible Galois representation $\rho_\lambda : \mathrm{Gal}(\overline{F}/F) \to GL_2(\overline{\mathbb{Q}_p})$ to λ satisfying the usual Eichler–Shimura relation. We say that ρ_λ is strongly irreducible if $\rho_\lambda|_H$ is irreducible for any open subgroup H of $\mathrm{Gal}(\overline{F}/F)$. Equivalently λ does not correspond to a CM form. We prove that for $\lambda : \mathbb{T} \to \overline{\mathbb{Q}_p}$ such that ρ_λ is strongly irreducible, there is a natural isomorphism of $G_{\mathbb{Q}}$-representations

$$H^d(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p})[\lambda] \cong \left(\otimes - \mathrm{Ind}_F^{\mathbb{Q}}\rho_\lambda\right)^{\oplus n_\lambda} \tag{1}$$

for some integer $n_\lambda > 0$, where $H^d(\mathrm{Sh}_{K,\overline{\mathbb{Q}}}, \overline{\mathbb{Q}_p})[\lambda]$ denotes the λ-isotypic part, and $\otimes - \mathrm{Ind}_F^{\mathbb{Q}}\rho_\lambda : G_{\mathbb{Q}} \to GL_{2^d}(\overline{\mathbb{Q}_p})$ denotes the tensor induction of ρ_λ from $\mathrm{Gal}(\overline{F}/F)$ to $G_{\mathbb{Q}}$. This recovers Nekovar's result under the strongly irreducible assumption because the tensor induction is known to be semisimple.

We also prove similar results for Hecke eigenspaces of the completed cohomology. Fix an open compact subgroup K^p of $GL_2(\mathbb{A}_f^p \otimes_{\mathbb{Q}} F)$. Recall that the p-adically completed cohomology introduced by Emerton is defined as

$$\tilde{H}^i := \varprojlim_n \varinjlim_{K_p} H^i(\mathrm{Sh}_{K^pK_p,\overline{\mathbb{Q}}}, \mathbb{Z}/p^n)$$

where K_p runs through all level subgroups at p. There is a natural action of $G_{\mathbb{Q}} \times \mathbb{T} \times GL_2(F \otimes_{\mathbb{Q}} \mathbb{Q}_p)$ on $\tilde{H}^d$. Denote by $\tilde{H}^{i,\mathrm{la}} \subseteq \tilde{H}^i \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ the subspace of $GL_2(F \otimes_{\mathbb{Q}} \mathbb{Q}_p)$-locally analytic vectors and $\tilde{H}^{i,\mathrm{la}}_{\overline{\mathbb{Q}_p}} := \tilde{H}^{i,\mathrm{la}} \otimes_{\mathbb{Q}_p} \overline{\mathbb{Q}_p}$. For $\lambda : \mathbb{T} \to \overline{\mathbb{Q}_p}$ such that $\tilde{H}^{d,\mathrm{la}}_{\overline{\mathbb{Q}_p}}[\lambda] \neq 0$, we can show that there is a natural isomorphism of $G_{\mathbb{Q}} \times GL_2(F \otimes_{\mathbb{Q}} \mathbb{Q}_p)$-representations

$$\tilde{H}^{d,\mathrm{la}}_{\overline{\mathbb{Q}_p}}[\lambda] \cong (\otimes - \mathrm{Ind}_F^{\mathbb{Q}}\rho_\lambda) \otimes_{\overline{\mathbb{Q}_p}} \Pi_\lambda \tag{2}$$

for some representation Π_λ of $GL_2(F \otimes_{\mathbb{Q}} \mathbb{Q}_p)$, assuming that ρ_λ is strongly irreducible and ρ_λ has distinct τ-Hodge-Tate-Sen weights for some embedding $\tau : F \to \overline{\mathbb{Q}_p}$.

The natural isomorphisms (1) and (2) are obtained by constructing some part of the plectic symmetry conjectured by Nekovar-Scholl in [2]. Let

$$G_F^{plec} := \mathrm{Aut}_F(F \otimes_{\mathbb{Q}} \overline{\mathbb{Q}})$$

denote the plectic Galois group of F which naturally contains $G_{\mathbb{Q}}$ as a subgroup. Nekovar-Scholl conjectured that the action of $G_{\mathbb{Q}}$ on the cohomology of Sh_K can be naturally extended to an action of G_F^{plec}. Under the above assumption on ρ_λ, we construct such an action on $\tilde{H}^{d,\mathrm{la}}_{\overline{\mathbb{Q}_p}}[\lambda]$ in our work. The key player in our construction is the so-called partial Sen operators. It was observed in [3] that when $d = 1$, the Sen operator on $\tilde{H}^{*,\mathrm{la}}\widehat{\otimes}_{\mathbb{Q}_p}\mathbb{C}_p$ can be constructed by studying the locally analytic functions on the modular curves of infinite level at p, where $\mathbb{C}_p$ denotes the p-adic completion of $\overline{\mathbb{Q}_p}$. This was later generalized to all Shimura varieties in [4]. Explicitly in the Hilbert case, the geometric construction produces d commuting operators, whose sum is the Sen operator. These are the partial Sen operators. We use a classical result of Sen to construct the Lie algebra action of the inertia subgroup of G_F^{plec} at p from the partial Sen operators. Finally, under our assumption on ρ_λ, we can extend this Lie algebra action to an action of the full plectic group.

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Integral Eichler–Shimura decompositions for ordinary p-adic automorphic forms

James Newton

(joint work with Ana Caraiani and Juan Esteban Rodríguez Camargo)

This is a report on work in progress. Our goal is to relate the ordinary part of completed cohomology [7, 8] to the higher Hida theory (coherent) cohomology groups of Boxer–Pilloni [3].

Ordinary completed cohomology. First we set up some notation. We will be working with Siegel modular varieties X_K for compact open subgroups $K \subset \mathrm{GSp}_{2g}(\mathbb{A}_f)$. We fix a Borel subgroup $B = TN$ in $\mathrm{GSp}_{2g}/\mathbb{Z}$, Siegel parabolic $B \subset P = MU$, and denote $N(\mathbb{Z}_p)$ by N_0. Then, following Hida, we define

$$\widetilde{H}^i_{c,\mathrm{ord}}(K^p, \mathbb{Z}_p) = \varprojlim_n \left(\varinjlim_{N_0 \le K_p} H^i_c(X_{K^pK_p,\overline{\mathbb{Q}}}, \mathbb{Z}/p^n) \right)^{\mathrm{ord}}.$$

The ordinary projector $(\cdot)^{\mathrm{ord}}$ is obtained by inverting the Hecke operators corresponding to elements $t \in T(\mathbb{Q}_p)$ which shrink N_0 under conjugation. For example, $\mathrm{diag}(p,1)$ in GL_2 shrinks the upper triangular N_0, and gives rise to the Hecke operator U_p acting on the cohomology of modular curves.

Our strategy to describe $\widetilde{H}^i_{c,\mathrm{ord}}(K^p, \mathbb{Z}_p)$ is to use Scholze's primitive comparison theorem [11] and the Hodge–Tate period map to decompose the cohomology according to the Bruhat stratification on the Hodge–Tate period domain. Related ideas have been used in several works on p-adic Eichler–Shimura decompositions (with p inverted) in the finite-slope setting [1, 5, 10, 6].

A theorem of Ohta. In the case of GL_2 (i.e. $g = 1$), the desired decomposition is provided by a theorem of Ohta [9] (which we restate using Boxer–Pilloni's point of view). We fix a complete and algebraically closed extension $C/\mathbb{Q}_p$.

There is a short exact sequence

$$\begin{aligned} 0 &\longrightarrow H^1_c(\mathfrak{Ig}^{\mathrm{tor}}_{K^p}, \mathcal{O}(-D))^{F\text{-ord}} \\ &\longrightarrow \widetilde{H}^1_{c,\mathrm{ord}}(K^p, \mathbb{Z}_p) \widehat{\otimes}_{\mathbb{Z}_p} \mathcal{O}_C \longrightarrow H^0(\mathfrak{Ig}^{\mathrm{tor}}_{K^p}, \Omega^1)^{U_p\text{-ord}}(-1) \longrightarrow 0 \end{aligned}$$

where D denotes the cuspidal divisor and $\mathfrak{Ig}^{\mathrm{tor}}_{K^p}/\mathcal{O}_C$ is the toroidal (partial) compactification of the Igusa tower, a $T(\mathbb{Z}_p)$-torsor over the ordinary locus in $X_{K^p\mathrm{GL}_2(\mathbb{Z}_p)}$.

This sequence becomes $T(\mathbb{Z}_p)$- and $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$-equivariant after equipping the coherent cohomology groups with suitable twisted actions. The operator F (cf. [2]) comes from the action of $w_0 \cdot \mathrm{diag}(p,1) = \mathrm{diag}(1,p) \in T(\mathbb{Q}_p)$. Taking the eigenspace corresponding to a dominant character of $T(\mathbb{Z}_p)$ then recovers the Hodge–Tate filtration on the ordinary part of étale cohomology of modular curves.

Expected theorem for general g. The decomposition in Ohta's theorem is indexed by the two Weyl group elements for GL_2. In general, we have the Bruhat decomposition

$$P\backslash \mathrm{GSp}_{2g} = \coprod_{w \in {}^MW} P\backslash PwB = \coprod_w C_w$$

where ${}^MW$ is the set of minimal length (Kostant) representatives for the cosets $W_M\backslash W$.

Viewing the analytification of $P\backslash\mathrm{GSp}_{2g}$ as the Hodge–Tate period domain for X_K, and filtering it by the closed subvarieties $\bigcup_{l(w)\le i} C_w$, we obtain a filtration on the complex of (almost) $\mathcal{O}_C$-modules $R\widetilde{\Gamma}_{c,\mathrm{ord}}(K^p,\mathbb{Z}_p)\widehat{\otimes}\mathcal{O}_C^a$ computing ordinary completed cohomology. This is analogous to the computation of the Jacquet module of a parabolic induction by the geometric lemma.

Each graded piece is a direct sum over the elements in ${}^MW$ of a fixed length. Using the dynamics of the Hecke operators at p, we show that these pieces can be computed on the fibre of the Hodge–Tate period map at w, which is essentially a perfectoid Igusa variety.

Our expected theorem identifies the piece indexed by w (in a $T(\mathbb{Z}_p)$- and $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$-equivariant way) with a (cuspidal) higher Hida theory complex

$$R\Gamma_w(K^p)$$

defined in [3], up to a cohomological shift, a Tate twist, and a twist by an automorphic vector bundle (cf. the Ω^1 appearing in Ohta's theorem).

Applications. Combined with a conjecture from [3] (a vanishing result for the groups $H^i(R\Gamma_w)$), our expected theorem would give a (torsion) vanishing result for the ordinary part of cohomology, analogous to the vanishing results of [4] but with 'ordinary' replacing 'generic'. Boxer and Pilloni proved their vanishing conjecture in the cases $g \leq 2$, so we can immediately deduce vanishing results for GSp_4.

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Eigenvarieties over CM fields and Galois representations

Vaughan McDonald

Fix a prime number p. In this talk we discussed a local-global compatibility result for Galois representations associated to finite slope p-adic automorphic forms. More specifically, the result we proved (see [11]) shows that for certain eigenvarieties for the group GL_n over a CM field F, the associated Galois representations are *trianguline* with the expected Sen weights.

Before providing a formal statement, we give some motivation (eliding some technicalities). For F a number field, the global Langlands program predicts the existence of a bijective map between (A): algebraic cuspidal representations π of $\mathrm{GL}_n(\mathbb{A}_F)$, and (B): irreducible *geometric* Galois representations $\rho : \mathrm{Gal}_F \to \mathrm{GL}_n(\mathbb{Q}_p)$, satisfying many compatibilities. Here *geometric* means ρ is unramified almost everywhere and also de Rham at p-adic places. There is much progress in constructing such a map $\pi \mapsto r_\pi$ (mainly in the *regular* case), but proving surjectivity of this map (i.e. modularity) is often quite difficult.

We are motivated by a p-adic approach to this question, which takes the following shape: let $\mathcal{E}$ be a space of p-adic automorphic forms, a so-called *eigenvariety* for GL_n/F, and $\mathcal{X}$ a rigid space of (not necessarily de Rham) Galois representations, and upgrade the map $\pi \mapsto r_\pi$ to a morphism of rigid spaces $\mathcal{R} : \mathcal{E} \to \mathcal{X}$. One approach to modularity is then as follows: (1) show $\mathcal{R}$ is an isomorphism of rigid spaces, and (2) ("Classicality") show that if $x \in \mathcal{E}$ is such that $\mathcal{R}_x$ is de Rham, then x comes from an automorphic representation. Arguably this approach sounds no less difficult, but we note that there is much progress on the "classicality" portion (see [4]).

Our goals are more modest: set up the architecture to make this approach plausible, in large generality. Let F be a CM field, which for technical reasons we assume contains an imaginary quadratic field in which p splits. Let $\mathcal{E}$ be an eigenvariety for GL_n/F, constructed as in Hansen [10] or Fu [7] (a variant of Emerton's construction [6]. Essentially, this is a parameter space of *finite-slope* p-adic automorphic forms, together with a choice of eigenvalue for the U_p-operator. In the case of $\mathrm{GL}_2/\mathbb{Q}$, Kisin showed that Galois representations associated to points on the eigenvariety are trianguline, en route to his proof of classicality in this setting. We generalize this result to the setting of GL_n/F, proving a conjecture of Hansen [10] in many cases.

Theorem 1. *Let $x \in \mathcal{E}(\overline{\mathbb{Q}}_p)$ be a "non–Eisenstein, decomposed generic" point. Then the associated Galois representation $R_x : \mathrm{Gal}_F \to \mathrm{GL}_n(\overline{\mathbb{Q}}_p)$ is trianguline at all p-adic places.*

In the talk, we also discussed a more deformation theoretic version of this result: when the point x has suitably generic U_p-eigenvalues, an open neighborhood of x maps to the trianguline variety of Breuil–Hellmann–Schraen [3]. Such a result will appear in a new version of the paper.

We note that such results were typically available for groups related to Shimura varieties, where standard techniques could prove that eigenvarieties $\mathcal{E}$ admit a

Zariski–dense set of "classical points," i.e. points corresponding to usual automorphic representations. The key novelty in our setting is this Zariski–density is *not* expected to hold for GL_n/F, $n \geq 2$.

We also discussed the proof strategy, which follows the lead of many other works in this field, especially the 10 author paper [1]. Namely, we realize the eigenvarieties for GL_n/F inside an eigenvariety for an auxiliary unitary group $\widetilde{G} = U(n,n)/F^+$. This group is chosen so that it has a maximal Siegel parabolic subgroup $P = MU$, where the Levi $M \simeq \mathrm{Res}_{F/F^+}\mathrm{GL}_n$ equals the group of interest. More specifically, these eigenvarieties are constructed by applying various (derived) functors for completed cohomology, and so the above relation motivates us to locate the cohomology of GL_n inside the boundary cohomology of $\widetilde{G}$. In practice, this amounts to a detailed study of the Lie algebra cohomology of certain locally analytic parabolic inductions.

The next key step is that in middle degree, the eigenvariety associated to $\widetilde{G}$ has the previously mentioned Zariski–density of classical points, so standard methods show the desired triangulinity result. Morally, this final density statement is telling us we can analytically approximate certain Eisenstein series on $\widetilde{G}$ (the classes coming from GL_n/F) via cusp forms on $\widetilde{G}$. A crucial ingredient to both steps are the generic torsion vanishing results of Caraiani–Scholze and Koshikawa [5, 9].

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On the distribution of modular points on deformation rings

John Bergdall
(joint work with Chengyang Bao, Brandon Levin)

Let p be an odd prime. Fix an odd and continuous Galois representation

$$\overline{\rho} : \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}_2(\overline{\mathbf{F}}_p).$$

Serre's conjecture implies $\overline{\rho}$ has *modular lifts* to level $N(\overline{\rho}) = N$, which are the Galois representations

$$\rho_f : \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}_2(\overline{\mathbf{Q}}_p)$$

attached to eigenforms $f \in S_k(\Gamma_1(N))$ such that $\overline{\rho}_f \simeq \overline{\rho}$. Let D be a decomposition group at p, and assume $\overline{r} = \overline{\rho}|_D$ has only scalar endomorphisms. Mazur defined ([9]) the universal deformation space $X_{\overline{r}}$. For $k \geq 2$ we consider the natural closed subscheme of crystalline deformations $X_{\overline{r}}^k \subseteq X_{\overline{r}}$, defined by Kisin ([5]), whose generic fiber parametrizes crystalline lifts of $\overline{r}$ that have Hodge–Tate weights $0 < k-1$ and cyclotomic determinant. Since N is prime to p, the restrictions $\rho_f|_D$ of modular lifts are crystalline and hence define *modular points* on $X_{\overline{r}}^k$ as k varies. Kisin showed ([6]), in landmark modularity results for $\mathrm{GL}_{2/\mathbf{Q}}$, each irreducible component of $X_{\overline{r}}^k$ supports a modular point. We aim to answer two questions:

(1) How many modular points are on a fixed component in $X_{\overline{r}}^k$?
(2) How many components does $X_{\overline{r}}^k$ have?

To answer Question 1, recall that Serre weights are the irreducible $\mathbf{F}_p$-linear $\mathrm{GL}_2(\mathbf{F}_p)$-representations $\sigma_{a,b} = \mathrm{Sym}^a(\det^b)$ with $a \leq p-1$. They enter the story in two ways. First, assuming $\sigma = \sigma_{a,b}$, we may define

$$N_{\mathrm{aut}}(\sigma) = \dim S_{a+2}(\Gamma_1(N))[\overline{\rho} \otimes \chi_{\mathrm{cyc}}^{-b}].$$

For context, the weight part of Serre's conjecture says that if $N_{\mathrm{aut}}(\sigma) \neq 0$, then σ is a Serre weight for $\overline{\rho}$. Second, writing $\overline{X}_{\overline{r}}^k \subseteq \overline{X}_{\overline{r}}$ for the special fibers, there are canonical one-dimensional *Breuil–Mézard cycles* $C(\sigma)$ in $\overline{X}_{\overline{r}}$. The geometric Breuil–Mézard conjecture of Emerton and Gee ([4]) implies the cycle of $\overline{X}_{\overline{r}}^k$ is

$$[\overline{X}_{\overline{r}}^k] = \sum_{\sigma} \begin{pmatrix} \text{Jordan–Hölder multiplicity} \\ \text{of } \sigma \text{ in } \mathrm{Sym}^{k-2} \end{pmatrix} \cdot C(\sigma),$$

and the Breuil–Mézard cycles are a $\mathbf{Z}$-basis of the cycles on $\overline{X}_{\overline{r}}^k$.

Each component $Z \subseteq X_{\overline{r}}^k$ has analogous data. Namely, we may consider the number of modular points $N_{\mathrm{aut}}(Z)$ on Z, and the cycle expansion

$$[\overline{Z}] = \sum_{\sigma} m_\sigma(\overline{Z}) C(\sigma) \qquad (m_\sigma(\overline{Z}) \in \mathbf{Z}).$$

Here is our answer to Question 1.

Theorem 1. *For fixed $k \geq 2$ and any irreducible component $Z \subseteq X_{\overline{r}}^k$,*

$$N_{\mathrm{aut}}(Z) = \sum_{\sigma} m_\sigma(\overline{Z})\, N_{\mathrm{aut}}(\sigma). \tag{1}$$

Theorem 1 is a *local-global distribution* result, since it describes the distribution of the global modular points among the components of the local deformation ring. The left-hand side of (1) is purely global, whereas the right-hand side is mixed. In a sense, the local special fiber geometry plays a larger role than the finite amount of seed data encoded in the $(N_{\mathrm{aut}}(\sigma))_\sigma$, which count modular lifts to Serre weights.

The proof of Theorem 1 builds upon the techniques of Kisin (and its precursors in work of Wiles and Taylor–Wiles and others). We specifically use patching methods to present global deformation rings as formally smooth extensions of local ones, uniformly in the weight k, acting on patched modules. What we observe is that the underlying algebra that comes out of the patching method allows one to relate modular point counts to global invariants on the special fibers of the patched deformation rings. In fact, the analysis naturally gives an interesting method also for counting automorphic points on automorphic components for n-dimensional Galois representations in terms of global special fiber invariants. The switch from global to local in Theorem 1 is mediated by Breuil–Mézard conjecture, which is only available in limited automorphic contexts (like $\mathrm{GL}_{2/\mathbf{Q}}$).

As presented, Theorem 1 could be interpreted as saying that the number of automorphic points on a component is governed by its special fiber geometry. We also may take the alternate perspective and try to use automorphic methods to control geometry. In this way, we answer Question 2 with the following purely local theorem. Let $I \subseteq D$ be the inertia subgroup and $c_{\mathrm{sm}}(k)$ be the number of *formally smooth* components on $X_{\overline{r}}^{k}$. Let ω be the cyclotomic character modulo p and $d_k(\sigma)$ be the Jordan–Hölder multiplicity of σ in Sym^{k-2}.

Theorem 2. *Assume $p > 7$ and $\overline{r}$ is non-split and reducible such that*

$$\overline{r}|_I \simeq \begin{pmatrix} \omega^{a+1} & * \\ 0 & 1 \end{pmatrix} \otimes \omega^b$$

with $2 \leq a \leq p-5$. Then,

$$c_{\mathrm{sm}}(k) \geq \left(1 - \frac{2}{p} - \frac{4}{p^{\frac{p-3}{2}}} - O(\log k/k)\right) \cdot d_k(\sigma_{a,b}).$$

For $\overline{r}$ as in Theorem 2, $\sigma_{a,b}$ is the unique Serre weight. The cycle $C(\sigma_{a,b})$ is just an irreducible component with multiplicity one. So, it follows from the geometric Breuil–Mézard conjecture that $c_{\mathrm{sm}}(k) \leq d_k(\sigma_{a,b})$. Theorem 2 thus implies the proportion of formally smooth components on $X_{\overline{r}}^{k}$ is at least roughly $1 - 2/p$. It would be interesting to determine whether the proportion is just 1, as $k \to +\infty$.

The first step in proving Theorem 2 is as follows. Given $Z \subseteq X_{\overline{r}}^{k}$, to check Z is formally smooth, it suffices to check $m_{\sigma_{a,b}}(\overline{Z}) = 1$. Paškūnas constructed local analogues of patched modules ([10, 3]) using the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbf{Q}_p)$, and applying our local-global theorem to those patched modules, we deduce Z is formally smooth if and only if Z has just one "modular" point coming from the local patched modules.

The assumptions on $\overline{r}$ enter into the second step of the proof, where we use the recent proof of the Bergdall–Pollack *ghost conjecture* ([1, 2]) by Liu, Truong, Xiao, and Zhao ([7, 8]). Those papers prove two remarkable facts for $\overline{r}$ as above. First, each component Z has a well-defined p-adic slope, which is a non-negative half-integer. Second, the collection of these slopes (counted with their local modular multiplicities) are exactly the slopes of an elementary *ghost series* Newton polygon that depends just on σ and k. (The original ghost conjecture did not propose this; the purely local version and its proof is due to Liu, Truong, Xiao, and Zhao.) We conclude that every time a slope appears without multiplicity on the ghost series Newton polygon, it must correspond to a formally smooth component of that slope. The estimate in Theorem 2 is then nothing more than a lower bound for the number of slopes on the ghost series Newton polygon that appear with multiplicity one.

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Eisenstein congruence at prime-square level

Jaclyn Lang

(joint work with Katharina Müller, Bharathwaj Palvannan, Robert Pollack, Preston Wake)

Let $N, p \geq 5$ be primes, and let $E_{2,N} \in M_2(\Gamma_0(N))$ denote the unique normalized Eisenstein series of weight 2 and level $\Gamma_0(N)$. We are interested in mod-p congruences between $E_{2,N}$ and cusp forms of weight 2 and N-power level.

We begin by defining the spaces that parametrize such congruences. Write $H'(N^i)$ for the anemic Hecke algebra acting on $M_2(\Gamma_0(N^i))$, which is generated as a $\mathbb{Z}_p$-algebra by T_ℓ for all primes $\ell \nmid N$. Let $\mathfrak{m}'$ denote the Eisenstein maximal ideal in $H'(N^i)$; that is, $\mathfrak{m}'$ is generated by p and $T_\ell - \ell - 1$ for all primes ℓ. Similarly, write $H(N^i)$ for the full Hecke algebra acting on $M_2(\Gamma_0(N^i))$, which is generated as a $\mathbb{Z}_p$-algebra by T_ℓ for all primes ℓ. Let $\mathfrak{m}$ denote the maximal ideal of $H(N^i)$ generated by p, T_N, and $T_\ell - \ell - 1$. Set $\mathbb{T}(N) := H'(N)_{\mathfrak{m}'}$ and $\mathbb{T}(N^2) := H(N^2)_{\mathfrak{m}}$. Let $r := \mathrm{rk}_{\mathbb{Z}_p} \mathbb{T}(N)$, which is the number of normalized Hecke eigenforms in $M_2(\Gamma_0(N))$ that are congruent to $E_{2,N}$ modulo a prime above p. Mazur proved the following fundamental result about such congruences.

Theorem 1 (Mazur [4]). *We have $r \geq 2$ (equivalently, $S_2(\Gamma_0(N))_{\mathfrak{m}'} \neq 0$) if and only if $N \equiv 1 \bmod p$.*

The key fact used to prove Mazur's theorem is that we know that the constant term of $E_{2,N}$ equals $\frac{N-1}{24}$. Henceforth assume that $N \equiv 1 \bmod p$, and set $\Delta := (\mathbb{Z}/N\mathbb{Z})^{\times, p\text{-part}}$. Let $\mathbb{Z}_p[\Delta]$ be the group ring of Δ and $\mathbb{Z}_p[\Delta]^+$ the subring of $\mathbb{Z}_p[\Delta]$ that is fixed under the involution that inverts group-like elements. The goal of the talk is to explain two related theorems, the first of which is the following.

Theorem 2 (L.–Pollack–Wake [1]).

(1) $r \geq 2$ (without using that $a_0(E_{2,N}) = \frac{N-1}{24}$).
(2) $r \geq 3$ if and only if $S_2(\Gamma_0(N^2))_{\mathfrak{m}}^{\mathrm{new}} \neq 0$.
(3) $\mathbb{T}(N^2)$ is a $\mathbb{Z}_p[\Delta]^+$-algebra, and it is a free $\mathbb{Z}_p[\Delta]^+$-module of rank r.

The last statement of Theorem 2 easily implies the first two by counting Eisenstein series.

To state the second theorem, let $B_2(x) = x^2 - x + 1/6$ be the second Bernoulli polynomial. We define

$$\zeta := \frac{-N}{2} \sum_{i=1}^{N-1} B_2(i/N)[i] \in \mathbb{Z}_p[(\mathbb{Z}/N\mathbb{Z})^\times].$$

This interpolates Dirichlet L-values in the sense that for any mod-N Dirichlet character χ we have

$$\chi(\zeta) = \begin{cases} L(-1,\chi) & \chi \neq 1 \\ (1-N)L(-1,1) & \chi = 1. \end{cases}$$

Let $\bar{\zeta} \in \mathbb{F}_p[(\mathbb{Z}/N\mathbb{Z})^\times]$ denote the reduction of ζ, and let $\mathcal{I}$ be the augumentation ideal in $\mathbb{F}_p[(\mathbb{Z}/N\mathbb{Z})^\times]$. It is easy to see that $\bar{\zeta} \in \mathcal{I}$. Let $\mathrm{ord}(\bar{\zeta})$ denote the largest integer m for which $\bar{\zeta} \in \mathcal{I}^m$. The second theorem is Theorem 3 below; the first statement was first proved by Merel [5]. It was later reproved and generalized by Lecouturier, who also proved the second statement using his theory of higher Eisenstein elements [3]. We give new, uniform, and transparent proofs of their theorems, in addition to proving the third point.

Theorem 3 (L.–Müller–Palvannan [2]).

(1) (Merel [5]) $\mathrm{ord}(\bar{\zeta}) = 1$ if and only if $r = 2$.
(2) (Lecouturier [3]) $\mathrm{ord}(\bar{\zeta}) = 2$ if and only if $r = 3$.
(3) Suppose that exactly one of $\mathrm{ord}(\bar{\zeta})$ and $r - 1$ equals 3. For simplicity, assume also that $p^2 \nmid N - 1$. Then $\mathrm{rk}_{\mathbb{Z}_p} S_2(\Gamma_0(N^2))_{\mathfrak{m}}^{\mathrm{new}} = \frac{(p-1)(r-2)}{2}$, and all eigenforms in this space are Galois conjugate.

We explain our proof of Theorem 2 and our proof of the first part of Theorem 3. The proof of Theorem 3 relies crucially on the freeness result of Theorem 2.

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Heights on non-split Cartan modular curves $X_{\mathrm{ns}}^+(p)$

Jan Vonk

(joint work with Jonathan Love, Élie Studnia / Henri Darmon)

This talk is about joint work with Jonathan Love, Élie Studnia, and Henri Darmon concerning height pairings of special cycles on the modular curves $X_{\mathrm{ns}}^+(p)$. We fix the following notation throughout:

- K is a quadratic number field of discriminant Δ,
- H is its (narrow) Hilbert class field, of degree $h(\Delta)$ over K,
- p is a prime that is *inert* in K.

We will consider, in turn, the CM case ($\Delta < 0$) and the RM case ($\Delta > 0$).

1. CM points ($\Delta < 0$)

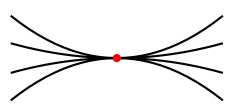
The classical work of Heegner on the class number one problem implicitly makes use of the construction of Heegner divisors P_Δ on $X_{\mathrm{ns}}^+(p)$, which under the running assumptions are divisors of degree $h(\Delta)$ defined over $\mathbf{Q}$. With Jonathan Love and Élie Studnia [5], we study the following question:

Question: Determine the arithmetic intersections $\langle P_1, P_2 \rangle$ on $X_{\mathrm{ns}}^+(p)$.

1.1. **Model over** $\mathrm{Spec}(\mathbf{Z})$**.** The first thing we need is to specify the regular model of the non-split Cartan modular curve $X_{\mathrm{ns}}^+(p)$. We show that the functor

$$S \longmapsto \{E/S \text{ ell. curve, } \mathbf{F}_{p^2} \hookrightarrow \mathrm{End}_S(E[p])\}/\sim$$

has a coarse moduli scheme $\mathcal{Y}_{\mathrm{ns}}^+(p)$ which is *smooth* over $\mathrm{Spec}(\mathbf{Z})$. We show that it has an open immersion into the minimal regular model [2], and its special fibre has the following shape:



where the singularity is the only point in the minimal regular model that is missing from $\mathcal{Y}_{\mathrm{ns}}^{+}(p)$. Here, the components are all isomorphic to $\mathbf{A}^1$, and are indexed by the finite set of supersingular elliptic curves over $\overline{\mathbf{F}}_p$ up to isomorphism. Their appearance is explained by noting that for a supersingular elliptic curve $E/S/\mathbf{F}_p$, the group scheme $E[p]$ represents a class in $\mathrm{Ext}^1_S(\boldsymbol{\alpha}_p, \boldsymbol{\alpha}_p)$. Given an element λ of $\mathrm{End}_S(\boldsymbol{\alpha}_p/S) \simeq \mathbf{A}^1(S)$ we obtain an endomorphism of $E[p]$ by composition

$$E[p] \longrightarrow \boldsymbol{\alpha}_p \xrightarrow{\lambda} \boldsymbol{\alpha}_p \longrightarrow E[p]. \tag{1}$$

This induces an action of $\mathbf{A}^1(S)$ on the set $\{\mathbf{F}_{p^2} \hookrightarrow \mathrm{End}_S(E[p])\}$ by adding this endomorphism to the image of a fixed algebra generator of $\mathbf{F}_{p^2}$.

1.2. **An arithmetic intersection formula.** Given the moduli interpretation of the model over $\mathrm{Spec}(\mathbf{Z})$, the arithmetic intersection numbers of two Heegner divisors P_1, P_2 may be computed as a counting problem in quaternion algebras. Contrary to the case of Gross–Zagier, the orders encountered here are not Eichler, but they are Bass. We prove the following formula for $\Delta_1, \Delta_2 < 0$ coprime:

$$\langle P_1, P_2 \rangle \quad = \quad \frac{w_1 w_2}{4} \quad \cdot \sum_{\substack{m \in \mathcal{S}(\Delta_1\Delta_2,\, p^2) \\ d \,|\, m,\ d>0}} \epsilon\left(\frac{m}{d}\right) \log(d). \tag{2}$$

Here we define:

- for $i = 1, 2$ the number of roots of unity $w_i := |\mathcal{O}_i^\times|$,
- for any positive integers $a, b \in \mathbf{Z}$ the finite set

$$\mathcal{S}(a,b) := \mathbf{Z}_{>0} \ \cap \ \left\{ \frac{a - x^2}{4b} : x \in \mathbf{Z} \right\}. \tag{3}$$

- ϵ to be the elementary character defined in Gross–Zagier [3, p. 192].

Qualitatively, this implies that all primes q at which an arithmetic intersection may take place must divide a positive integer of the form

$$q \mid \frac{\Delta_1\Delta_2 - x^2}{4p^2}.$$

Note that $q = p$ can, and frequently does, appear as a prime of intersection.

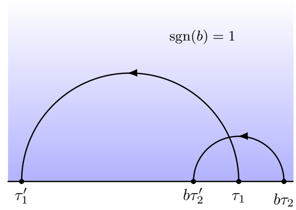
2. RM geodesics ($\Delta > 0$)

For positive discriminants $\Delta > 0$ we obtain for each quadratic form an associated geodesic P_Δ on the Riemann surface $X_{\mathrm{ns}}^{+}(p)$. In spite of their lack of algebraic structure, attempts can be made to define (frequently algebraic) invariants.

2.1. **RM Singular moduli.** In joint work with Henri Darmon [1], we constructed a p-adic quantity $\Theta_p^\times[P_1, P_2]$ associated to a pair of such geodesics. The general invariants are defined in terms of *rigid cocycles* of meromorphic functions on the p-adic half plane $\mathfrak{H}_p$. A special concrete case of the construction (accounting only for *norms* to $\mathbf{Q}_p$ of the general invariants) may be defined explicitly as:

$$\Theta_p^{\times}[P_1, P_2] := \lim_{n\to\infty} \prod_{\substack{b \in \mathcal{O}_1^* \backslash M_2(\mathbf{Z})/\mathcal{O}_2^* \\ \det(b) = p^{n!}}} \left(\frac{(\tau_1 - b\tau_2)(\tau_1' - b\tau_2')}{(\tau_1 - b\tau_2')(\tau_1' - b\tau_2)} \right)^{\mathrm{sgn}(b)}.$$

where $\mathrm{sgn}(b) \in \{0, 1, -1\}$ is the intersection of the geodesics associated to P_1 and bP_2 (connecting the roots $\{\tau_1, \tau_1'\}$ and $\{b\tau_2, b\tau_2'\}$ of their associated quadratic forms, with canonical orientation) in the complex upper half plane.



Several conjectures about these invariants were formulated in [1]. When

$$p \in \{2, 3, 5, 7, 13\},$$

they were conjectured to be algebraic, defined over the compositum H_1H_2 of Hilbert class fields. In this case, their valuation at a prime $\mathfrak{q}$ was conjectured to be equal to the q-weighted intersection number $m_{\mathfrak{q}}$ of a certain pair of geodesics $P_1^\flat, P_2^\flat$ on the Shimura curve X_{pq} associated to the indefinite quaternion algebra ramified at p and q. In particular, any prime $\mathfrak{q}$ dividing $\Theta_p^{\times}[P_1, P_2]$ should lie above a prime q which divides a positive integer of the form

$$q \mid \frac{\Delta_1\Delta_2 - x^2}{4p}.$$

These conjectures were verified numerically in hundreds of examples, thanks to the work of Rickards [6] who developed algorithms for computing intersection numbers of geodesics on the Shimura curves X_{pq}. The conjectures can be paraphrased by saying that $\Theta_p^{\times}[P_1, P_2]$ mirrors all the properties (switching ∞ and p) of the CM case described by Gross–Zagier, for the differences of singular moduli

$$\Theta_\infty^{\times}[P_1, P_2] := j(\tau_1) - j(\tau_2).$$

2.2. **RM height pairings.** In ongoing work with Henri Darmon, we define for any prime p and any pair P_1, P_2 as above, a p-adic *height symbol*

$$h(P_1, P_2) \in \mathbf{Q}_p$$

which relates to the RM singular moduli of [1] via an equality of the form

$$h(P_1, P_2) = -\log_p \Theta_p^{\times}[P_1, P_2] + \sum_{\mathfrak{q}} m_{\mathfrak{q}} \log_p u_{\mathfrak{q}}.$$

where $u_{\mathfrak{q}}$ is a certain Gross–Stark unit in $\mathcal{O}_{H_1H_2}[1/q]^{\times} \otimes \mathbf{Q}$, defined over $\mathbf{Q}_p$. The main result discussed in the talk is a modularity statement for the generating series of this height symbol. We explain how this relates to the conjectures in [1] for $p \in \{2, 3, 5, 7, 13\}$, and suggests an interpretation of the invariant for general p.

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Reduction to depth-zero for $\bar{\mathbb{Z}}[1/p]$-representations of p-adic groups

Jessica Fintzen

The category of smooth complex representations of p-adic groups decomposes into Bernstein blocks, and by a recent joint result with Adler, Mishra and Ohara [1, 2] we know that under some explicit tameness assumptions each Bernstein block is equivalent to a depth-zero Bernstein block. Depth-zero representations correspond, roughly speaking, to representations of finite groups of Lie type, as we explained in the talk via an explicit example and by indicating the general theory. This result allows one to reduce a lot of problems about representations of p-adic groups and the Langlands correspondence to their depth-zero counterparts that are often easier to solve or already known.

For number theoretic applications one would like to have a similar result when working with representations whose coefficients are a more general ring than the complex numbers. Thus, after recalling the above complex setting for motivation, we then presented analogous results for R-representations of p-adic groups where R is any commutative $\bar{\mathbb{Z}}[\mu_{4p^\infty}, \frac{1}{\sqrt{p}}]$-algebra, for example, R could be an algebraically closed field of characteristic different from p, or the ring $\bar{\mathbb{Z}}[1/p]$. This is joint work with Jean-François Dat [3]. While the result is analogous to the result with complex coefficients (except that the resulting subcategories are larger than Bernstein blocks), the proof is of a very different nature. In the complex setting the proof is achieved via type theory and an isomorphism of Hecke algebras, which are techniques that are not available for general R-representations. We sketched how we deal with the category of R-representations instead, using appropriate coefficient systems on the Bruhat–Tits building. Moreover, the subcategories can in general no longer be indexed by equivalence classes of Levi subgroups and cuspidal representations of those Levi subgroups. Instead, we parameterize the subcategories via wild inertia parameters and a cohomological invariant, whose definitions we outlined in the talk. This provides an immediate connection with the Langlands program.

For more details, we refer the interested reader to the survey article [4].

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Algebraic specialisations of the elliptic gamma function

Luis García

(joint work with Nicolas Bergeron and Pierre Charollois)

A number field K is said to be almost totally real (ATR) if it has exactly one complex place. Conjectures of Stark predict the existence of a collection of units in abelian extensions of ATR fields generalising the main properties of elliptic units. The talk discussed our recent proposal [1] that, for cubic K, such units (conjecturally) arise as special values of the elliptic gamma function.

Stark's (rank one, ATR) conjecture. Let K be an ATR number field with ring of integers $\mathcal{O}_K$. Fix a complex embedding $\iota_{\mathbb{C}} : K \hookrightarrow \mathbb{C}$ and an ideal $\mathfrak{f} \subset \mathcal{O}_K$. Let $K(\mathfrak{f})$ and $\mathrm{Cl}(\mathfrak{f})$ be the narrow ray class field and narrow ray class group of conductor $\mathfrak{f}$ respectively; we assume that $K(\mathfrak{f})$ is totally complex. Denote by $\mathrm{Art} : \mathrm{Cl}(\mathfrak{f}) \to \mathrm{Gal}(K(\mathfrak{f})/K)$ the Artin reciprocity isomorphism. To each Galois automorphism $\sigma \in \mathrm{Gal}(K(\mathfrak{f})/K)$ corresponds a partial zeta function

$$\zeta_{\mathfrak{f}}(\sigma, s) = \sum_{\substack{\mathfrak{b} \subseteq \mathcal{O}_K \\ (\mathfrak{b},\mathfrak{f})=1,\ \mathrm{Art}(\mathfrak{b})=\sigma}} \mathrm{N}\mathfrak{b}^{-s}, \qquad \mathrm{Re}(s) > 1,$$

with meromorphic continuation to $s \in \mathbb{C}$ that is regular at $s = 0$. For simplicity in stating the conjecture below, when K is quadratic we assume that $\mathfrak{f}$ is divisible by at least two prime ideals. Write $w_{\mathfrak{f}}$ for the number of roots of unity in $K(\mathfrak{f})$.

Conjecture 1 (Stark [10]). *Fix an embedding $K(\mathfrak{f}) \hookrightarrow \mathbb{C}$ extending $\iota_{\mathbb{C}}$. There exists a unit $u(\mathfrak{f})_{\mathrm{St}} \in \mathcal{O}_{K(\mathfrak{f})}^{\times}$ such that $K(\mathfrak{f})(u(\mathfrak{f})_{\mathrm{St}}^{1/w_{\mathfrak{f}}})$ is abelian over K and satisfying*

- $\zeta'_{\mathfrak{f}}(\sigma, 0) = -\frac{1}{w_{\mathfrak{f}}} \log |\sigma(u(\mathfrak{f})_{\mathrm{St}})|^2$ *for every* $\sigma \in \mathrm{Gal}(K(\mathfrak{f})/K)$,
- $|u(\mathfrak{f})_{\mathrm{St}}|_v = 1$ *for every archimedean place v of $K(\mathfrak{f})$ not lying above $\iota_{\mathbb{C}}$.*

Note that the conjecture specifies all valuations of $u(\mathfrak{f})_{\mathrm{St}} \in K(\mathfrak{f})^{\times}$ and so determines it up to a root of unity in $K(\mathfrak{f})$; we can remove this ambiguity by considering the unit

$$u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}} := u(\mathfrak{f})_{\mathrm{St}}^{\mathrm{Art}(\mathfrak{a})} / u(\mathfrak{f})_{\mathrm{St}}^{\mathrm{N}\mathfrak{a}}$$

for a well-chosen ideal $\mathfrak{a}$. However, the conjecture does not provide an analytic construction for $u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}$, only for its absolute values $|u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}|_v$ for archimedean v.

Question. Are there meromorphic (transcendental) functions that allow to compute $u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}$ by specialising at inputs in K?

When K is (imaginary) quadratic this question has a well-known positive answer: the relevant functions are (finite products of) classical theta functions

$$\theta(z,\tau) = \prod_{j\geq 0}(1-e^{2\pi i(j\tau+z)})(1-e^{2\pi i((j+1)\tau - z)}), \quad z\in\mathbb{C}, \quad \mathrm{Im}(\tau)>0,$$

which recover $u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}$ by specialisation to quadratic irrationals. For K of higher degree conjectural formulas for $u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}$ have been proposed in work of Ren–Sczech [9] (K cubic, using Shintani's method) and Charollois–Darmon [2] (K quartic containing a real quadratic subfield).

The elliptic gamma function. For complex numbers z,τ,σ with $\mathrm{Im}(\tau)$ and $\mathrm{Im}(\sigma)$ positive, the elliptic gamma function is defined by the convergent doubly infinite product

$$\Gamma(z,\tau,\sigma) = \prod_{j,k=0}^{\infty} \frac{1-e^{2\pi i(-z+(j+1)\tau+(k+1)\sigma)}}{1-e^{2\pi i(z+j\tau+k\sigma)}}. \tag{1}$$

This product defines a meromorphic function of z with simple poles and zeroes. It has interesting automorphy properties under the group

$$\Gamma_3 = \mathrm{SL}_3(\mathbb{Z}) \ltimes \mathbb{Z}^3$$

that were studied in detail in [4]. These properties were reinterpreted in a more geometric way in [3]: let X_3 denote the complex 3-dimensional manifold given by the total space of the dual tautological line bundle over $\mathbb{P}^2(\mathbb{C})-\mathbb{P}^2(\mathbb{R})$. The space X_3 admits a natural action of Γ_3 and a natural Γ_3-equivariant open cover $\mathcal{U}$. The function (1) arises as a component of an explicitly constructed cocycle representing a class

$$[\Gamma] \in \mathrm{H}^1(\mathcal{U},\Gamma_3,\mathcal{M}^\times/\mathcal{O}^\times)$$

in the first equivariant Čech cohomology group. This viewpoint allows to think of the elliptic Gamma function as a "Γ_3–analog" of the theta function $\theta(z,\tau)$, whose automorphy properties under $\mathrm{SL}_2(\mathbb{Z})\ltimes\mathbb{Z}^2$ are well-known.

Complex cubic fields. Fix a complex cubic field K, a complex embedding $\iota_{\mathbb{C}}: K\hookrightarrow\mathbb{C}$ and an ideal $\mathfrak{f}$ such that $K(\mathfrak{f})$ is totally complex. We also fix ideals $\mathfrak{a},\mathfrak{b}$ of $\mathcal{O}_K$ and an element $h\in K$ subject to certain coprimality and congruence conditions. In [1] we explain how to attach to these data a finite collection of elements $(z_i,\tau_i,\sigma_i)\in K^3$, $i\in I$, such that a certain finite (signed) product

$$\mathbf{\Gamma}_{\mathfrak{a},h}(\mathfrak{f}\mathfrak{b}^{-1}) = \prod_{i\in I}\Gamma(z_i,\tau_i,\sigma_i)^{\pm 1}$$

is a non-zero complex number.

Conjecture 2. [1, §2.3] *Fix a complex embedding* $K(\mathfrak{f}) \hookrightarrow \mathbb{C}$ *extending* $\iota_{\mathbb{C}}$. *Then:*

- *The number* $u_{\mathfrak{a}}(\mathfrak{f}\mathfrak{b}^{-1}) = \mathbf{\Gamma}_{\mathfrak{a},h}(\mathfrak{f}\mathfrak{b}^{-1})$ *is independent of* h *and lies in* $\mathcal{O}_{K(\mathfrak{f})}^{\times}$.
- *If* v *is an archimedean place of* $K(\mathfrak{f})$ *not dividing* $\iota_{\mathbb{C}}$, *then* $|u_{\mathfrak{a}}(\mathfrak{f}\mathfrak{b}^{-1})|_v = 1$.
- *The following reciprocity law holds:* $u_{\mathfrak{a}}(\mathfrak{f})^{\mathrm{Art}(\mathfrak{b})} = u_{\mathfrak{a}}(\mathfrak{f}\mathfrak{b}^{-1})$ *if* $(\mathfrak{b}, \mathfrak{f}) = 1$.

The conjecture is supported by numerous numerical examples provided in [1]. Note also that the conjecture specifies that $|u_{\mathfrak{a}}(\mathfrak{f})|_w = 1$ for every place w of $K(\mathfrak{f})$ except for places lying above the complex place of K. For the remaining places we prove the following theorem, which is a higher degree analog of the classical Kronecker limit formula.

Theorem 3. [1] *For any ideal* $\mathfrak{b}$ *coprime to* $\mathfrak{f}$ *we have*

$$\zeta_{\mathfrak{f}}'(\mathrm{Art}(\mathfrak{a}\mathfrak{b}), 0) - \mathrm{N}\mathfrak{a} \cdot \zeta_{\mathfrak{f}}'(\mathrm{Art}(\mathfrak{b}), 0) = \log |\Gamma_{\mathfrak{a},h}(\mathfrak{f}\mathfrak{b}^{-1})|^2.$$

The proof proceeds by defining an Eisenstein series and computing its periods along two different 1-chains. Comparing the two conjectures one concludes that the Stark unit $u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}}$ and the conjectural unit $u_{\mathfrak{a}}(\mathfrak{f})$ are related by

$$u_{\mathfrak{a}}(\mathfrak{f})_{\mathrm{St}} = u_{\mathfrak{a}}(\mathfrak{f})^{-w_{\mathfrak{f}}}.$$

The elliptic gamma function is part of a family of a functions having analogous transformation properties under $\mathrm{SL}_n(\mathbb{Z}) \ltimes \mathbb{Z}^n$ for any $n \geq 2$ [7, 8]. Using these functions, in his PhD thesis Pierre Morain generalises the conjectures in [1] to higher degree ATR fields; he provides ample numerical evidence when $[K : \mathbb{Q}] \leq 6$ (see [5, 6]).

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Arithmetic of Fourier coefficients on G_2

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(joint work with Petar Bakić, Siyan Daniel Li-Huerta, Naomi Sweeting)

In 1973, Shimura [9] associated with a classical modular form f of weight $2k$, a holomorphic modular form $\mathcal{F}_f$ of weight $k+\frac{1}{2}$ such that $T_{p^2}\mathcal{F}_f = T_p f$. In the early 80s, Waldspurger [7] reinterpreted his construction as a theta lift and discovered a remarkable relationship between the Fourier coefficients of $\mathcal{F}_f$ and the L-values of quadratic twists of f: for $D > 0$ squarefree and coprime to the level of f,

$$|a_D(\mathcal{F}_f)|^2 = L(f \times \chi_{-D}, k) \cdot \mathrm{disc}(\mathbb{Q}(\sqrt{-D}))^{k-\frac{1}{2}} \cdot (*)$$

and $(*)$ is a constant which can be made explicit. Here, χ_{-D} is the quadratic character associated with $\mathbb{Q}(\sqrt{-D})$, i.e. $\mathrm{Ind}_{\mathbb{Q}(\sqrt{-D})}^{\mathbb{Q}} 1 = 1 \oplus \chi_{-D}$.

This led to several arithmetic applications. To name a few, Waldspurger's result was used by

(1) Tunnell [8] to give a (partial) resolution of the classical congruent number problem,
(2) Skinner–Ono [10] to give qualitative results about non-vanishing of quadratic twists of L-functions of modular forms,
(3) Gross–Kohnen–Zagier [4]/Gross [5] used it to study variation of Heegner cycles associated with imaginary quadratic fields on modular curves/genus 0 curves.

The goal of our work is to study a similar phenomenon for *cubic* twists of L-functions of modular forms, i.e. we would like to replace the imaginary quadratic field with a cubic field E. Somewhat surprisingly, it turns out that Fourier coefficients of certain modular forms on the split algebraic group G_2 over $\mathbb{Q}$ seem to have the analogous properties for real cubic fields.

Our first result is the analogue of Shimura's theorem.

Theorem 1 ([1]). *Associated with a dihedral holomorphic cuspidal eigenform f of weight $2k$ such that $L(f, k) \neq 0$ (and some extra data), there exists a quaternionic modular form $\mathcal{G}_f$ on G_2 of weight k such that all the local representations of the automorphic representation of $\mathcal{G}_f$ are explicitly determined in terms of those of f (and the extra data). More specifically, the Arthur multiplicity formula holds for long-root A-packets associated with such f.*

The term *quaternionic modular form* here replaces the classical notion of *holomorphic* modular form: since $G_2(\mathbb{R})$ has no holomorphic discrete series, the next best thing is to ask for the representations at infinity to be quaternionic discrete series.

Our second result is the analogue of Waldspurger's theorem.

Theorem 2 ([2]). *For a totally real cubic field E (satisfying a condition at p dividing the level of f),*

$$|a_{\mathcal{O}_E}(\mathcal{G}_f)|^2 = L(f \times V_E, k) \cdot \mathrm{disc}(E)^{k-\frac{1}{2}}.$$

Here, V_E is the quadratic character associated with E, i.e. $\mathrm{Ind}_E^{\mathbb{Q}} 1 = 1 \oplus V_E$.

This relationship was conjectured by Gross in 2000 for any cusp eigenforms f (although in this generality $\mathcal{G}_f$ exists only conjecturally). Here, $a_{\mathcal{O}_E}(\mathcal{G}_f)$ are the Fourier coefficients of the quaternionic modular form $\mathcal{G}_f$ introduced in [3, 6] which are labeled by cubic rings.

It would be interesting to study some arithmetic applications of this result. In the final part of the talk, I described the analogue of application (3) above. Classically, the work of Waldspurger allowed Gross–Kohnen–Zagier and Gross to study the variation of Heegner divisors $\Delta(D)$ associated with $D \geq 1$. More specifically, they proved that the generating series

$$(\text{constant term}) + \sum_{D \geq 1} [\Delta(D)] q^D \in \mathrm{Pic}(X)[[q]]$$

is the q-expansion of a modular form of weight $\frac{3}{2}$ valued in $\mathrm{Pic}(X)$, where:

- ([4]) X is a modular curve,
- ([5]) X is a genus 0 curve associated with a definite quaternion algebra.

In ongoing work, I have been studying Heegner cycles associated with totally real cubic fields. More specifically, for an imaginary quadratic field K, I consider a definite (projective) unitary group in 3 variables. The associated Shimura variety for is 0-dimensional and parametrizes certain abelian threefolds with $\mathcal{O}_K$-multiplication. I define a certain rational fourfold X living over it, which plays the role of Gross's genus 0 curve. Associated with any totally real cubic ring A, I then define a Heegner cycle $\Delta(A)$ of codimension 4 on X. Roughly, it parametrizes abelian threefolds A with $\mathcal{O}_K$-mulplication which have CM by the larger ring $A \otimes_{\mathbb{Z}} \mathcal{O}_K$.

Theorem 3 (expected)**.** *The generating series*

$$\textit{(constant term)} + \sum_A [\Delta(A)] q^A$$

defines a quaternionic modular form of weight 4 *on* G_2 *valued in* $\mathrm{CH}^4(X)$*.*

This result initiates the study of a version of the Kudla program where the generating series is a modular form an exceptional group (rather than a symplectic group). This possibility was suggested to me by Luis Garcia and Jens Funke.

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Greenberg's question for Siegel modular forms

Shaunak Deo
(joint work with Bharathwaj Palvannan)

A famous question posed (independently) by Greenberg and Coleman is as follows:

Question 1 (Greenberg and Coleman)**.** *Let f be a p-ordinary cuspidal eigenform with weight $k \geq 2$ and level $\Gamma_1(N)$ with Hecke eigenvalues in $\overline{\mathbb{Q}_p}$. Let ρ_f be the p-adic Galois representation attached to it. If $\rho_f |_{G_{\mathbb{Q}_p}}$ is* decomposable*, then does f have* complex multiplication*?*

Ghate–Vatsal studied an analogue of this question by considering a Λ-adic Hida family F, instead of a single ordinary cuspidal eigenform f. They proved the following theorem.

Theorem (Ghate–Vatsal [3])**.** Suppose that the following hypotheses hold for the residual Galois representation $\overline{\rho}_F$ associated to the Λ-adic Hida family F:

(i) The restriction of $\overline{\rho}_F$ to $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}(\sqrt{(-1)^{(p-1)/2}p})$ is absolutely irreducible.
(ii) The restriction of $\overline{\rho}_F$ to the decomposition group at p is p-distinguished.

Then, the following statements are equivalent.

(1) F is a CM Hida family.
(2) There are infinitely many classical weight one specializations.
(3) The restriction $\rho_F |_{G_{\mathbb{Q}_p}}$, of the Λ-adic Galois representation ρ_F associated to the Hida family F to the decomposition group at p, is decomposable.

In this report, we address the following question:

Question 2. *Does an analogue of the theorem of Ghate–Vatsal hold in the setting of Siegel modular forms of genus 2?*

Set-up. We now describe the set-up and objects with which we will be working. Let $p \geq 5$ denote a prime. Let K be a real quadratic field where the prime p splits as $\mathfrak{p}_1\mathfrak{p}_2$. Denote its ring of integers by O_K and its discriminant by Δ_K. We fix embeddings $\iota : \overline{\mathbb{Q}} \hookrightarrow \mathbb{C}$ and $\iota_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}_p}$. There are two embeddings

$\sigma_1, \sigma_2 : K \hookrightarrow \overline{\mathbb{Q}}$. The prime $\mathfrak{p}_1$ is determined by $\iota_p \circ \sigma_1$. We choose the ordering of the weights of Hilbert modular forms over K with respect to $\iota \circ \sigma_1$ and $\iota \circ \sigma_2$ respectively. Let M be a *squarefree* integer *co-prime* to $p\Delta_K$ and $\mathfrak{M}$ be the ideal of O_K generated by M.

Let g_0 be a *p-ordinary* cuspidal Hilbert modular newform over K, with weight (κ_1, κ_2), *trivial nebentypus* and level $\mathfrak{M}$. Here p-ordinary means that the Hecke eigenvalues of the *normalized* Hecke operators $T_{\mathfrak{p}_1}$ and $T_{\mathfrak{p}_2}$ are p-adic units (under the embedding ι_p). We further assume that $\kappa_1 \equiv \kappa_2 \pmod 2$ and $\kappa_1 \geq \kappa_2 \geq 2$. Let ρ_{g_0} be the p-adic Galois representation attached to g_0 and let $\rho_0 = \mathrm{Ind}_{G_K}^{G_\mathbb{Q}} \rho_{g_0}$. So ρ_0 is unramified outside primes dividing $pM\Delta_K$. Let $\bar{\tau}_0$ be the residual mod p representation associated to g_0 and $\bar{\rho}_0 = \mathrm{Ind}_{G_K}^{G_\mathbb{Q}} \bar{\tau}_0$. Suppose both $\bar{\rho}_0$ and $\bar{\tau}_0$ are *absolutely irreducible*.

Stable Yoshida lifts: Under the assumptions above, it follows, from the works of Yoshida, Böcherer–Schulze-Pillot, Roberts and Hsieh–Namikawa, that there exists a *p-ordinary* cuspidal Siegel modular eigenform f_0 of genus 2 which is a *stable Yoshida lift* (or *automorphic induction*) of g_0. Note that the weight of f_0 is $\left(\frac{\kappa_1 + \kappa_2}{2}, \frac{\kappa_1 - \kappa_2}{2} + 2\right)$ and its level is $\Gamma_0^{(2)}(M\Delta_K)$ (i.e. the Siegel congruence subgroup of level $M\Delta_K$). Here, stable Yoshida lift means that if ℓ is a prime not dividing $pM\Delta_K$, then the ℓ-th Hecke polynomial of f_0 is the characteristic polynomial of $\rho_0(\mathrm{Frob}_\ell)$.

Hecke algebras: Let $\mathbb{T}$ be the local component of the ordinary GSp_4-Hida Hecke algebra passing through f_0 with tame level $\Gamma_0^{(2)}(M\Delta_K)$. This Hecke algebra is a local ring and is generated by the Hecke operators $T_{\ell,0}$, $T_{\ell,1}$ and $T_{\ell,2}$ for all primes $\ell \nmid pM\Delta_K$, along with the Hecke operators $U_{p,1}$ and $U_{p,2}$ (over a suitable finite extension of $\mathbb{Z}_p$). Note that $\mathbb{T}$ is a finite integral extension of $\Lambda = \mathbb{Z}_p[\![T_1, T_2]\!]$ where T_1 and T_2 are 'weight variables'.

Let η be a minimal prime of $\mathbb{T}$. We call η a *Yoshida prime* if every *classical* specialization of $\mathbb{T}/\eta$ with *cohomological* weight is an automorphic induction of a Hilbert modular eigenform over K as described above. We call the corresponding component as Yoshida component. Let I be the intersection of all Yoshida primes of $\mathbb{T}$ and let $\mathbb{T}^{\mathrm{Yos}} := \mathbb{T}/I$.

Galois representations: Let S be the set of primes dividing $pM\Delta_K$ and $G_{\mathbb{Q},S}$ be the Galois group of the maximal extension of $\mathbb{Q}$ unramified outside of S over $\mathbb{Q}$. Thus it follows, from a standard argument involving the density of classical points, that there exists a representation $\rho : G_{\mathbb{Q},S} \to \mathrm{GL}_4(\mathbb{T})$ lifting $\bar{\rho}_0$ (satisfying the usual nice properties).

Let $G_{K,S}$ be the image of G_K under the natural surjective map $G_\mathbb{Q} \twoheadrightarrow G_{\mathbb{Q},S}$ and ρ^{Yos} be the $\mathbb{T}^{\mathrm{Yos}}$-valued representation obtained by composing ρ with the natural surjective map $\mathbb{T} \twoheadrightarrow \mathbb{T}^{\mathrm{Yos}}$. From the definition of $\mathbb{T}^{\mathrm{Yos}}$, it follows that there exists a representation $\tau : G_{K,S} \to \mathrm{GL}_2(\mathbb{T}^{\mathrm{Yos}})$ lifting $\bar{\tau}_0$ such that $\rho^{\mathrm{Yos}} \simeq \mathrm{Ind}_{G_{K,S}}^{G_{\mathbb{Q},S}} \tau$. Fix a lift σ_0 of the non-trivial element of $\mathrm{Gal}(K/\mathbb{Q})$ in $G_{\mathbb{Q},S}$ and let $\tau^c : G_{K,S} \to \mathrm{GL}_2(\mathbb{T}^{\mathrm{Yos}})$ be the representation given by $\tau^c(g) := \tau(\sigma_0 g \sigma_0^{-1})$.

Selmer group: Let $V = \mathrm{Hom}_{\mathbb{T}^{\mathrm{Yos}}}(\tau^c, \tau)$. So V is a free module over $\mathbb{T}^{\mathrm{Yos}}$ of rank 4 with an action of $G_{K,S}$. Let $(\mathbb{T}^{\mathrm{Yos}})^\vee$ be the Pontryagin dual of $\mathbb{T}^{\mathrm{Yos}}$. Define $\mathrm{Sel}(V \otimes (\mathbb{T}^{\mathrm{Yos}})^\vee)$ to be the subgroup of $H^1(G_{K,S}, V \otimes (\mathbb{T}^{\mathrm{Yos}})^\vee)$ consisting of all cohomology classes which are *trivial* when restricted to $\mathfrak{p}_1$ and $\mathfrak{p}_2$ and *unramified* at all other places. Denote the Pontryagin dual of $\mathrm{Sel}(V \otimes (\mathbb{T}^{\mathrm{Yos}})^\vee)$ by $\mathrm{Sel}(V \otimes (\mathbb{T}^{\mathrm{Yos}})^\vee)^\vee$.

Theorem 3 (D-Palvannan)**.** *Suppose we are in the set-up as above. Moreover, assume the following:*

(1) $\bar{\rho}_0$ is p-distinguished and it has 'big' image,
(2) The tame Artin conductor of $\bar{\tau}_0$ is $\mathfrak{M}$ and the restriction of $\bar{\tau}_0$ to both local decomposition groups $G_{K_{\mathfrak{p}_1}}$ and $G_{K_{\mathfrak{p}_2}}$ is indecomposable*,*
(3) primes dividing $M\Delta_K$ satisfy certain congruence conditions,
(4) Sel$(V \otimes (\mathbb{T}^{Yos})^\vee)^\vee$ is a pseudo-null *Λ-module and a* cyclic *$\mathbb{T}^{Yos}$-module.*

Let η be a minimal prime of $\mathbb{T}$. Then $\rho \pmod{\eta}|_{G_{\mathbb{Q}_p}}$ is decomposable if and only if η is a Yoshida prime.

We refer the reader to the introduction of [2] for precise hypotheses. Note that this is an analogue of the equivalence (1) $\iff$ (3) of the theorem of Ghate and Vatsal in our setting.

It is natural to ask what can be concluded if we relax the cyclicity hypothesis on $\mathrm{Sel}(V \otimes (\mathbb{T}^{\mathrm{Yos}})^\vee)^\vee$. In this direction, we prove the following theorem

Theorem 4 (D-Palvannan)**.** *Suppose we are in the set-up as above. Assume the hypotheses* (1)-(3) *of Theorem 3 hold. Assume Sel$(V \otimes (\mathbb{T}^{Yos})^\vee)^\vee$ is a* pseudo-null *Λ-module. If $\mathbb{T} \not\simeq \mathbb{T}^{Yos}$, then there exists a minimal prime η of $\mathbb{T}$ such that $\rho \pmod{\eta}|_{G_{\mathbb{Q}_p}}$ is* not *decomposable.*

Note that the analogue of weight 1 modular forms in our setting is Siegel modular forms of genus 2 with weight $(k, 2)$. In the setup above, we also prove, after replacing the Selmer group above with a slightly bigger one, a characterization of Yoshida components in terms of density of points with weight $(k, 2)$ whose associated p-adic Galois representations are de Rham at p. This is an analogue of the equivalence (1) $\iff$ (2) of the theorem of Ghate and Vatsal. We refer the reader to [2] for more details.

The method of Ghate–Vatsal crucially uses two arithmetic inputs. The first arithmetic input is the fact that Galois representations associated to classical weight one elliptic modular eigenforms are Artin representations. The second arithmetic input is a modularity lifting theorems of Buzzard and Buzzard–Taylor.

These inputs are not available in our setting. So we use Ribet's method. A similar approach in the GL_2-case was taken by Castella–Wang-Erickson ([1]). However, we need to overcome various obstacles which are not present in the GL_2 case. Proving a *minimal* $R = \mathbb{T}$ theorem, where R is the minimal, p-ordinary GSp_4-valued deformation ring of $\bar{\rho}_0$, is a key step in the proofs of Theorems 3 and 4. This $R = \mathbb{T}$ theorem is similar to the ones proved by Genestier–Tilouine and Pilloni and most of our assumptions are required to prove this theorem. The assumptions on Selmer groups are similar to the ones appearing in the work of Castella–Wang-Erickson. We also have an example to illustrate that just the pseudo-nullity assumption on the Selmer group and Ribet's method are not sufficient to get the characterization of the Yoshida components obtained in Theorem 3.

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Intersection Cohomology of p-adic Shimura varieties

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(joint work with Ana Cariani, Mingjia Zhang)

We fix a Shimura datum (G,X) with reflex field $E/\mathbb{Q}$, which we assume to be of PEL type AC. For $K\subset \mathsf{G}(\mathbb{A}_f)$ a sufficiently small compact open subgroup, we consider the Shimura varieties attached to (G,X) together with their projective minimal compactifications

$$j_K:\mathrm{Sh}(\mathsf{G},\mathsf{X})_K\hookrightarrow \mathrm{Sh}^*(\mathsf{G},\mathsf{X})_K.$$

This defines a tower of varieties of dimension d over $\mathrm{Spec}(E)$ equipped with an action of $\mathsf{G}(\mathbb{A}_f)$. For $?\in\{!,*,!*\}$ and ℓ a prime number, we can consider the ℓ-adic cohomology groups

$$R\Gamma_?(\mathsf{G},\mathsf{X})_K := R\Gamma(\mathrm{Sh}^*(\mathsf{G},\mathsf{X})_{K,\overline{E}}, j_{K?}(\overline{\mathbb{Q}}_\ell[d])),$$

which will be modules under the absolute Galois group $\mathrm{Gal}(\overline{E}/E)$ and the smooth Hecke algebra H_K of level K.

We are interested in understanding the local structure of these complexes with respect to these two actions at a fixed prime $p\neq l$. To this aim, we fix an isomorphism $\overline{\mathbb{Q}}_p\simeq\mathbb{C}$, and let $\mathfrak{p}|p$ be the associated place of E defined by it. We write $E_\mathfrak{p}$ for the completion of E at the place $\mathfrak{p}$, and let $C:=\hat{\overline{E}}_\mathfrak{p}$ denote the completion of its algebraic closure. We let $G:=\mathsf{G}_{\mathbb{Q}_p}$ be the local group at p. Writing $K^pK_p=K$ for $K_p\subset G(\mathbb{Q}_p)$ a compact open, we may consider the complexes

$$R\Gamma_?(\mathsf{G},\mathsf{X})_{K^p} := \mathrm{colim}_{K_p\to 1}\, R\Gamma_?(\mathsf{G},\mathsf{X})_{K^pK_p}(d/2) \tag{1}$$

of $\overline{\mathbb{Q}}_\ell$-vector spaces. It is equipped with a smooth $G(\mathbb{Q}_p)$-action and a continuous action of $W_{E_\mathfrak{p}}$, the Weil group of $E_\mathfrak{p}$. We denote the derived category of such complexes by $\mathrm{D}(G(\mathbb{Q}_p),\overline{\mathbb{Q}}_\ell)^{BW_{E_\mu}}$.

The point of restricting to the local information at p is that it is captured in a more flexible geometric object witnessing new structures on the cohomology. Namely, we write

$$\mathcal{S}_{K_p} := \mathrm{Sh}(\mathsf{G},\mathsf{X})^{\mathrm{ad}}_{K^pK_p,C} \text{ and } \mathcal{S}^*_{K_p} := \mathrm{Sh}(\mathsf{G},\mathsf{X})^{*,\mathrm{ad}}_{K^pK_p,C}$$

for the adic spaces over C attached to the Shimura varieties. Taking inverse limits over the level at $K_p \subset G(\mathbb{Q}_p)$ and completing the structure sheaf, we obtain a natural $G(\mathbb{Q}_p)$-equivariant open immersion

$$j : \mathcal{S}_\infty \hookrightarrow \mathcal{S}^*_\infty$$

of Shimura varieties at infinite level, equipped with a natural $G(\mathbb{Q}_p)$-equivariant period morphism

$$\pi_{\mathrm{HT}} : \mathcal{S}^*_\infty \to \mathcal{F}\ell_{G,\mu^{-1}} \tag{2}$$

recording the Hodge-Tate filtration on the p-adic cohomology of the abelian varieties that $\mathcal{S}_\infty$ parametrizes [1]. Here $\mu \in \mathbb{X}_*(G_{\overline{\mathbb{Q}}_p})$ is the Hodge cocharacter of (G,X) transported under the isomorphism $i : \overline{\mathbb{Q}}_p \xrightarrow{\simeq} \mathbb{C}$, and $\mathcal{F}\ell_{G,\mu^{-1}}$ is the associated flag variety. We may pass to the stack quotient by $G(\mathbb{Q}_p)$ and (2) gives us a map

$$[\mathcal{S}^*_\infty/\underline{G(\mathbb{Q}_p)}] \xrightarrow{\pi} [\mathcal{F}\ell_{G,\mu^{-1}}/\underline{G(\mathbb{Q}_p)}] \xrightarrow{h^{\rightarrow}} [\mathrm{Spd}(C)/\underline{G(\mathbb{Q}_p)}]. \tag{3}$$

Now, we have an identification

$$\mathrm{D}_{\mathrm{lis}}([\mathrm{Spd}(C)/\underline{G(\mathbb{Q}_p)}], \overline{\mathbb{Q}}_\ell) \simeq \mathrm{D}(G(\mathbb{Q}_p), \overline{\mathbb{Q}}_\ell)$$

between suitably normalized lisse-étale sheaves [2, VII.7] on the classifying stack and the derived category of smooth $G(\mathbb{Q}_p)$-representations. It is not too hard to check[1] that the collection of sheaves $\{j_{K^pK_p?}(\overline{\mathbb{Q}}_\ell[d])\}_{K_p\to\{1\}}$ for $? \in \{!, *, !*\}$ descend to sheaves on this stack quotient $[\mathcal{S}^*_\infty/\underline{G(\mathbb{Q}_p)}]$ such that, upon applying the pushforward $h^{\rightarrow}_*\pi_*$, it computes precisely the complexes (1) as a smooth $G(\mathbb{Q}_p)$-representation.

This reinterpretation of (1) allows us to observe a new structure. Indeed, the stack $[\mathcal{F}\ell_{G,\mu^{-1}}/\underline{G(\mathbb{Q}_p)}]$ parametrizes flags of C-vector spaces, and C is the residue field of a certain curve X_C known as the Fargues-Fontaine curve at a fixed closed point $\infty \in X_C$. Beauville-Laszlo gluing and the Bialynicka-Birula isomorphism therefore allow us to reinterpret this as a moduli space of modifications $\mathcal{E} \dashrightarrow \mathcal{E}_1$ of type μ at ∞. The map $h^{\rightarrow}$ remembers the trivial G-bundle $\mathcal{E}_1$. In particular, we may interpret $[\mathrm{Spd}(C)/\underline{G(\mathbb{Q}_p)}] \simeq \mathrm{Bun}^1_{G,C} \xrightarrow{j_1} \mathrm{Bun}_{G,C}$, as the locus defined by the trivial G-bundle inside the moduli stack $\mathrm{Bun}_{G,C}$ of G-bundles on X_C [2]. Similarly, we may consider the map

$$[\mathcal{F}\ell_{G,\mu^{-1}}/\underline{G(\mathbb{Q}_p)}] \xrightarrow{h^{\leftarrow}} \mathrm{Bun}_G. \tag{4}$$

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[1] Indeed, one can define the descent of these sheaves by taking appropriate extensions along $j : [\mathcal{S}_\infty/\underline{G(\mathbb{Q}_p)}] \hookrightarrow [\mathcal{S}^*_\infty/\underline{G(\mathbb{Q}_p)}]$, as essentially follows from the fact that $[\mathcal{S}_\infty/\underline{G(\mathbb{Q}_p)}] \to [\mathcal{S}_\infty/\underline{K_p}]$ is finite étale and the fact that that $[\mathcal{S}_\infty/\underline{K_p}] \to \mathcal{S}_{K_p}$ is cohomologically proper of cohomological amplitude 0 if K_p is sufficiently small.

remembering the bundle $\mathcal{E}$. The set of G-bundles $\mathcal{E}_b$ on X_C are parametrized by elements b in an explicit set $B(G)$. In particular, the stack Bun_G admits a locally closed stratification $j_b : \mathrm{Bun}_G^b \hookrightarrow \mathrm{Bun}_G$ indexed by $b \in B(G)$ such that one has an identification $\mathrm{D}_{\mathrm{lis}}(\mathrm{Bun}_G^b, \overline{\mathbb{Q}}_\ell) \simeq \mathrm{D}(G_b(\mathbb{Q}_p), \overline{\mathbb{Q}}_\ell)$ with smooth representations of G_b, an inner form of a Levi of a quasi-split inner form of G attached to b. Pulling this stratification back along $\pi \circ h^{\leftarrow}$, we obtain a $G(\mathbb{Q}_p)$-equivariant stratification on $[\mathcal{S}_\infty^{b,*}/\underline{G(\mathbb{Q}_p)}] \hookrightarrow [\mathcal{S}_\infty^*/\underline{G(\mathbb{Q}_p)}]$ which will only be non-empty if $b \in B(G, \mu)$ (I.e the Harder-Narasimhan slopes of $\mathcal{E}_b$ are bounded by μ). By applying excision with respect to this stratification, for $? \in \{!, *, !*\}$ we obtain a filtration in $\mathrm{D}(G(\mathbb{Q}_p), \overline{\mathbb{Q}}_\ell)^{BW_{E_\mu}}$ on (1) which we refer to as "Mantovan's filtration" with graded denoted by

$$\mathrm{gr}_{b,?} \in \mathrm{D}(G(\mathbb{Q}_p), \overline{\mathbb{Q}}_\ell)^{BW_{E_\mu}} \tag{5}$$

for $b \in B(G, \mu)$. The basic question we are interested in is: "What is the structure of $\mathrm{gr}_{b,?}$?" , especially when $? = !*$ (i.e when we are studying the filtration on intersection cohomology). This may be understood through the following diagram of spaces with Cartesian squares introduced by Zhang [3] and further refined by Kim [4][2]

$$\begin{array}{ccccccc}
\left[\mathcal{S}_\infty/\underline{G(\mathbb{Q}_p)}\right] & \xrightarrow{j} & \left[\mathcal{S}_\infty^*/\underline{G(\mathbb{Q}_p)}\right] & \xrightarrow{\pi} & \left[\mathcal{F}\ell_{G,\mu^{-1}}/\underline{G(\mathbb{Q}_p)}\right] & \xrightarrow{h^{\rightarrow}} & \left[\mathrm{Spd}(C)/\underline{G(\mathbb{Q}_p)}\right] \\
\downarrow & & \downarrow{\scriptstyle \tilde{h}^{\leftarrow}} & & \downarrow{\scriptstyle h^{\leftarrow}} & & \\
\mathrm{Igs} & \xrightarrow{j_{\mathrm{Igs}}} & \mathrm{Igs}^* & \xrightarrow{\overline{\pi}} & \mathrm{Bun}_G, & &
\end{array}$$

where Igs^* is a certain stack capturing information about Igusa varieties. Now the map $\tilde{h}^{\leftarrow}$ is smooth[3], so $*$-pullback satisfies base-change against $j_{\mathrm{Igs}?}$ for $? \in \{!, *, !*\}$ this in particular allows us to descend the relevant sheaves on $[\mathcal{S}_\infty^*/\underline{G(\mathbb{Q}_p)}]$ to obtain sheaves on Igs^*, and, by pushing forward along $\overline{\pi}$, we obtain sheaves $\mathcal{F}_? \in \mathrm{D}_{\mathrm{lis}}(\mathrm{Bun}_G, \overline{\mathbb{Q}}_\ell)$ for $? \in \{!, *, !*\}$, which will satisfy $h_*^{\rightarrow} h^{\leftarrow *}\mathcal{F}_?[d](\frac{d}{2}) \simeq R\Gamma_?(\mathsf{G}, \mathsf{X})_{K^p}$. If we set $j_b^*\mathcal{F}_? := V_{b,?} \in \mathrm{D}_{\mathrm{lis}}(\mathrm{Bun}_G^b, \overline{\mathbb{Q}}_\ell) \simeq \mathrm{D}(G_b(\mathbb{Q}_p), \overline{\mathbb{Q}}_\ell)$ for $? = \{!, *, !*\}$ then, by applying excision with respect to the locally closed stratification by $j_b : \mathrm{Bun}_G^b \hookrightarrow \mathrm{Bun}_G$, we obtain

$$\mathrm{gr}_{b,?} := V_{b,?} \otimes_{\mathcal{H}(J_b)} R\Gamma_c(G, b, \mu), \tag{6}$$

where $R\Gamma_c(G, b, \mu)$ computes the cohomology (suitably normalized) of the space of modifications $\mathcal{E}_b \dashrightarrow \mathcal{E}_0$ of type μ on X_C at $\infty \in X_C$ equipped with a smooth commuting actions of $G_b(\mathbb{Q}_p)$ and $G(\mathbb{Q}_p)$ and a continuous action of $W_{E_\mathfrak{p}}$. Therefore, to answer our original question it suffices to compute the complexes $V_{b,?}$. This has a long history.

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[2]Strictly speaking, one needs to assume here that the codimension of the boundary of the minimal compactification is greater than or equal to 2, and we will implicitly do this in what follows.

[3]It is essentially a fibration in the flag variety $\mathcal{F}\ell_{G,\mu}$ by virtue of the diagram being Cartesian

Theorem 1. *For each $b \in B(G,\mu)$, there exists a quasi-affine perfect variety $\mathrm{Ig}^b/\overline{\mathbb{F}}_p$ known as an Igusa variety with affine closure $g_b : \mathrm{Ig}^b \hookrightarrow \mathrm{Ig}^{b,*}$ which arises as an inverse limit of finite type varieties $g_{b,m} : \mathrm{Ig}^b_m \hookrightarrow \mathrm{Ig}^{b,*}_m$ parametrizing abelian varieties with a trivialization of their p^m-torsion. For each $b \in B(G,\mu)$, we have the following description of the complexes $V_{b,?}$ in (6), as $G_b(\mathbb{Q}_p)$-representations.*

(1) [5, 6, 7] $V_{b,*} \simeq R\Gamma(\mathrm{Ig}^b, \overline{\mathbb{Q}}_\ell)$.
(2) [8] $V_{b,!} \simeq R\Gamma(\mathrm{Ig}^{b,*}, g_{b!}(\overline{\mathbb{Q}}_\ell))$.
(3) [9] $V_{b,!*} \simeq \mathrm{colim}_{m\geq 1} R\Gamma(\mathrm{Ig}^{b,*}_m, g_{b,m!*}(\overline{\mathbb{Q}}_\ell)$.

While (1) and to some extent (2) can be treated in the realm of more classical rigid/algebraic geometry, for the intersection cohomology we found the perspective on Mantovan's filtration described above indispensable to get a clean picture. To illustrate this, the above reasoning tells us that to compute the sheaf $j_b^*\mathcal{F}_?$ we must by base-change look at the fiber product $\mathrm{Igs}^* \times_{\mathrm{Bun}_G,\overline{\pi}} \mathrm{Bun}_G^b$. Up to a unipotent part which we will ignore, this fiber product is given by $[\overline{\mathrm{Ig}^{b,*,\diamond}}/\underline{G_b(\mathbb{Q}_p)}]$. The space $\mathrm{Ig}^{b,*,\diamond}$ is a certain analytic space that captures information about the rigid generic fiber over C of the Witt vector lift of the perfect scheme $\mathrm{Ig}^{b,*}/\overline{\mathbb{F}}_p$. Moreover, its sheaf theory can be used to understand nearby cycles between $\mathrm{Ig}^{b,*}$ and this rigid generic fiber, as described in [10], and these ideas can be used to show that $j_b^*\mathcal{F}_{!*}$, which, a priori came from something over C can be described in terms of intersection cohomology of algebraic varieties over $\overline{\mathbb{F}}_p$. This is one of the many ways this perspective helps us study Mantovan's filtration in the case of intersection cohomology.

While Mantovan's filtration is already interesting, what a fortiori becomes of more interest when understanding it from the above point of view is the sheaf $\mathcal{F}_{!*} \in \mathrm{D}_{\mathrm{lis}}(\mathrm{Bun}_G, \overline{\mathbb{Q}}_\ell)$ described above. We recall this satisfies the property

$$h_*^{\rightarrow} h^{\leftarrow *}\mathcal{F}_{!*} \simeq R\Gamma_{!*}(\mathsf{G},\mathsf{X})_{K^p}$$

or equivalently

$$j_1^* T_\mu \mathcal{F}_{!*}[d](\frac{d}{2}) \simeq R\Gamma_{!*}(\mathsf{G},\mathsf{X})_{K^p}, \tag{7}$$

where $T_\mu : \mathrm{D}(\mathrm{Bun}_G) \to \mathrm{D}(\mathrm{Bun}_G)^{BW_{E_\mathfrak{p}}}$ is a Hecke operator in the sense of geometric Langlands. This can be understood in terms of the categorical local Langlands conjecture [2, Conjecture 9.1]. We recall [12, 13] that, after fixing an isomorphism $j : \overline{\mathbb{Q}}_\ell \xrightarrow{\simeq} \mathbb{C}$, the RHS of (7) becomes isomorphic to the L^2-cohomology of the real analytic space attached to (G,X), which by work of Borel-Casselman [11] breaks up as a direct sum indexed by global discrete automorphic representations tensored by certain $(\mathfrak{g}, K_\infty)$-cohomology groups. It turns out that there are certain special sheaves (See [14]) called sheared eigensheaves, predicted by the categorical local Langlands conjecture, which, when applying $j_1^*T_\mu$, should give rise to similar behavior as this $(\mathfrak{g}, K_\infty)$-cohomology via their relation to the Arthur SL_2 associated to a global automorphic representation (See [15, Proposition 9.1]). In particular, this suggests that the sheaf $\mathcal{F}_{!*}$ should also break up as such a direct sum of sheared eigensheaves, which would in turn give a very conceptual explanation for the structure of the filtration $\mathrm{gr}_{b,!*}$ as well as the extensions between them in terms of global multiplicity formula and the categorical local Langlands conjecture.

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$\mathbf{Aut}(\mathbb{A}^1_{\mathbf{dR}})$

Peter Scholze
(joint work with Joseph Ayoub)

The goal of this talk was to introduce a new "nonlinear"[1] motivic Galois group $\mathbb{G}_{\mathbb{Q}} = \mathrm{Aut}(\mathbb{A}^1_{\mathrm{dR}})$ and describe three applications:

(1) A stacky interpretation of Nori motives;
(2) A Nori-motivic version of geometric Langlands;
(3) A possible approach to Simpson's conjecture that rigid local systems are of geometric origin.

[1] or "nonabelian" in the spirit of nonabelian Hodge theory

1. Nonabelian Hodge theory

For a smooth projective variety X over $\mathbb{C}$, Hodge theory gives an action of $\mathbb{C}^\times$ on $H^*(X,\mathbb{C})$, acting as $z \mapsto z^p\overline{z}^q$ on (p,q)-forms, under the Hodge decomposition. Simpson observed that by passing from "abelian" cohomology to "nonabelian cohomology" $H^1(X,\mathrm{GL}_n(\mathbb{C}))$, understood to be the $\mathbb{C}$-points of the moduli stack $\mathrm{LocSys}_{X,n}$ of rank n local systems on X, there is also a $\mathbb{C}^\times$-action on $\mathrm{LocSys}_{X,n}(\mathbb{C})$. Namely, you can first use the Corlette–Simpson correspondence to pass to Higgs bundles, then rescale the Higgs field, and pass back. An important result is that the $\mathbb{C}^\times$-fixed points of this action are precisely complex variations of Hodge structure. In particular, any rigid local system, i.e. isolated point of $\mathrm{LocSys}_{X,n}$, underlies a complex variation of Hodge structure. Simpson conjectured that it should in fact be of geometric origin.

Standard philosophy says that all structures in Hodge theory should have "motivic" incarnations. In particular, working over a field k of characteristic 0, there is a motivic Galois group G_k^{mot} — a (pro-)algebraic group — that acts on the cohomology of any variety X over k. While Grothendieck's original construction of the motivic Galois group was based on the standard conjectures, unconditional constructions (of slightly different versions) have been given by André, Nori, and Ayoub. The action of the motivic Galois group generalizes the $\mathbb{C}^\times$-action on $H^*(X,\mathbb{C})$ (where $\mathbb{C}^\times$ gets turned into the algebraic group $\mathrm{Res}_{\mathbb{C}/\mathbb{R}}\mathbb{G}_m$, also known as the Deligne torus).

However, Simpson's $\mathbb{C}^\times$-action on $\mathrm{LocSys}_{X,n}(\mathbb{C})$ has no known motivic version; part of the issue is that this action is not algebraic in nature, not even after substituting $\mathbb{C}^\times$ by the Deligne torus. Nonetheless, we will construct an "upgrade" of G_k^{mot} that does act on $\mathrm{LocSys}_{X,n}$, at least if X is a curve (and optimistically in general), generalizing Simpson's $\mathbb{C}^\times$-action.

For simplicity, we will focus on the case $k=\mathbb{Q}$ below; but everything adapts to arbitrary fields of characteristic 0.

2. Gestalten

The new "nonlinear motivic Galois group" $\mathbb{G}_{\mathbb{Q}} = \mathrm{Aut}(\mathbb{A}^1_{\mathrm{dR}})$ lives in an enlarged category of geometric objects, the Gestalten introduced in joint work with Stefanich [3]. Roughly, a Gestalt (for our applications, always over $\mathrm{Spec}(\mathbb{Q})$), is a geometric object X that gives rise to the following algebraic invariants:

(0) A commutative ring $\mathcal{O}(X)_0 \in \mathrm{CAlg}(D(\mathbb{Q}))$ of "functions on X".
(1) A presentable symmetric monoidal $(\infty,1)$-category $\mathcal{O}(X)_1 \in \mathrm{CAlg}(1\mathrm{Pr}_{\mathbb{Q}})$ of "quasicoherent sheaves on X". This has a unit object $1 \in \mathcal{O}(X)_1$, and one can recover global functions as endomorphisms of 1, i.e. $\mathcal{O}(X)_0 = \mathrm{End}_{\mathcal{O}(X)_1}(1)$.
(2) A presentable symmetric monoidal $(\infty,2)$-category $\mathcal{O}(X)_2 \in \mathrm{CAlg}(2\mathrm{Pr}_{\mathbb{Q}})$ of "sheaves of linear categories on X. This again has a unit object $1 \in \mathcal{O}(X)_2$, and $\mathcal{O}(X)_1 = \mathrm{End}_{\mathcal{O}(X)_2}(1)$.
(3) ...

Thus, this has higher (∞, n)-categorical information at all levels $n \geq 0$; the resulting algebraic object $(\mathcal{O}(X)_0, \mathcal{O}(X)_1, \mathcal{O}(X)_2, \ldots)$ is what I call a "Stefanich ring", introduced in Stefanich's thesis [4]. Often, $\mathcal{O}(X)_{m+1} = \mathrm{Mod}_{\mathcal{O}(X)_m}(m\mathrm{Pr})$ for $m \geq n$; this condition isolates the full subcategory of n-affine Stefanich rings, which is equivalent to $\mathrm{CAlg}(n\mathrm{Pr})$. We will in fact mostly need the case of 1-affine Stefanich rings, which are equivalently "just" presentable symmetric monoidal $(\infty, 1)$-categories. However, the general theory of Gestalten critically needs the category of all Stefanich rings, and the proofs of our main theorems specifically require many 2-affine (and not 1-affine) Stefanich rings.

Usually, the passage from geometry to algebra loses information — for example $\mathbb{P}^1_{\mathbb{Q}}$ is indistinguishable from a point using just global functions. However, if one keeps track of information at all levels, an "extreme Tannaka" duality result is possible. Namely, defining Gestalten formally as the opposite of Stefanich rings, it turns out that this is, for all practical purposes, an $(\infty, 1)$-topos in the sense of Lurie; i.e. one can perform arbitrary gluings and they behave as expected.[2]

3. The Gruppengestalt $\mathbb{G}_{\mathbb{Q}}$

There is a functor from algebraic stacks to Gestalten. In particular, one can consider the ring stack $\mathbb{A}^1_{\mathrm{dR}}$ (always working over $\mathbb{Q}$), governing the theory of algebraic D-modules, as a Ringgestalt, and then take its (internal) automorphism group

$$\mathbb{G}_{\mathbb{Q}} := \mathrm{Aut}(\mathbb{A}^1_{\mathrm{dR}})$$

as a Ringgestalt; this defines a group object in Gestalten, i.e. a Gruppengestalt. The following theorem summarizes our main results.

Theorem 1.

(1) The Gestalt $\mathbb{G}_{\mathbb{Q}}$ is 1-affine and 0-prim (i.e., the unit $1 \in \mathcal{O}(\mathbb{G}_{\mathbb{Q}})_1$ is compact). Thus, it is equivalently given by a Hopf algebra $\mathcal{O}(\mathbb{G}_{\mathbb{Q}})_1$ in $1\mathrm{Pr}_{\mathbb{Q}}$. There is a description of $\mathcal{O}(\mathbb{G}_{\mathbb{Q}})_1$ in terms of generators and relations.

(2) The global functions $\mathcal{O}(\mathbb{G}_{\mathbb{Q}})_0 \in D(\mathbb{Q})$ agree with Ayoub's motivic Hopf algebra; in particular they are connective (and conjecturally in degree 0). Thus, there is a natural surjective map $\mathbb{G}_{\mathbb{Q}} \to G^{\mathrm{mot}}_{\mathbb{Q}}$ (which further surjects onto the usual profinite absolute Galois group). Let $\mathbb{G}^N_{\mathbb{Q}} = \mathbb{G}_{\mathbb{Q}} \times_{G^{\mathrm{mot}}_{\mathbb{Q}}} G^{\mathrm{mot,cl}}_{\mathbb{Q}}$ be the fibre product with the classical subgroup $G^{\mathrm{mot,cl}}_{\mathbb{Q}}$ of $G^{\mathrm{mot}}_{\mathbb{Q}}$. It is known that $G^{\mathrm{mot,cl}}_{\mathbb{Q}}$ is Nori's motivic Galois group.

(3) The category of representations of $\mathbb{G}^N_{\mathbb{Q}}$ is the (derived) category of Nori motives over $\mathbb{Q}$, i.e.

$$\mathcal{O}(*/\mathbb{G}^N_{\mathbb{Q}})_1 = \mathrm{Nori}(\mathbb{Q}).$$

[2] The precise statement requires more careful bookkeeping of certain set-theoretic bounds, but it is largely irrelevant in practice.

(4) *More generally, for any algebraic variety X over $\mathbb{Q}$, the group $\mathbb{G}_{\mathbb{Q}}$ acts on the Gestalt X_{dR}, and*

$$\mathcal{O}(X_{\mathrm{dR}}/\mathbb{G}_{\mathbb{Q}}^{N})_1 = \mathrm{Nori}(X),$$

giving a "stacky interpretation of Nori motives".

(5) *Let X be a smooth projective curve over $\mathbb{Q}$, and let $\mathrm{LocSys}_{X,n}$ denote the algebraic stack parametrizing rank n vector bundles with connection on X. There is a natural action of $\mathbb{G}_{\mathbb{Q}}$ on $\mathrm{LocSys}_{X,n}$ considered as a Gestalt. Moreover, the geometric Langlands equivalence between tempered D-modules on $\mathrm{Bun}_{\mathrm{GL}_n}$ and $\mathrm{QCoh}(\mathrm{LocSys}_{X,n})$ is $\mathbb{G}_{\mathbb{Q}}$-equivariant, and in particular yields an equivalence between tempered Nori motives on $\mathrm{Bun}_{\mathrm{GL}_n}$ and $\mathcal{O}(\mathrm{LocSys}_{X,n}/\mathbb{G}_{\mathbb{Q}}^{N})_1$.*

The proof of the theorem relies on Aoki's thesis [1] relating motives to Ring-gestalten, and work of Jacobsen–Terenzi [2] relating Ayoub's approach to Nori motives and the usual definition of Nori motives.

For part (5), we note that one would expect $\mathrm{LocSys}_{X,n}$ to be given by the mapping Gestalt $\mathrm{Map}(X_{\mathrm{dR}}, */\mathrm{GL}_n)$. There is in fact a natural closed immersion

$$\mathrm{LocSys}_{X,n} \hookrightarrow \mathrm{Map}(X_{\mathrm{dR}}, */\mathrm{GL}_n)$$

but the target is much larger; it is 1-affine, corresponding to a category known as $\mathrm{Rep}(\mathrm{GL}_n)_{\mathrm{Ran}}$ in geometric Langlands. Note that on the target, there is visibly an action of $\mathbb{G}_{\mathbb{Q}}$ by acting on X_{dR}. Part (5) says that the subspace $\mathrm{LocSys}_{X,n}$ is stable under the action of $\mathbb{G}_{\mathbb{Q}}$. In fact, the formalism of the spectral action makes $D(\mathrm{Bun}_{\mathrm{GL}_n,\mathrm{dR}})$ into a sheaf of categories over $\mathrm{Map}(X_{\mathrm{dR}}, */\mathrm{GL}_n)$; this is even $\mathbb{G}_{\mathbb{Q}}$-equivariant as the construction of the spectral action is "of geometric origin" (induced by Hecke operators). Now geometric Langlands implies that the support of $D(\mathrm{Bun}_{\mathrm{GL}_n,\mathrm{dR}})$ is precisely the subspace

$$\mathrm{LocSys}_{X,n} \hookrightarrow \mathrm{Map}(X_{\mathrm{dR}}, */\mathrm{GL}_n).$$

This proves the $\mathbb{G}_{\mathbb{Q}}$-stability of $\mathrm{LocSys}_{X,n}$, and simultaneously makes the geometric Langlands equivalence $\mathbb{G}_{\mathbb{Q}}$-equivariant.

One can hope that for all smooth projective X, the subspace

$$\mathrm{LocSys}_{X,n} \hookrightarrow \mathrm{Map}(X_{\mathrm{dR}}, */\mathrm{GL}_n)$$

is stable under the $\mathbb{G}_{\mathbb{Q}}$-action. If this is true, then rigid local systems would be fixed under a finite index subgroup of $\mathbb{G}_{\mathbb{Q}}$; ignoring this finite index subgroup and other details, this means that they descend from X_{dR} to $X_{\mathrm{dR}}/\mathbb{G}_{\mathbb{Q}}$, thus yielding a Nori motive on X; this would prove Simpson's conjecture.

We note that although we can show that $\mathbb{G}_{\mathbb{Q}}$ must be very large, as it surjects onto $G_{\mathbb{Q}}^{\mathrm{mot}}$ and hence $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, we know only one explicit element. Namely, we can construct a complex conjugation automorphism of $\mathbb{A}^1_{\mathrm{dR},\mathbb{C}_{\mathrm{liq}}}$. Roughly, the construction reinterprets holomorphic bundles with holomorphic connection in terms of C^∞-bundles with holomorphic and antiholomorphic connection; in this latter picture, complex conjugation can act.

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Local-Global compatibility in low weight

George Boxer

(joint work with Frank Calegari, Toby Gee, Vincent Pilloni)

The p-adic Galois representations associated to modular forms of weight ≥ 2, constructed by Eichler, Shimura, and Deligne, realize in the p-adic étale cohomology of the modular curve. Meanwhile the Galois representations associated to modular forms of weight 1 constructed by Deligne–Serre are constructed via congruences.

In this work we consider a natural class of automorphic representations that generalize weight 1 modular forms. They are the so called weakly regular, odd, essentially conjugate self dual, algebraic, cuspidal automorphic representations π of GL_n/L for L a CM field. We refer to [2, §9.1] for the precise definition but roughly:

- **Weakly regular**: the infinitesimal character
$$\lambda = (\lambda_{1,\tau}, \ldots, \lambda_{n,\tau})_{\tau \in \mathrm{Hom}(L,\mathbf{C})}$$
with $\lambda_{1,\tau} \geq \cdots \geq \lambda_{n,\tau}$ has the property that for fixed τ each $\lambda_{i,\tau}$ occurs with multiplicity at most 2.
- **Odd** In the case of $\mathrm{GL}_2/\mathbf{Q}$ and trivial infinitesimal character, we consider weight 1 modular forms and not Maass forms.

In the case that π is actually regular (meaning that the $\lambda_{i,\tau}$ are pairwise distinct, generalizing weight ≥ 2) then the Galois representations associated to π may (usually) be found in the p-adic étale cohomology of unitary Shimura varieties and so the local properties at p of these Galois representations may be understood using p-adic comparison theorems.

When π is weakly regular but not regular, then π still realizes in the coherent cohomology of unitary Shimura varieties and this realization may be used to construct congruences with regular weight forms, and thus construct Galois representations. In this generality this is due to Pilloni–Stroh (see [2, Thm 9.10]). However it is difficult to understand the local properties at p via this construction, and in this generality it is not known that these Galois representations are Hodge–Tate, let alone de Rham with the expected Weil–Deligne representation. Our main theorem establishes this in many cases.

Theorem 1. *Let π be a weakly regular, odd, algebraic, essentially (conjugate) self dual, cuspidal automorphic representation of GL_n/L with infinitesimal character*

$\lambda = (\lambda_{1,\tau}, \ldots, \lambda_{n,\tau})_{\tau \in \mathrm{Hom}(L,\mathbf{C})}$ *with* $\lambda_{1,\tau} \geq \cdots \geq \lambda_{n,\tau}$. *Then for each isomorphism* $\imath : \overline{\mathbf{Q}}_p \simeq \mathbf{C}$ *there is a continuous semisimple Galois representation* $\rho_{\pi,\imath} : G_L \to \mathrm{GL}_n(\overline{\mathbf{Q}}_p)$ *such that:*

(1) If $v \nmid p$ *is a finite place of* L*, then*

$$\imath\mathrm{WD}(\rho_{\pi,\imath}|_{G_{L_v}})^{\mathrm{ss}} \cong \mathrm{rec}(\pi_v \otimes |\det|_v^{\frac{1-n}{2}})^{\mathrm{ss}};$$

(2) Let $v|p$ *be a place of* L*, and assume either that*
  (a) π_v *is potentially unramified and* $\rho_{\pi,\imath}$ *is irreducible, or*
  (b) π_v *is a principal series and* $\mathrm{rec}(\pi_v)$ *contains no character with multiplicity greater than two.*

Then $\rho_{\pi,\imath}|_{G_{L_v}}$ *is de Rham with* $\imath^{-1} \circ \tau$*- Hodge–Tate weights* $(-\lambda_{n,\tau} + \frac{n-1}{2}, \cdots, -\lambda_{1,\tau} + \frac{n-1}{2})$*, and*

$$\imath\mathrm{WD}(\rho_{\pi,\imath}|_{G_{L_v}})^{\mathrm{ss}} \simeq \mathrm{rec}(\pi_v \otimes |\det|_v^{\frac{1-n}{2}})^{\mathrm{ss}}.$$

We make some remarks:

- In the case that we replace (b) with the stronger condition that π_v is a principal series and $\mathrm{rec}(\pi_v)$ is a sum of distinct characters (we say π_v is "p-distinguished") then we previously proved this in [1, Thm 6.11.2]. The key novelty in the paper [1] is that we show that the realization of π in coherent cohomology may be used to put π in a Coleman family. Then we may conclude using results of Kisin on interpolation of p-adic periods along eigenvarieties, as in the work of Jorza [3]. Thus the main novelty in the present work is that we go beyond the p-distinguished case and the principal series case.
- We view case (a) of the above theorem as most interesting. Note that conjecturally $\rho_{\pi,\imath}$ is always irreducible. However if π_v is $\imath$-ordinary then case (b) applies.

Let us briefly indicate the idea of the proof of the theorem in the case that f is a weight one modular form of level prime to p (where it was proved in a different way by Deligne–Serre). In this case the theorem says that ρ_f is unramified at p and Frob_p has characteristic polynomial the Hecke polynomial $x^2 - a_p x + \chi(p)$. In case this polynomial has two distinct roots $\alpha \neq \beta$ (the "p-distinguished case) we may proceed as follows: there are two oldforms at level $\Gamma_0(p)$ f_α, f_β with $U_p f_\alpha = \alpha f_\alpha$, $U_p f_\beta = \beta f_\beta$. We may put f_α and f_β in Hida families and conclude $\rho_f|_{G_{\mathbf{Q}_p}}$ has unramified quotients on which Frobenius acts by α and β, and hence must be unramified as $\alpha \neq \beta$.

In the non p-distinguished case, there will be a two dimensional space of oldforms at level $\Gamma_0(p)$ on which U_p acts non-semisimply. The idea of the proof is then to consider the inertia co-invariants of a Galois representation valued in a Hida Hecke algebra, and show that if ρ_f ramifies at p then U_p acts semisimply on this space of oldforms.

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Euler systems and the relative Satake isomorphism

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(joint work with Li Cai, Yangyu Fan)

One technical aspect in the construction of Euler systems is the choice of certain test vectors so that the resulting Galois cohomology classes satisfy the correct norm relations. In this talk, we will describe a conceptual and uniform method to prove the existence of such test vectors by applying unramified harmonic analysis on spherical varieties.

Let E be a number field and let $\rho : \mathrm{Gal}_E \to \mathrm{GL}(V)$ be a Galois representation with coefficient field $\mathbb{Q}_p$ for simplicity. Let $T \subseteq V$ be a Galois-stable $\mathbb{Z}_p$-lattice. An (tame) *Euler system* for T is a consists of the following data

- A collection of abelian extensions $\{E[\mathfrak{m}]\}_{\mathfrak{m}\in\mathcal{R}}$, where $\mathcal{R}$ is a set of square-free ideals.
- A collection of cohomology classes $\{z_\mathfrak{m} \in \mathrm{H}^1(E[\mathfrak{m}], T)\}_{\mathfrak{m}\in\mathcal{R}}$ satisfying the *norm relations*
$$\mathrm{Tr}_{E[\mathfrak{m}]}^{E[\mathfrak{m}\ell]} z_{\mathfrak{m}\ell} = P_\ell(\mathrm{Frob}_\ell^{-1}) z_\mathfrak{m}, \quad \ell \nmid \mathfrak{m}$$
where $P_\ell(X)$ is the characteristic polynomial of the geometric Frobenius Frob_ℓ acting on V.

Of particular interests are the following two cases:

- Rubin: $E = \mathbb{Q}$ and $E[m] = \mathbb{Q}(\mu_m)$ is the cyclotomic extension. Examples include Kato's Euler system. They are useful for showing the vanishing of certain Selmer groups.
- Split anticyclotomic: E is a CM field and $E[\mathfrak{m}]$ is a ring class field. Examples include Kolyvagin's Euler system of Heegner points. They are useful for showing certain Selmer groups have rank 1.

Euler systems are closely related to L-values, and the polynomial $P_\ell(X)$ gives local L-factors for a particular automorphic representation. Consequently most known constructions are based on re-interpreting integral representations of L-functions in arithmetic geometry.

In the relative Langlands program, such integral representations are indexed by a reductive group G acting on an affine variety X (with a large list of conditions). We take this as the starting point and describe our construction of Euler systems.

Step 1: Construct a $G(\mathbb{A}_f)$-equivariant map
$$C_c^\infty(X(\mathbb{A}_f), \mathbb{Z}_p) \to \mathrm{H}^*_{\mathcal{M}}(\mathrm{Sh}_G, \mathbb{Z}_p(*)).$$

In homogeneous cases, where $X = H\backslash G$ and H also admits Shimura varieties, this is the standard special cycles construction. Moreover, the Eisenstein classes considered by Faltings and Kings can be indexed this way with $(G, X) = (\mathrm{GSp}_{2g}, \mathbb{A}^{2g})$.

Step 2: Insert character twists through a G-equivariant torus bundle

$$\nu : \widetilde{X} \to X.$$

The torus would be $\mathbb{G}_m$ for a Rubin-style Euler system and $\mathrm{U}(1)$ for a split anticyclotomic Euler system. Starting with a motivic theta series for $\tilde{G} = G \times \mathbb{G}_m$ and varying the level structure only on the torus, we obtain classes over $E[\mathfrak{m}]$.

The above steps reduce the tame norm relations to a purely local, function-theoretic question. At an unramified place ℓ, let

$$K_0 = G(\mathcal{O}_\ell) \times \mathcal{O}_\ell^\times, \quad K_1 = G(\mathcal{O}_\ell) \times (1 + \varpi\mathcal{O}_\ell)$$

be two compact open subgroups of $\tilde{G}(F_\ell) = G(F_\ell) \times F_\ell^\times$. We need to solve the equation

$$\mathrm{Tr}_{K_0}^{K_1} \Phi_1 = \mathcal{H}_\ell * \Phi_0, \quad \Phi_1 \in C_c^\infty(\tilde{X}(F_\ell), \mathbb{Z}_p)^{K_1}$$

Here, Φ_0 is the indicator function of $\tilde{X}(\mathcal{O}_\ell)$, and $\mathcal{H}_\ell$ is an unramified Hecke operator for the group $\tilde{G}$, whose Satake transform gives the expected L-factor for the Euler system.

Step 3: Study the image of the trace map. Using the work of Gaitsgory–Nadler, we identify $\tilde{X}(F_\ell)/K_0$ with a certain cone $\Lambda_X^+ \times \mathbb{Z}$. Under this identification, we show that the image of the trace map contains all functions which are divisible by $\ell - 1$ along walls corresponding to spherical roots of type T.

The local picture for the Heegner point case is $X = \mathbb{G}_m\backslash\mathrm{PGL}_2$, then $\Lambda_X^+ = \mathbb{Z}_{\leq 0}$. Our result means that the image contains all functions Φ such that $\Phi(0) \in (\ell - 1)\mathbb{Z}_p$. In this case, a concrete calculation shows that this condition characterizes the image.

Step 4: Apply the works of Sakellaridis on the relative Satake isomorphism. As an impressionistic sketch, it describes the value of $\mathcal{H}_\ell * \Phi_0$ as the Laurent series coefficients of a rational function

$$\mathcal{L}_\ell \cdot \frac{\mathtt{ad}}{\mathtt{L}} \in \mathbb{C}[A_X]$$

where $\mathcal{L}_\ell$ is the local L-factor, and $\mathtt{ad}, \mathtt{L}$ are concrete polynomials coming from the asymptotics of the basic function.

While the entire theory is developed over $\mathbb{C}$, it is basically formal that all coefficients are polynomials in $\mathbb{Z}[\ell^{\pm 1}]$, so to verify divisibility by $\ell - 1$, it suffices to set $\ell = 1$. At this point, we make two observations:

- If $\mathcal{L}_\ell$ exactly corresponds to the L-function attached to X, then $\frac{\mathcal{L}_\ell}{\mathtt{L}}$ cancels into a polynomial.
- The function is anti-symmetric under a spherical reflection of type T (and symmetric for type G).

Combining both facts, we see that its coefficients along a wall of type T must be 0, as required by the previous step.

A Riemann–Hilbert functor in p-adic analytic geometry

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(joint work with Johannes Anschütz, Arthur-César Le Bras, Peter Scholze)

1. Introduction

Prismatic cohomology of Bhatt–Scholze [2] and its geometric interpretation given by the prismatization of Drinfeld [3] and Bhatt–Lurie [1], have been very prolific in the study of p-adic cohomology theories of p-adic formal schemes. In joint work in progress with Anchütz, Le Bras and Scholze we develop a variant of this theory in (rational) analytic geometry called the *analytic prismatization.* Quite contrary to what occurs in the integral theory, the relationship of rational analytic prismatic cohomology with differential operators is immediate from its definition, and analytic geometry gives us enough tools to profit of this feature and compare with several classical constructions in p-adic Hodge theory.

In this talk I will focus in one of these classical comparisons, namely, the Riemann–Hilbert functor of Scholze as in [5]. Our approach to recover Scholze's Riemann–Hilbert functor is rather geometric and barely uses explicit computations in the proof, except in few key cases and in the comparison with the classical functor. Nonetheless, the complexity of the statement is hidden behind the formalism; the whole framework of analytic geometry and quasi–coherent six functor formalism is supported in highly non-trivial advancements of the last ten years, in particular in the more common use of higher category theory in arithmetic geometry. We also expect that our interpretation of the Riemann–Hilbert functor recovers that announced by Bhatt–Lurie, also by putting all the difficulty in the formalism and not the construction itself.

2. Main construction

To motivate the problem we recall Scholze's construction, we use the notation of [5]. Let X be a smooth rigid analytic variety over $\mathbb{Q}_p$ and let X_{proet} denote the proétale site of X. There is a list of period sheaves on X_{proet}, the most important for us being the completed structural sheaf $\widehat{\mathcal{O}}_X$ sending an affinoid perfectoid $\mathrm{Spa}(R,R^+)\to X$ to $\widehat{\mathcal{O}}_X(R,R^+)=R$, and the de Rham period sheaf $\mathbb{B}_{\mathrm{dR}}^+$ sending $\mathrm{Spa}(R,R^+)$ to the de Rham ring $\mathbb{B}_{\mathrm{dR}}^+(R)$ obtained as the formal completion of the degree 1-divisor determined by R in the Fargues–Fontaine curve of $(R^\flat, R^{+,\flat})$. There is a natural map of rings $\mathbb{B}_{\mathrm{dR}}^+\to\widehat{\mathcal{O}}_X$ whose kernel is an invertible $\mathbb{B}_{\mathrm{dR}}^+$-module. Thus, $\mathbb{B}_{\mathrm{dR}}^+$ carries a natural adic filtration $\mathrm{Fil}^i(\mathbb{B}_{\mathrm{dR}}^+)$ whose graded pieces are given by the *Tate twists*

$$\mathrm{gr}^i(\mathbb{B}_{\mathrm{dR}}^+)=\widehat{\mathcal{O}}_X(i).$$

We let ξ denote a local generator of $\mathrm{Fil}^1\mathbb{B}_{\mathrm{dR}}^+$ and write $\mathbb{B}_{\mathrm{dR}} := \mathbb{B}_{\mathrm{dR}}[\frac{1}{\xi}]$. We also write B_{dR}^+ for the de Rham ring associated to $\mathbb{C}_p$.

We have the following theorem.

Theorem 1 (Scholze)**.** *Let* $(\mathrm{Fil}, V, \nabla)$ *be a filtered vector bundle on* X *with flat connection satisfying Griffiths transversality. Then there is a natural* $\mathbb{B}_{\mathrm{dR}}^+$*-local system* $\mathbb{M}(\mathrm{Fil}, V, \nabla)$ *over* X_{proet} *of same rank as* V*. Moreover, the following holds:*

(1) The formation of $(\mathrm{Fil}, V, \nabla) \mapsto \mathbb{M}(\mathrm{Fil}, V, \nabla)$ *is a fully faithful functor from filtered vector bundles with flat connections satisfying Griffiths transversality into* $\mathbb{B}_{\mathrm{dR}}^+$*-local systems.*

(2) Suppose that X *is proper, then there is a natural filtered and* $\mathrm{Gal}_{\mathbb{Q}_p}$*-equivariant isomorphism of cohomologies*

$$R\Gamma_{\mathrm{proet}}(X_{\mathbb{C}_p}, \mathbb{M}(\mathrm{Fil}, V, \nabla))[\frac{1}{\xi}] = R\Gamma_{\mathrm{dR}}(X, V) \otimes_{\mathbb{Q}_p} B_{dR}.$$

where the left term carries the adic filtration on $\mathbb{M}(\mathrm{Fil}, V, \nabla)$*, and the right term the convolution filtration between the Hodge filtration of the de Rham cohomology of* V*, and the adic filtation of* B_{dR}*.*

Scholze's construction of the Riemann–Hilbert functor uses an auxiliary period sheaf denoted $\mathcal{O}\mathbb{B}_{\mathrm{dR}}^+$, it is a formal completion of the tensor of the structural sheaf $\mathcal{O}_X$ of X and the period sheaf $\mathbb{B}_{\mathrm{dR}}^+$ along the kernel of the natural map to $\widehat{\mathcal{O}}_X$. This sheaf also carries a natural filtration and a flat connection. Define $\mathcal{O}\mathbb{B}_{\mathrm{dR}} := \mathcal{O}\mathbb{B}_{\mathrm{dR}}^+[\frac{1}{\xi}]$, one then has that

$$\mathbb{M}(\mathrm{Fil}, V, \nabla) := (\mathcal{O}\mathbb{B}_{\mathrm{dR}}^+ \otimes_{\mathcal{O}_X} V)^{\nabla=0}.$$

Our strategy to recover and generalize the previous construction is to replace all the previous previous sheaves by their corresponding analytic stacks, and by using as a bridge a piece of the analytic prismatization, that is, the formal completion at the Hodge–Tate point. More precisely, we construct morphisms of analytic stacks

$$X^{\mathbb{B}_{\mathrm{dR}}^+} \to X^{\widehat{\mathrm{HT}}} \to X^{\widehat{\mathrm{cone}}}$$

with the following features:

- Quasi-coherent sheaves on $X^{\mathbb{B}_{\mathrm{dR}}^+}$ are precisely solid quasi-coherent sheaves of $\mathbb{B}_{\mathrm{dR}}^+$-modules on X_{proet}. In particular, its quasi-coherent cohomology computes proétale cohomology. There is a relative version over B_{dR}^+ that we denote $X^{\mathbb{B}_{\mathrm{dR}}^+/B_{\mathrm{dR}}^+}$.
- The stack $X^{\widehat{\mathrm{HT}}}$ is the formal completion of the prismatization at the Hodge–Tate point. There is a relative version over B_{dR}^+ that we denote $X^{\widehat{\mathrm{HT}}/B_{\mathrm{dR}}^+}$.
- $X^{\widehat{\mathrm{cone}}}$ is the *formal cone of* X, it is an extension of the deformation to normal cone construction from Zariski closed immersions to arbitrary stacks. There is a relative version over B_{dR}^+ that we denote $X^{\widehat{\mathrm{cone}}/B_{\mathrm{dR}}^+}$.

The following theorem summarizes the extension of the Riemann–Hilbert functor.

Theorem 2 (A-LB-RC-S). *Keep the previous notation, the following hold:*

(1) The pullback along $X^{\mathbb{B}^+_{\mathrm{dR}}} \to X^{\widehat{\mathrm{HT}}}$ *induces a fully faithful functor*

$$\mathrm{QCoh}(X^{\widehat{\mathrm{HT}}}) \hookrightarrow \mathrm{QCoh}(X^{\mathbb{B}^+_{\mathrm{dR}}}).$$

Moreover, this holds after any base change on $X^{\widehat{\mathrm{HT}}}$.

(2) There is a natural equivalence between the categories of vector bundles on $X^{\widehat{\mathrm{cone}}}$ *and triples* $(\mathrm{Fil}, V, \nabla)$ *consisting of a filtered vector bundle endowed with a flat connection as above. Moreover, the quasi-coherent cohomology of* $X^{\widehat{\mathrm{cone}}}$ *computes de Rham cohomology endowed with the Hodge–Filtration.*

(3) Consider the map $X^{\widehat{\mathrm{HT}}/B^+_{\mathrm{dR}}} \to X^{\widehat{\mathrm{cone}}/B^+_{\mathrm{dR}}}$ *of relative stacks over* B^+_{dR}*, then the pullback map induces a fully faithful functor*

$$\mathrm{QCoh}(X^{\widehat{\mathrm{cone}}/B^+_{\mathrm{dR}}}) \hookrightarrow \mathrm{QCoh}(X^{\widehat{\mathrm{HT}}/B_{\mathrm{dR}+}}).$$

(4) The composite functor

$$\mathrm{QCoh}(X^{\widehat{\mathrm{cone}}/B^+_{\mathrm{dR}}}) \hookrightarrow \mathrm{QCoh}(X^{\widehat{\mathrm{HT}}/B_{\mathrm{dR}+}}) \hookrightarrow \mathrm{QCoh}(X^{\mathbb{B}^+_{\mathrm{dR}}/B^+_{\mathrm{dR}}})$$

extends Scholze's original Riemann–Hilbert functor of Theorem 1.

Part (1) of the theorem is a reminiscent of the idea of *decompletion by locally analytic vectors of completed sheaves* occurring in the work of Pan [4]. With the introduction of additional stacks, one can identify sheaves on $X^{\widehat{\mathrm{cone}}}$ with analytic completed filtered D-modules with locally nilpotent Higgs field. One can also construct a *solution functor*

$$\mathrm{Sol}\colon \mathrm{QCoh}(X^{\mathbb{B}^+_{\mathrm{dR}}/B^+_{\mathrm{dR}}}) \to \mathrm{QCoh}(X^{\widehat{\mathrm{cone}}/B^+_{\mathrm{dR}}})$$

by taking !-pushforwards. We expect that when restricted to Zariski constructible sheaves (with some additional de Rhamness conditions on points) the solution functor recovers Bhatt–Lurie's Riemann–Hilbert functor.

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Reporters: Carlo Kaul, Devansh Sehta

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